Pre-Algebra PoW Packet

A Pound of Valentine's Chocolates

February 7, 2011 • http://mathforum.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in February 2006. This is Library Problem #3644. The text of the problem is included below. A print-friendly version is available using the "Print" link from the blue-shaded box on the problem page.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. To access the discussions [log in using your PoW username/password], choose one of these methods:

- use the link to "PoW Member Discussions" from your **My PoW Work as a Teacher** area
- go to *prealgpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=527
- from the blue-shaded box, use the **Tips/Ideas from Teachers** link.

Are you making the most of your PoW Membership? If you have an Individual Teacher Membership, consider registering for one of our (free) Orientation Sessions to learn more about the features of your membership. Teachers with Class or School or District Memberships are welcome to take the free Orientation Session but also are encouraged to register for one of our online courses. View information, dates, and links to register at http://mathforum.org/pd/.

Standards

In ***A Pound of Valentine's Chocolates***, students are asked to find how long a one pound box of chocolates should last if Isolde eats it alone. The **key concepts** are fractions and proportional reasoning.

If your state has adopted the Common Core State Standards, this alignment might be helpful:

Grade 6: Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

Grade 7: Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Additional alignment information can be found through the Write Math with the Math Forum service, where teachers can browse by NCTM and individual state standards, as well as popular textbook chapters, to find related problems.

The Problem

A Pound of Valentine's Chocolates

When Tristan visits Isolde on Valentine's Day, he's planning to bring her a one pound box of chocolates. He knows that when they share a one pound box of chocolates, it usually lasts for two weeks. When he eats a one pound box of chocolates on his own it lasts for six weeks.

How long should a one pound box of chocolates last if Isolde eats it alone?

Remember to explain how you solved the problem and show how you know you are correct.

Note: Assume they don't eat faster or slower when they are alone.

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Isolde would eat a one pound box of chocolates in 3 weeks.

If your answer **doesn't** match ours,

- did you try drawing pictures to think about how much was eaten?
- did you try organizing the information using a table?
- did you think about how much of the box of chocolates Tristan eats when he and Isolde eat the chocolates together in two weeks?
- this page from *Ask Dr. Math* might help you think about working together to eat the chocolate.
  http://mathforum.org/dr.math/faq/faq.working.together.html
- this page from *Ask Dr. Math* is another way to think about it that involves fractions.
  http://mathforum.org/library/drmath/view/58597.html
- did you check your arithmetic?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

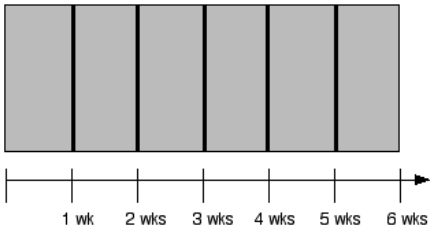
- have you clearly shown and explained the work you did?
- are you confident that you could solve another problem like this successfully?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Number Lines and Parts of a Whole

We are told that Tristan can eat the whole box in 6 weeks. I drew a number line showing the six weeks. Above the number line I drew a box and thought about how much he might eat each week.



If I have a box divided into six equal parts, each part represents 1/6. In 1 week, Tristan will eat 1/6 of the box of chocolates.

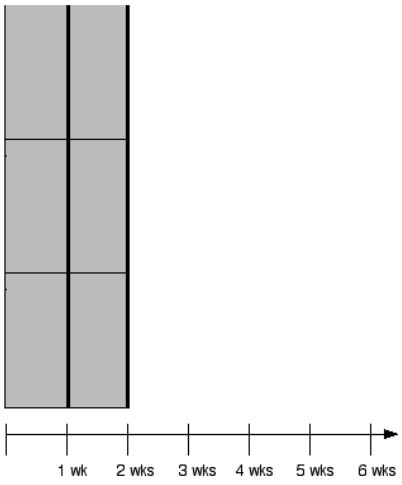
We're also told that together, Tristan and Isolde will eat the whole box in 2 weeks.

I made another number line and above the two week span of time I arranged the 6 parts of the box of chocolates to show that together they ate it all in the two weeks – 3/6 in one week and 3/6 in the other week. So in 1 week, they'll eat 3/6 or 1/2 of the box

Since I already know that Tristan eats 1/6 of the box in a week, Isolde must be eating the rest of it.

Isolde eats 3/6 - 1/6 of the chocolate, which is 2/6 or 1/3.

Since she is eating 1/3 of a box in 1 week, she would eat the whole box by herself in 3 weeks (1/3 + 1/3 + 1/3).



Method 2: Organize in a Table

After reading the problem we know that Tristan can eat the whole box in 6 weeks. In 1 week, he eats 1/6 of a box, and in two weeks he eats 2/6 of a box, which is 1/3 of a box. Together in 2 weeks, Isolde and Tristan eat one whole box.

	6 weeks	1 week	2 weeks
Tristan	1 box	1/6 box	2/6 or 1/3 box
Isolde & Tristan			1 box

So in 2 weeks, Isolde eats what Tristan doesn't eat, or 1 - 1/3, or 2/3 of a box.

	6 weeks	1 week	2 weeks
Tristan	1 box	1/6 box	2/6 or 1/3 box
Isolde & Tristan			1 box
Isolde			2/3 box

This means that in 1 week she eats 1/3 of a box.

	6 weeks	1 week	2 weeks
Tristan	1 box	1/6 box	2/6 or 1/3 box
Isolde & Tristan			1 box
Isolde		1/3 box	2/3 box

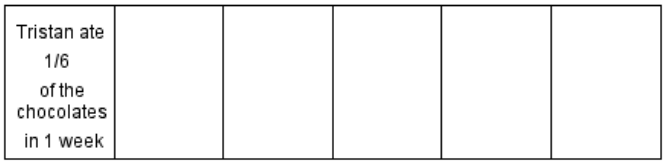
It will take Isolde 3 weeks to eat the whole thing by herself.

Method 3: Logical Reasoning

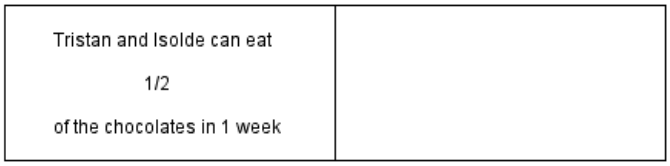
We know that Tristan can eat 1 box in 6 weeks. Together they eat 1 box in 2 weeks. So in 6 weeks, they would eat 3 boxes. If Tristan eats one of those boxes in those 6 weeks, then Isolde eats the other 2 boxes. That must mean that it takes her 3 weeks to eat 1 box by herself.

Method 4: Fractions of a Box

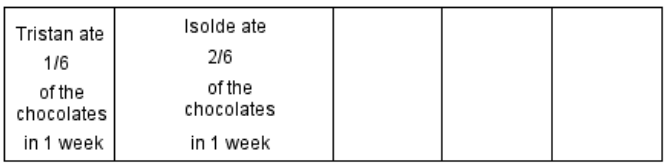
We know that Tristan can eat 1 box in 6 weeks and if I draw the box of chocolates, I can think of it this way:



We also know that Isolde and Tristan together can eat 1 box in 2 weeks:



When I look at the two fraction bars, I notice that Isolde must eat 2/6 (or 1/3) of the box in 1 week.



Now I can think about Isolde eating the box of chocolates by herself:

Isolde ate 2/6 of the chocolates in 1 week	Isolde ate 2/6 of the chocolates in 1 week	Isolde ate 2/6 of the chocolates in 1 week

It should take her 3 weeks!

Method 5: Using a Table to Think of Rates

If I think about eating chocolate as "work" that Isolde and Tristan have to do (isn't that fun!) with the information given in the problem, then I can organize it in a table:

	rate = amount of box of chocolate eaten per week	number of weeks	amount eaten per week
Tristan	$\frac{1}{6}$	1	$\frac{1}{6} \cdot 1$
Isolde	$\frac{1}{n}$	1	$\frac{1}{n} \cdot 1$
Tristan & Isolde	$\frac{1}{2}$	1	$\frac{1}{2} \cdot 1$

Now I can make an equation to solve for n:

$$\frac{1}{6} \cdot 1 + \frac{1}{n} \cdot 1 = \frac{1}{2} \cdot 1$$
$$\frac{1}{6} + \frac{1}{n} = \frac{1}{2}$$
$$\frac{1}{n} = \frac{1}{2} - \frac{1}{6}$$
$$\frac{1}{n} = \frac{3}{6} - \frac{1}{6}$$
$$\frac{1}{n} = \frac{2}{6} = \frac{1}{3}$$

If Isolde eats one third of a box in 1 week, then it takes her 3 weeks to eat the box of chocolates alone.

Method 6: Algebraic

After reading the problem I know:

- Tristan can eat all of the chocolates in 6 weeks which means he eats $\frac{1}{6}$ of the chocolate in one week.
- Tristan and Isolde together can eat a box of chocolates in two weeks, which means they eat $\frac{1}{2}$ of the chocolate in one week.

If I let $\frac{1}{n}$ represent the amount Isolde would eat in one week, then I can add the amount Tristan eats in a week and she eats in a week.

$$\frac{1}{6} + \frac{1}{n} = \frac{1}{2}$$
$$\frac{1}{6} + \frac{1}{n} = \frac{3}{6}$$
$$\frac{1}{n} = \frac{2}{6}$$
$$\frac{1}{n} = \frac{1}{3}$$

If it takes Isolde 1 week to eat $\frac{1}{3}$ of the chocolates, it will take her 3 weeks to eat the chocolates by herself.

Method 7: Working Together Formula

There is a formula to find the answer to this. If a is the length of time that it takes one person to do the job and b is the time it takes a second person to do the same job, then the amount of time that it will take the two of them to do the job together can be found with:

$$\frac{a \bullet b}{a + b}$$

In this problem, we don't know how long it would take Isolde, so we can call her time b. Tristan's time is 6, since it would take him 6 weeks. And the formula would be equal to 2, since we know that together it will take them 2 weeks. We can put these values in and then solve for b.

$$\frac{6 \bullet b}{6 + b} = 2$$
$$6b = 12 + 2b$$
$$4b = 12$$
$$b = 3$$

It will take Isolde 3 weeks to eat the chocolates by herself.

Teaching Suggestions

When we first offered this problem we heard comments ranging from "At first I thought it was impossible!" to "After I found out how to work it, it was not difficult." When students have these thoughts it's an indication to me that you definitely want to start with the Scenario [pdf] before giving out the full problem. The more noticing and wondering that they do, the more likely you won't hear the "I don't get it" refrain.

As students start talking about the information in the problem suggest that they clearly label what is being measured. These labels can definitely help students particularly if they think of organizing the information using a table.

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the methods that we have thought to use for this problem that there are multiple ways to approach this problem. And, we may not have thought of them all.

I also encourage you to explore activities in the Change the Representation strategies in our Activity Series. Something there may provide the impetus that your students need.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the Current Problems as we add them throughout the 2010-11 season, including a calendar, consider bookmarking this page (a link to the page is always available in the left menu when you're logged in):

http://mathforum.org/pow/support/

Sample Student Solutions

focus on **Strategy**

In the solutions below, I've provided the scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill, or doesn't provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: • change the representation • logical reasoning • make a table	Might use a formal algebraic method or the working together formula or discuss several different ways to generate possible solutions.

Zakiah
age 14

Strategy
Novice

i think it will take 6 weeks toeat the chocalate

i divided the days that it willtake and the the number of chocolate

I notice that Zakiah used the numbers he could find in the problem and an operation that seemed reasonable. This reminds me of students who see numbers in a problem and decide (perhaps, frantically) to add them or multiply them and have no real reason except that they're just not sure what else to do.

I might start talking with Zakiah to find out what numbers he divided. Can he make a list of what he notices in the problem?

Dexter
age 13

Strategy
Novice

My final answer is 6 weeks.

It would take 6 weeks because they would take the same amount weeks to eat the pound of chocolate by themselves.

Dexter like Zakiah doesn't seem to have gotten very far into the problem. It seems that he used the information that Tristan takes 6 weeks to eat one box of chocolate alone and assumes that Isolde would do the same.

I wonder how he would respond if I brought to his attention the idea that together Isolde and Tristan can eat the box in 2 weeks. I might ask him how that changes things.

Minkyung
age 12

Strategy
Novice

When Isolde eats alone, the one pound chocolate should last 4 weeks.

When Tristan shares the one pound chocolate with Isolde, it lasts 2 weeks. Then, if Tristan eats alone, it takes 6 weeks to finish the chocolate. Then, I did 6 - 2 = 4 weeks that the chocolate would last if Isolde eats alone. Therefore, it would take 4 weeks for Isolde to finish her chocolate alone.

I notice that Minkyung has written a little more but has taken numbers from the problem and subtracted them.

I wonder if adding labels to the numbers might mix things up a bit. 6 (Tristan alone) - 2 (Isolde and Tristan together) = 4 (Isolde alone).

Can you subtract "together" from "alone" and get a different "alone"?

Hannah
age 12

Strategy
Apprentice

I found out that it takes her 4 weeks to eat one pound of chocolates.

i figured out if they eat it together: it takes them 2 weeks so they eat 1/2 in one week. so they both eat 1/4 of the 1/2 in one week. than i know that she eats 1/4 in one week. you need 4/4 so i did 1/4=1 week so 4/4= 4 weeks.

I notice that Hannah has started to think in terms of fractions. She used the idea given about Isolde and Tristan.

I wonder if she saw the other fact given in the problem about how long Tristan can eat the box of chocolates on his own.

Hayley
age 12

Strategy
Apprentice

It would take Isolde 1 week to finish the box of chocolates on her own.

When Tristan and Isolde share a 1 pound box of chocolates, it takes them 2 weeks to finish. But when Tristin eats the box by himself, it takes him 6 weeks to finish. 2 is 1/3 of 6. If Tristin eats 1/3 of the chocolates, then Isolde would be eating the rest of the box (2/3). That means that she would be eating more chocolates at a faster pace than Tristan. She is eating twice as much as Tristan, and she is eating them twice as fast as him. So you would have to divide 2 weeks by 2, which equals 1 week.

Hayley has such a good start on her logical reasoning strategy. It's true that 2 is 1/3 of 6 but it doesn't follow from her statement that Tristan eats the chocolates alone in 6 weeks. I might ask Hayley if Tristan eats the chocolates in 6 weeks, what fraction of the box would he have eaten in one week.

Matthew
age 13

Strategy
Practitioner

42 - {(42 / 42) * 14)} = 28 28 / 14 = 2 42 / 2 = 21

First, I counted how many chocolates were in the picture which was 42. Then, I found how many Tristan eats a day in his six weeks. I realize he restrains himself to one chocolate a day. Next, I found out how many he eats in two weeks and found that to be 14. After that, I subtracted 14 from 42 and found that Isolde eats 28 chocolates in two weeks. Then, to find out how many she eats in one day, I divided 28 by 14 to get 2 chocolates a day. Finally, I divided the box of 42 chocolates by 2 chocolates a day to find that she eats the entire box in only 21 days or three weeks.

Interestingly when I wrote this problem it never occurred to me to count the number of chocolates featured in the picture! Matthew seems to have counted but conveniently used 42 (6 weeks with 7 days worth of choclate) so that he can conclude that Tristan only eats one chocolate a day–that is restraint!

Matthew's miscounting might lower his Interpretation score. He could note that he made this "assumption" rather than physically counting the chocolates in the picture (actually numbering 32 and not 42).

I would suggest to Matthew that he separate his work using commas, for example, or otherwise it looks like 42 equals 21! Because he has nicely explained his thinking I know that he doesn't really mean that. It's a common clarity error but simple to fix.

Caleb
age 12

Strategy
Practitioner

She will eat the pound of chocolates in 3 weeks.

I knew that Tristan would take 6 weeks to eat the chocolate, so I made a 5x6 grid. Since 30/6=5, I knew Tristan would eat 5/30 of the chocolate per week. In two weeks, he would eat 10/30, which leaves 20/30 for Isolde. If Isolde eats 20/30 in two weeks, then she eats 10/30 in one week, and 30/30 in 3 weeks.

I notice that Caleb, for some reason, thought in terms of thirtieths.

It seems to work for him but I wonder why he started with a 5x6 grid. I wonder if the 5 represents the number of days in a week. I also wonder if this strategy works for numbers other than 5. Why or why not?

Theron
age 12

Strategy
Practitioner

It will take 3 weeks for Isolde to eat the pound of chocolates.

First, I figured out how much Tristan eats in a week. If he can eat 1 pound in 6 weeks, then he can eat 1/6 of a pound in 1 week. If the shared box usually last 2 weeks, then Tristan would eat 1/3 of the pound of chocolate. Then Isolde must eat 2/3 of the chocolates because 1-(1/3)=2/3. So Isolde eats 1/3 of a pound per week because (2/3)/2 = 1/3. Since 1/(1/3)=3, it will take 3 weeks for Isolde to eat the pound of chocolates.

Theron's strategy is similar to both Matthew's and Caleb's except he works in weeks rather than days and with reduced fractions.

I might suggest breaking is one paragraph into two for ease of reading but overall he's done a nice job.

Kyle
age 12

Strategy
Expert

A one pound box of chocolates should last Isolde 3 weeks if he eats it alone.

I found the information from the problem that was important to find the solution.

= pounds
T (Tristan) = 1#/6 WEEKS
I(isolde) and T(Tristan) together eat 1#/2 weeks

I had to use the formula I(Isolde) = (I+T)-T in order to find out how much chocolate Isolde would eat by herself.

I= 1/2 -1/6
I= 2/6 or 1/3# per week

It would take Isolde 3 weeks to eat the pound of cholcolate all by herself. I know I am correct in determining how much cholcolate Isolde eats by herself becasue substitution would confirm my answer:

	Amount of Chocolate per week (#'s)			
	Week One	Week Two	Week Three	
Isolde	1/3	1/3	1/3	= 1 # every 3 wks
Tristan	1/6	1/6	1/6	=1/2 # every 3 wks

X= nyumber of weeks for 1 pound of cholcolate

1=x(1/6 +1/3)
1=x(3/6)
1= .5 x
1(2)= .5x (2)

x = 2 weeks of Tristan and Isolde eating the one pound box of cholcoates together

I notice that Kyle lists the information that he knows from reading the problem. He uses that information and goes one more step to check his thinking.

Samuel
age 12

Strategy
Expert

It will take her three weeks to finish it.

I made an equation. In one week, Tristan would be done with 1/6 of his chocolate and Isolde would be done with 1/n of her chocolates. The two numbers added together must equal 1/2 because it would take them 2 weeks to finish it all together. 1/6 + 1/n = 1/2. n=3 weeks.

I notice Samuel has an algebraic approach to solving the problem. Although I would mark him an expert in strategy, I might suggest that he use a little formatting to set off his equation as well as show at least one interim step to show how he found his final answer to improve his Clarity score.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Suzanne <suzanne@mathforum.org>