

Writing to Develop Understanding: The Math Forum @ Drexel's PoWs

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What can we ask students about their problem solving that will help them the most? At the Math Forum @ Drexel we ask students to write solutions that explain both their process and their reasoning. Mentors then use a scoring rubric, which covers several components of problem solving and mathematical communication, to provide feedback designed to get the students to revise and improve their work. This iterative approach that includes initial writing, consideration of supportive feedback, and revision is where significant improvement in problem solving takes place. Since 1994, the Problems of the Week (PoWs) have provided this individualized mentoring to more than one hundred thousand students. Evaluative research (Renninger, et al. 2000, 1998) has demonstrated that both weak and strong students who submit solutions will make more connections to, generate more effective strategies for, and work more independently on mathematics challenges.

In the professional development workshops that we provide locally and also nationally at conferences, we often encounter teachers who view student work as either “right” or “wrong” but miss the opportunity to recognize and build on the particular understandings that each student has demonstrated.

We have two questions that we ask ourselves to help us focus on the students’ thinking and not rush to interventions when we are preparing to give students feedback on their work.

1. What does the student know of the mathematics of the problem?
2. What am I still wondering about?

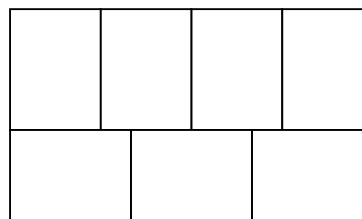
One of our favorite problems to use during workshops is *Congruent Rectangles*, written by Annie Fetter and originally introduced as a Geometry Problem of the Week (GeoPoW) September 2, 2002. We find that the problem

appeals to a wide range of teachers at various grade levels. As we work through the problem, teachers can identify with the examples of solutions that we show.

Here is the problem:

Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is the perimeter?



What follows are four solutions by actual students (their names have been changed) that were submitted to the GeoPoW. Following each of the students’ solutions are examples of what we might think about the solution and ideas we might communicate to the student.

James, age 15

The perimeter is 166 units. I knew that the area was length times width. So I knew that 27 times 28 gave me 756. So I added 28 plus 28 plus 27 plus 27 to get my answer of 166.

An initial reaction might be to point out to James the errors that he has made. Instead of pointing out any errors, however, James might realize more about the problem if he is praised for what he understands and then asked some questions about what is still in question about his solution. Because James’ solution is quite short, it would be a good idea to select one thing to praise and one thing to

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ask more about. Here are some ideas that we might consider.

If we look closely at the work of James we can praise his understanding that:

- ✓ area is length times width;
- ✓ perimeter is measured in units rather than square units;
- ✓ perimeter of a rectangle is length + length + width + width.

We could communicate to James that we are still wondering:

- ✓ When deciding to use 27 and 28 as the measurements for the length and width of the larger rectangle, how he knew those two numbers were correct and not 378 and 2 or 189 and 4 or some other two numbers that multiply to give 756.
- ✓ Whether James knows about factors and whether he considered other numbers that multiply to 756.
- ✓ If he thought the seven smaller rectangles are important to the problem.
- ✓ If he knows what the word “congruent” means or if he would like some help understanding that word.

Once we have acknowledged James’ understanding of the difference between area and perimeter and asked him about other possible factors of 756, he might revise his solution to consider additional factors but still have no understanding of how the small rectangles come into play. At that point we would ask some questions about the seven rectangles. It might take several exchanges before James ties together his various understandings into a more effective solution.

Here is a sample of an exchange:

Hi James,

Thanks for starting to work on this problem. I can see that you know how to find the area and perimeter of a rectangle. Do you think 27 and 28 are the only two numbers that can be multiplied to get 756? Are there other possibilities that you can think of?

Talk to you again soon.
Suzanne

Revision James, age 15

The perimeter is 166 units. I knew that the area was length times width. So I knew that 27 times 28 gave me 756. So I added 28 plus 28 plus 27 plus 27 to get my answer of 166.

I found some other numbers that can be multiplied to get 756. I found 2 and 378, 4 and 189, 63 and 12, 21 and 36. I used my calculator to find them.

Notice that James revises his solution by adding to it rather than editing it. Often when students revise for the first time they don’t fully understand the idea of editing a draft. If the teacher can draw an analogy to the writing process, that can help. Also, with time as the mentor encourages students, they catch on to the process. The mentor might reply to the revision with this response to James:

Hi James,

Thanks for finding those other numbers that can be multiplied to get 756. Are you sure now that 27 and 28 are the length and width of the larger rectangle? Why do you think the problem talks about the seven “congruent” rectangles? Do you need help to understand what “congruent” means?

Talk to you again soon.
Suzanne

You can see that this could lead to several more exchanges. Even if James doesn’t reach a final answer, the thinking that he’s done with this problem will help him as he tries the next one.

Mentoring that is done well effectively bridges the distance between what the learner understands and what still needs to be understood by focusing the learner and fading task-informed feedback. (Renninger, et al., in press)

Dre, age 12

The perimeter of the rectangle is 111.2 units^2 . First I tried to solve it algebraically. I used $xy = 756$ but there were too many answers so I tried finding the area of one of the smaller squares. I did this by dividing the area of the big square by seven because there are seven little squares in the big one. $756/7 = 108$ but that got me no where.

Then I saw the answer. It takes 3 of the rectangles layed across the long way to go from one side to the other the long way but it takes 4 rectangles stacked up the tall way to get from one side to the other long way.

Hmmm I thought I now have 2 equations and 2 unknowns. They are $3/4x = y$ and $xy = 756$. All I

need to do now is solve. I plugged in $3/4x$ into y in the second equation. I now have $x(3/4x) = 756$. I then multiplied $x \cdot 3/4x$ and got $3/4x^2 = 756$. Then I divided 756 by $3/4$ and got $x^2 = 1008$. Next I took the square root of 1008 and got $x = \text{approx. } 31.8$. To solve for y you must multiply by $3/4$ because $3/4x = y$. Therefore $y = \text{approx. } 23.8$.

There are 2 x 's and 2 y 's for both sides of the rectangle. When you add those up you get $31.8 \cdot 2 + 23.8 \cdot 2 = 111.2$. The perimeter of the rectangle is 111.2 units².

Another initial reaction as teachers view student solutions is that they can get locked into one solution path, usually their own. The beauty of the Congruent Rectangles problem is that there are several solution paths and using it can affirm the merits of each one.

To help students overcome misconceptions, effective mentors ask questions of the student rather than telling or directing. This gives the student another opportunity to connect to what they *do* understand about the mathematics.

If Dre can come to realize that the x and y that she is using are actually the dimensions of each small rectangle, she could easily revise her solution to find the correct perimeter measurement of the larger rectangle. She would have $(3x/4)x = 108$ and when she solved for x she would get $x = 12$. Once she had that she would know that $y = 9$ and then if she looked at her diagram she would be able to calculate that the longer side of the larger rectangle equals 36 units and the shorter side of the larger rectangle equals 21 units. Adding $36 + 36 + 21 + 21$ would get 114 units as the perimeter of the larger rectangle.

Should we explain to Dre where she made mistakes? No! It's much better for her problem-solving development if we value the mathematical understanding she has demonstrated in her solution and then ask some questions so that she can realize that the solution she chose needs some tweaking.

If we look closely at the work of Dre we can praise her understanding that:

- ✓ the formula for the area of a rectangle is xy ;
- ✓ congruency would mean that the seven small rectangles all have the same area and so that can be found by dividing the area of the larger rectangle by 7;
- ✓ there is a 3:4 ratio and it's a great idea to set up the equation $(3/4)x = y$.

We could communicate to Dre that we are still wondering:

- ✓ How should those unknowns be identified? Is x the long side of a small rectangle or is it the long side of the larger rectangle. What about y ?
- ✓ If Dre is firm on what she meant by x and y , did she just make a careless mistake of what area to put when she multiplied x by y ?

Matteo, age 15

3024 units. If 756 units² is the area, and area is calculated by multiplying width times length, then the width and height must both be 756 units. Then, if the perimeter is calculated by $2L + 2W$ the perimeter must be 3024 units.

At first glance it might seem that Matteo has no idea about how to solve this problem and yet there are ideas that we can praise and value in his work as well as questions that we can ask him to move him forward in his mathematical thinking. Similar to James' solution Matteo has written very little. We would want to restrain ourselves and write one sentence of praise and one question about what we are wondering about. Our goal is to engage Matteo so that we can take small steps. Even if he doesn't follow through on this problem, he may start to understand the idea of problem solving as a process rather than a one time event when he attempts the next problem.

If we look closely at the work of Matteo we can praise his understanding that:

- ✓ the area of a rectangle is calculated by multiplying width times length;
- ✓ the perimeter of a rectangle is found using the formula $2L + 2W$ and he calculated that accurately using 756;
- ✓ if a rectangular area is given as something squared, then one possibility for the dimensions is the number that was squared.

We could share with Mateo wonderings such as:

- ✓ What is the difference between a measure-

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ment given in square units and a measurement given as a number squared? How might this matter?

- ✓ How does one recognize the differences?
- ✓ What does 756 unit^2 mean?
- ✓ Does this answer fit the other information in the problem?

Chung, age 13

The perimeter of the larger rectangle is 57 units. Let the dimensions of the small rectangles be a , b (length a , width b), then the area of the large rectangle is:

$$4b(a + b) = 756$$

From the graph, we can see that $4b = 3a$, so $b =$

$$(3/4)a, \text{ then we have } 3a(a + (3/4)a) = 756$$

$$(7/4)a^2 = 252$$

$$a^2 = 144$$

$$a = 12 \text{ (-12 rejected)}$$

$$b = (3/4)a = (3/4) \cdot 12 = 9$$

The perimeter of the larger rectangle is $4b + a + b$

$$\begin{aligned} &= a + 5b \\ &= 12 + 5 \cdot 9 \\ &= 12 + 45 \\ &= 57 \end{aligned}$$

Chung has used the ratio concept, the area of the larger rectangle concept, and calculating a square root to find the dimensions of the smaller rectangles, namely 12 units and 9 units. If we let Chung know that he has found those two dimensions and then bring him back to focus on the diagram, might he not be able to realize by himself that his statement about the perimeter of the larger rectangle is inaccurate? He has half of the perimeter but he needs to include two more sides of the rectangle. When he does, he'll get double 57 as his answer or in other words 114 units. Chung has done a great job with his use of algebra!

How might you praise Chung? What would you point out in his solution to make him realize where he is in his mathematical thinking? What questions could you ask him that don't "give" him the answer but lead him to reconsider the one misstep in his solution?

Valuing each individual's current level of mathematical understanding and asking questions to encourage taking the next step is the basis of the Math Forum's Problem of the Week experience. Writing provides the venue for seeing the work. The lessons that we have learned can be applied to any problem solving in the mathematics classroom. Our research

shows that learning is significantly enhanced when students revise their solutions. Feedback is motivating and provides the scaffolding many students need to improve their problem solving. Think of it as you would the writing process—brainstorm, rough draft, feedback, revise, and repeat those steps of feedback and revision until the final draft stage is reached. In the course of that journey mathematical understanding, literacy skills, and confidence in mathematics will all increase.

References

Renninger, K. A., Lillian S. Ray, Swarthmore College, Ilana Luft, State University of New York at Albany, and Erica L. Newton, University of Pennsylvania, 2006. *Coding Online Content-Informed Scaffolding of Mathematical Thinking*. (in press)

Renninger, K. A., L. Farra, and C. Feldman-Riordan. *The Impact of the Math Forum's Problems of the Week on Students' Mathematical Thinking*. Proceedings of ICLS, 2000. <mathforum.org/articles/rennin2_2000.html> (May 31, 2006).

Renninger, K. A. and W. Shumar. *Why and How Students Work with The Math Forum's Problem(s) of the Week: Implications for Design*. Proceedings of ICLS, 1998: 348–350.

Related Links at the Math Forum Web Site

The Math Forum @ Drexel's Problems of the Week service: mathforum.org/problems_puzzles_landing.html

Write Math with The Math Forum: mathforum.statestandards.org/wmmf/

Current Problems: mathforum.org/pow/

Problems of the Week Library: mathforum.org/library/problems/

Product Information: mathforum.org/pow/productinfo.html

PoWs in the Classroom: mathforum.org/pow/teacher/process.html

How FunPoW Submissions are Scored: mathforum.org/funpow/rubrics/

The Math Fundamentals PoW Scoring Rubric for Students: mathforum.org/funpow/rubrics/rubric_funpow.pdf

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