Extending the Enneagon

Extend the sides AB and ED of the regular enneagon ABCDEFGHI until they intersect. What is the measure of the acute angle at this intersection?

Extra: Extend sides AB and EF. Now what’s the measure of that acute angle at the intersection?

After students submit their solution, they can choose to “check” their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn’t agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

The measure of acute angle at the intersection is 60°.

If your answer does not match our answer,
• did you notice that an “enneagon” has nine sides?
• did you notice that the extended sides form a concave quadrilateral with two sides of the enneagon?
• did you try finding the measure of the interior angles of the enneagon?

If any of these questions help you, you might revise your answer, and then leave a comment that tells us what you did. If you’re still stuck, leave a comment that tells us where you think you need help.

If your answer does match ours,
• did you give mathematically sound reasons for each of your statements?
• do you think you could solve another problem like this, say with an octagon?
• did you try the Extra?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Exterior Angles and Triangles

I wasn’t sure what an enneagon was, but I noticed that it had nine vertices, so it must be a nine-sided polygon. I know that as a nonagon. I drew a picture and labeled the points in order, then extended sides AB and ED. The picture on the left is what I got.

There are two exterior angles of the polygon formed by the red lines. I also added a line, extending DC, to have another exterior angle of the polygon. I know the exterior angles of a regular polygon can be found by doing 360/n, where n is the number of sides. So here it’s 360/9 = 40°. This gives us the picture on the right.

Now I have two triangles. I’m looking for the angle at the top right. I know the third angle of the triangle with the two 40° angles will be 100° (since the angles of a triangle must add up to 180°). For the right-hand triangle, the upper left angle must be 80°, since it’s supplementary to the 100° angle. This means that the angle I’m looking for must be 60°, since the other two angles in the triangle are 100° and 40°.

Method 2: Central Angles and Triangles

I drew a picture. I knew the figure had to have nine sides because it has nine vertices. I extended AB and ED, and then added some segments from the center of the circle (O) to B, C, and D, and continued the one on to the intersection point, which I labeled Z.
Since it’s a regular polygon, I know that the central angles at 0 are equal to 360/9, since there are nine vertices. So each one is 40 degrees. Then triangles OBC and ODC are isosceles, since the green edges are radii, so the two base angles of each are \((180° - 40°)/2\), which is 70°. You can see this in the picture on the left below.

The other angle at B and at D will be 40°, since those three angles at B and at D must add up to 180°. The two angles at C are each 110°, since they are supplementary to the 70° angle. So the angles at Z must be 30°, since the other two angles in each triangle are 40° and 110°, and they all have to add to 180°, as in the picture on the right above.

**Method 3: Interior Angles and a Quadrilateral**

I drew a picture and noticed that I had a quadrilateral stuck on the outside of the enneagon. I know that the interior angles of a quadrilateral have to add up to 360°, so if I can find the three angles next to the enneagon, I’ll know the angle that we’re looking for.

I know that the interior angles of a regular polygon with n sides can be found with the formula \(180(n - 2)/n\). That means the interior angles of the enneagon will be 140°. Those are the angles at B, C, and D. The other angles at B and C will be 40° since they’re supplementary to the 140° angles. Then the other angle at C will be 220, since the two angles that meet at C have to total 360°.

For the Extra, I added ray FE to the picture.
You can see that a triangle is formed from E, the earlier intersection, and the new intersection. I know that the other angle at E is 40°, since it is supplementary to the 140° angle. The top angle of this new triangle is 120°, because it’s supplementary to the 60° angle. So the third angle, the one we want, must be 20°, since the three angles of the triangle must total 180°.

**Method 4: A Big Equilateral Triangle**

I drew a picture and extended sides AB and FE. I noticed that if I extended side GH, it would mean that I have extended every third side – there are nine sides, and extending every third means there are three equally spaced sides that have been extended. Since the enneagon is regular, I know that the three sides form an equilateral triangle. So the angle that we’re looking for must be 60°.

This problem might seem a bit scary at first, probably because kids don’t know what an “enneagon” is. That’s somewhat intentional – the idea is to help kids develop strategies for working things out when a problem might initially seem confusing. Students could certainly look up “enneagon” online or perhaps in a dictionary, but they might also look to the additional clue of the name of this figure. Since it has nine vertices, it will have nine sides.

Students might also feel that they have to draw an accurate picture. While I wouldn’t discourage this, it’s worth having them thinking about whether this is true or not. In fact, a less-than-perfect picture can help students focus on the mathematical facts of the situation instead of how things “look” or what a protractor or ruler might tell you. Here’s one of my favorite pictures that was included in a submission to this problem:
This student has really focused on the “facts”, and not how things look. That’s a great skill to develop, as students need to get away from reasons such as, “It looks like….” and move to reasons like, “I know this because…”

In fact, it’s worth talking about whether you really need to draw the whole figure. Is a picture like the one on the right by itself sufficient? If you do want to draw a picture, what tools could students use to make sure it’s as accurate as you want it to be?

If students need help getting started, check out the resources in our Change the Representation and Solve a Simpler Problem Activity Series strategies. These support the idea of adding auxiliary lines to your drawing to break it into manageable shapes. You’ll find everything linked from the Activity Series link in the left menu bar when you’re logged in.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the Current Problems as we add them throughout the 2010-11 season, including a calendar, consider bookmarking this page (a link to the page is always available in the left menu when you’re logged in):

http://mathforum.org/pow/support/

Sample Student Solutions focus on Strategy

In the solutions below, I’ve provided the scores the students would have received in the Strategy category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

<table>
<thead>
<tr>
<th>Novice</th>
<th>Apprentice</th>
<th>Practitioner</th>
<th>Expert</th>
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<tbody>
<tr>
<td>Has no ideas that will lead them toward a successful solution. Shows no evidence of their strategy.</td>
<td>Picks an incorrect strategy, or relies on luck to get the right answer. Draws an accurate picture by hand and measures the angle with a protractor, or uses dynamic geometry software to measure the angle.</td>
<td>Picks a sound strategy—success achieved through skill, not luck. This might include: • using properties of the angles of quadrilaterals • adding segments to the picture to split it into triangles • reasoning using properties of regular polygons, triangles, and perhaps quadrilaterals</td>
<td>Might try to solve the problem two different ways, or provides a very solid explanation for a correct solution to the Extra.</td>
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Jennifer age 15

Strategy Novice

The acute angle of the intersection is 80 degrees.

I subtracted 1 from 9 since there is 9 sides and then multiplied 8 and 180 which equals 1440 and then divided that by 9 and then divided by 2 which gave me 80 degrees.

It’s not entirely clear what she has done, though she multiplies by 8 instead of 7 (when finding the total number of degrees in the interior angles). I would start by asking her to tell me why she divided by 2, as that might tell me more about what she is looking at.

Adi age 14

Strategy Novice

The Angle of intersection of this virtual figure is 60 degrees. from symetric reason and angle calculation

It’s possible that Adi has a sound solution – symmetric reasoning and angle calculation could certainly be part of that! But he’s said too little for us to know. I would ask him to tell me more about the symmetric reasoning he used.
Brandon
age 14
Strategy
Apprentice

The measure of the angle created by the intersection of line segments AB and ED is approximately 54 degrees.

I found this by drawing a regular enneagon or a nine-sided polygon. The number of sides is equal to the number of vertices, since there are nine vertices A, B, C, D, E, F, G, H, and I, then there are nine sides. The sum of the measures of the interior angles of a convex n-gon is \((n-2)(180)\) (Polygon Interior Angles Theorem (6.1)). Since there are nine sides \((9-2)(180)\) would be the equation. Then you would solve it, \(7(180)\), then 1260 degrees. (The sum of the measures of the interior angles) then you would divide by 9 \(\{9\text{ angles}\} / \# \text{of angles} = \# \text{of sides} = \# \text{of vertices}\). That would leave you with 140 degrees per angle. Using any measure for the measurement of the sides (All enneagon are similar if sides proportionate, and all angles are congruent), I made a picture then measured the angle created.

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Brian
age 17
Strategy
Apprentice

answer is 100 degree

each angle of the regular n-gon is \(180 \text{ degree} \times (n-2) / n\) so each angle is \(180 \times 7/9 = 140\) the triangle made by those extended lines is BOD and angle OBD and angle ODB are both 40 degree, for the reason of a line is 180 degree so the acute angle of BOD is 180-80=100 degree

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Jason
age 13
Strategy
Practitioner

The measure of the acute angle at the intersection is 60 degrees.

First, an enneagon is a polygon with 9 sides; thus, each angle in a regular enneagon is \(180(9-2)/9=140\) degrees. Denoting the intersection by J, we have BJD is 360-JBC-JDC-BCD(on the outside) because the sum of all angles in a quadrilateral is 360. By the linear pair theorem, JBC=JDC=40 degrees. Because there are 360 degrees around a point, the outside of BCD measures 220 degrees. Therefore angle BJD is 60 degrees.

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Brandon has done a good job explaining how he found the interior angle measures of the enneagon, and he makes a good point about the side length not being important. But then he measures the desired angle with (presumably) a protractor! I would point out that that’s a great way to start, but that next he needs to find a way to actually calculate that angle measure. I might ask him what shape was formed outside the enneagon and whether he can find any of the angle measures of that figure based on the angles he has already calculated.

Brian has found the interior angle of the enneagon correctly, but I’m confused by the fact that he describes the new shape as a triangle. I might point out to him that in my picture, extending those sides didn’t form a triangle, and suggest that he take a closer look at the shape.

Jason tackles the outside shape as a quadrilateral. He’s given a rationale for each of his statements. I would encourage him to try the Extra question.
The measure of the angle formed by the intersection of the extended line segments AB and ED is 60 degrees. Bonus: The measure of the angle formed by the intersection of the extended line segments AB and FE is 20 degrees.

An Enneagon has 9 vertexes of 140 degrees. I had trouble figuring out what to do until I extended the line segments BC and DC to intersect the line extensions of ED and AB.

The angle LDC and LCD = 180 - 140 = 40 degrees.

The angle CLD = 180 - 40 - 40 = 100 degrees.

The next line segment I added was KC.

The angle KCL = 1/2(140) = 70 degrees.

The angle KLC = 180 - 100 = 80 degrees.

The angle of CKL = 180 - 70 - 80 = 30 degrees.

The angle MKL = 60 degrees

Bonus: The angle JEK = 180 - 140 = 40 degrees

The angle EKJ = 180 - 60 = 120 degrees

The angle KJE = 180 - 40 - 120 = 20 degrees.

This was a fun problem.

Lisa

Lisa has used a solid strategy to solve the problem. I would encourage her to add some words to each of her steps to explain how she knows each one is a valid mathematical decision. That would make her explanation more convincing.

A problem-specific rubric can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A generic student-friendly rubric can be downloaded from the Teaching with PoWs link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Geometry Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Annie