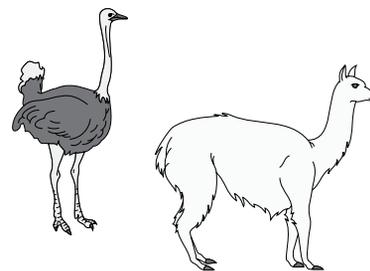


Ostrich Llama Count—Examining Solution Methods

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CONCEPTS: Number Sense, Algebra and Functions, Mathematical Reasoning

SKILLS: Understanding and discussing a variety of methods for solving a problem, making sense of and persevering in solving problems, constructing arguments and critiquing the reasoning of others

MATHEMATICS STANDARDS: Gr 6: NS 1.3, AF 1.2, MR 1.1, 1.2, 1.3, 2.3, 2.4, 2.5, 3.1, 3.2, 3.3; Gr 7: AF 1.0, MR 1.1, 1.2, 1.3, 2.2, 2.3, 2.4, 2.5, 3.1, 3.2, 3.3; Alg1: 4.0, 5.0

STANDARDS FOR MATHEMATICAL PRACTICE: MP1, MP3

GRADES: 6–8

MATERIALS: Activity Sheets (pages 39–40)

BACKGROUND

In the Ostrich Llama Count problem, students are given the combined number of heads and legs of ostriches and llamas. Using that information they are asked to explain how Raul and Esteban might have figured out how

many ostriches and llamas there were on the farm.

In this activity, the focus is shifted from solving the problem to reflecting on methods used by others to solve the problem. Students are asked to embrace one of the methods, make sense of it, and present it as a reasonable method to use in solving the problem. In addition, students are asked to compare the method to a method they would have used. How does it compare? What are the advantages of each method? What are the challenges?

Classroom activities that offer a variety of problem-solving methods (or strategies) provide the scaffolding for students to make sense of problems, to focus on different problem-solving methods, and to persevere in solving problems. This activity does not need to be done in one class period. By spreading it out over several class periods, students are encouraged to persevere.

See *Ostrich Llama Count*, page 37 > >

Ostrich Llama Count (cont. from page 34)

DIRECTIONS

Step One

Take a maximum of five minutes to introduce the scenario version of the problem to students (Figure 1).

Ostrich Llama Count

Raul and Esteban just started working at their uncle's farm on the weekend. Their first task was to count the ostriches and llamas. When they reported to their uncle,



Raul said, "I counted 47 heads."
Esteban said, "I counted 122 legs."



Figure 1

Ask them what they notice about the problem and what they wonder about the problem. You may either read the prompt to the students, or project it and give students time to read it for themselves. After students have heard or read the prompt, ask them, "What did you hear?" Or, "What do you notice?" You may ask students to respond in writing and then orally. Call on students so that they respond. The idea behind this initial activity is to get the students to buy in to the context of the problem, to feel comfortable with what is happening, and to be engaged. The goal at this point is NOT to solve the problem.

Step Two

When students are familiar with the problem-solving context, take a maximum of two min-

utes to present them with the full problem (Figure 2). Then explain to them that they will be discussing a variety of methods used to solve the problem, but will not be solving the problem at this time.

Step Three

Cut out the five "Methods" that are given on Activity Sheets 1 and 2 (pages 39–40). These describe different methods for solving the problem. Give each group a different method sheet, but give each student in the group a copy of the method sheet for that group. Methods include:

Method 1: Guess and Check—Make a Chart

Method 2: More Sophisticated Guess and Check—Make a More Detailed Chart

Method 3: Logical Reasoning

Method 4: Algebraic Reasoning

Method 5: Algebra

Directions for each group:

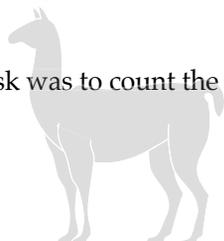
1. Read the "Method" assigned to your group and discuss the following questions:
 - ✓ What did they do?
 - ✓ Why do you think they did that?
 - ✓ What do you think they figured out?
 - ✓ Why does it work?
 - ✓ What does their result mean?

Ostrich Llama Count

Raul and Estaban just started working at their uncle's farm on the weekend. Their first task was to count the ostriches and llamas. When they reported to their uncle,



Raul said, "I counted 47 heads."
Estaban said, "I counted 122 legs."



"How many are ostriches? How many are llamas?" asked their uncle.
"It's getting dark and I promised your mother I'd get you home for dinner. There's no time to count again. You'll have to figure out how many ostriches and how many llamas there are from that information when you get home. Can you give me a call after dinner and let me know your answer?"

How did Raul and Estaban figure out how many ostriches and how many llamas there were?

Figure 2

2. Discuss the question: Is this a problem-solving method you have used before?
3. Prepare a report to present to the class. Include answers to the following questions:
 - ✓ How does the name of the method match the explanation?
 - ✓ Are there other problems done recently that used this method?
 - ✓ What did your group learn as you discussed this method?
 - ✓ Compare the method to a method you would have used. How does it compare?
 - ✓ What are the advantages of this method?
 - ✓ What are the challenges?

Step Four (optional)

Pair groups together and ask each group to present their method to the other group. While one group is presenting, the students in the other group should be prepared to ask the following questions:

- ✓ How do you know?
- ✓ How does that make sense?
- ✓ How can you say that in a different way?

Step Five (optional)

Each group presents their method to the rest of the class with the ideas that students will:

- ✓ Value each of the different methods as valid approaches to solving the problem.
- ✓ Identify at least one method that they might have thought of using.
- ✓ Identify at least one method that they might not have thought of using, but they can see the value in using.

EXTENSIONS

- ◆ The Ostrich Llama Count Problem—including the scenario, scoring rubric, teacher packet, and the methods included in this activity—is one of several Sample Problems of the Week from mathforum.org/pow/teacher/samples.html. In the second

half of each of the Teacher Packets, actual student work is included. Teachers can design a similar activity using the student work as the focus—including how to take a “novice” solution to an “apprentice” level and how to take an “apprentice” solution to a “practitioner” level.

- ◆ Using the resources at mathforum.org/pow/teacher/samples.html, similar activities can be created for the following problems:
 - ✓ The Eagle and the Bear (grades K–2)
 - ✓ Dad’s Cookies (grades 3–5)
 - ✓ Math Club Mystery (Algebra 1)
 - ✓ Broken Pottery (Geometry)

References

The Math Forum. “Samples of the Problems of the Week.” <http://mathforum.org/pow/teacher/samples.html>.

Activity Sheets, pages 39–40 . . .

Ostrich Llama Count 1

Methods 1, 2 and 3

by Suzanne Alejandre

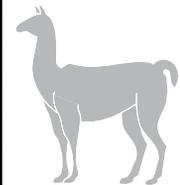
Method 1: Guess and Check—Make a Chart

I created 4 columns showing number of ostriches, number of llamas, number of legs for ostriches and number of legs for llamas. Then, through trial and error, I put numbers in these columns so that the animals numbered 47 and the legs equaled “whatever.”

Here are the number columns I used:



| Ostriches | Llamas | Ostrich legs | Llama legs |
|-----------|--------|--------------|------------|
| 20 | 27 | 40 | 108 |
| 27 | 20 | 54 | 80 |
| 30 | 17 | 60 | 68 |
| 31 | 16 | 62 | 64 |
| 32 | 15 | 64 | 60 |
| 33 | 14 | 66 | 56 |

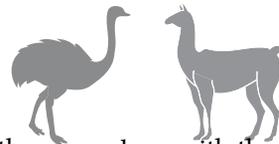


When the proper combination of numbers equaled 47 animals and 122 legs I knew I had the correct combination of animals.

Method 2: More Sophisticated Guess and Check—Make a More Detailed Chart

I found out how to get my answer by matching two numbers, like 14 llamas and 33 ostriches, to get 47 heads and 122 legs. I used this information stated in the problem:

- There are 47 heads (so 47 animals)
- There are 122 legs altogether
- Ostriches have 2 legs and llamas have 4 legs



I needed to figure out how many animals there were and to match those numbers with the correct number of legs and heads that there were. I used a guess and check method. To make this easier to visualize, I made a chart.

| Number of llamas | Number of llama legs | Number of ostriches | Number of ostrich legs | Total animal heads | Total legs | Does it match 122 legs, 47 heads? |
|------------------|----------------------|---------------------|------------------------|--------------------|------------|-----------------------------------|
| 15 | 60 | 32 | 64 | 47 | 124 | no |
| 13 | 52 | 34 | 68 | 47 | 120 | no |
| 14 | 56 | 33 | 66 | 47 | 122 | yes |

I guessed a number of llamas, multiplied it by 4 because each llama has 4 legs. The reason I guessed 15 llamas as a number to start with was just a lucky guess, it turned out to be a pretty close number to the actual answer. I subtracted the guessed number of llamas from 47 (heads) to find the number I would use for ostriches. I multiplied that number by 2 because ostriches have 2 legs.

My next step was to add the number of legs together to see if it would equal 122. We knew the number of animals because we had already done the subtraction step to get an amount equal to 47 heads. This is how Raul and Esteban might have figured out how many ostriches and llamas there were.

Method 3: Logical Reasoning

There were 47 heads, which means there were 47 animals. Ostriches have 2 legs and llamas have 4 legs. Both ostriches and llamas have at least two legs. So, I multiplied 47 heads \times 2 legs = 94 legs.

Because Esteban counted a total of 122 legs, I subtracted the 94 legs from 122 and the difference was 28 legs. Now I have 28 legs left to attach to heads. These 28 legs must be attached by 2s to make 4-legged llamas. So, I divided the remaining 28 legs by 2, which equals 14 pairs, and so there were 14 animals with 4 legs (llamas).

I had found out there were 14 llamas. Since llamas and ostriches have 1 head each, I subtracted the 14 llamas from the 47 heads and found out there were 33 heads with 2 legs each (ostriches).

I checked my work by multiplying 2 legs \times 33 ostriches, which equals 66 legs. Next I multiplied 4 legs \times 14 llamas, which equals 56 legs. Then I added 66 legs + 56 legs and it equaled 122 legs.

Ostrich Llama Count 2

Methods 4 and 5

by Suzanne Alejandre

Method 4: Algebraic Reasoning

Because there were 47 heads total and each animal has one head, the total number of heads would be:

$$\text{number of ostrich heads} + \text{number of llama heads} = 47$$

We also know that ostriches have two legs and llamas have four, so:

$$\text{two times (number of ostrich heads)} + \text{four times (number of llama heads)} = 122$$

How do we use these two number sentences to find out the number of each animal? We can think of the first one like this:

$$\text{number of ostrich heads} = 47 - (\text{number of llama heads})$$

And, now I can use that with the second number sentence:

$$\text{two times } [47 - (\text{number of llama heads})] + \text{four times (number of llama heads)} = 122$$

If I use the distributive property,

$$94 - [\text{two times the number of llama heads}] + [\text{four times the number of llama heads}] = 122$$

$$94 + [\text{two times the number of llama heads}] = 122$$

$$[\text{two times the number of llama heads}] = 28$$

the number of llama heads equals 14

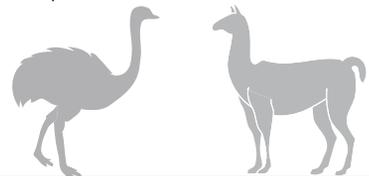
If I use "14" in my first number sentence instead of the "number of llama heads," then I would have

$$\text{two times (number of ostrich heads)} + \text{four times (14)} = 122$$

$$\text{two times (number of ostrich heads)} + 56 = 122$$

$$\text{two times (number of ostrich heads)} = 66$$

the number of ostrich heads equals 33



Method 5: Algebra

There are 33 ostriches and 14 llamas.

Let x = number of ostriches

y = number of llamas

(I would like to choose "o" and "l" but didn't since they look too much like numbers.)

Because there were 47 heads total and each animal has one head (we assume. . .) the sum can be represented as ostriches plus llamas: $x + y = 47$.

Ostriches have two legs and llamas have four, so we can multiply the variables by 2 and 4, respectively, to represent the total number of legs: $2x + 4y = 122$.

Now we can solve the system of two variables with two equations, and there are several ways to do this. I've chosen substitution.

$$x + y = 47$$

$$y = 47 - x$$

$$2x + 4y = 122$$

$$2x + 4(47 - x) = 122$$

$$2x + 188 - 4x = 122$$

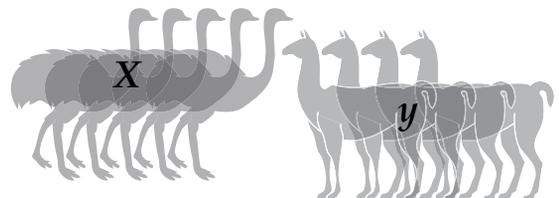
$$-2x = -66$$

$$x = 33$$

$$y = 47 - x$$

$$y = 47 - 33$$

$$y = 14$$



Therefore, the number of ostriches is 33, and the number of llamas is 14.