A Single Term Formula
for
Approximating the Circumference of an Ellipse

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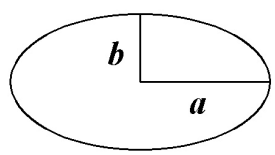
1 Introduction.

There is no simple formula with high accuracy for calculating circumference of an ellipse. There are simple formulas but they are not exact, on the other hand, there are exact formulas but they are not simple. The must famous formulas (approximations) were presented before by Kepler (1609), Euler (1773), Muir (1883), Lindner (1904), Ramanujan (1914), Hudson (1917), and Cantrell (2001). In this note, I will present one of the highest in accuracy and also the most simple of formulas (approximations) for calculating the circumference of an ellipse.

2 Illustration of formula.

For an ellipse having semiaxes of lengths a and b, the perimeter (P) is approximately

$$P = 4 \times (a+b) \times \left(\frac{\pi}{4}\right)^{\left(\frac{4ab}{(a+b)^2}\right)}$$



As easily could be seen, this approximation is "extreme-perfect" (as David W. Cantrell named so). That is, it gives the perimeter of the ellipse perfectly at both extremes of eccentricity. In other words, it gives the circumference of ellipse exactly both when $b=a$ (I.e. when ellipse is then a circle or *eccentricity=0*) and when $b=0$ (I.e. when ellipse is then a pair of lines (of length a) or *eccentricity=1*).

Between the extremes of eccentricity, this formula estimates the perimeter of the ellipse, with the maximum *worst* $|$*relative error*$|$ less than 0.0017 that occurs when $a \approx 0.27 * b$ or $e \approx 0.96$.

Consider that the unique specification of this formula is that it has not an absolute constant coefficient such as other formulas' terms had have, instead it includes a variable function of π that varies from *1* to *3.1415…* regarding the shape (eccentricity) of ellipse. That was the basic assumption that the formula was built on it.

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3 Advantages of new approximation.

The most important question is “What are the advantages of the new approximation compared to well known approximations? “. There are two main answers. First, it is as simple as possible. It has only one term and so could be easily kept in mind. It is suitable for students in high schools and for using in computational programs. Second, it is significantly accurate.

In other words, it is notable for both of its’ great simplicity and substantial accuracy simultaneously. Furthermore, its’ form seems to be a guidance to solve the complicated elliptical integral in new method.

References

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