Using Homemade Algebra Tiles to Develop Algebra and Prealgebra Concepts

Algebra for all really is possible. Unfortunately, too many students attempt to learn algebra by using memorization, and too many teachers use instructional methods that encourage memorization. Students struggle to learn algebra, and teachers struggle with how to best teach algebra so that it makes sense to their students and so that students will remember it beyond next week’s test.

We must change our thinking about what it means not only to learn mathematics but also to teach it. As teachers, we do not need to be convinced of the importance of the subject, but we may need to be convinced that algebra for all is attainable. How can we structure our classrooms and lessons to foster an environment that is conducive to meeting the goal of successful algebra learning for all students?

One way to teach for understanding is to use concrete models to introduce concepts rather than concentrate only on the abstract or symbolic (Lawson 1990; NCTM 1991). The method with which we have had success uses algebra tiles as concrete models in classrooms of middle school and secondary school students, with college-level introductory algebra students, and with prison inmates studying for General Educational Development (GED) tests. These students encompass a full range of intellectual and motivational levels. They come from both advantaged and disadvantaged homes. We have seen their successes in learning algebraic topics with which many of them had previously been unsuccessful.

WHAT ARE ALGEBRA TILES?
The algebra tiles, which students help make, consist of small squares, large squares, and rectangles. The number 1, the unit tile, is represented by the small square; the large square represents $x^2$; and the rectangle represents $x$. The sizing of these homemade tiles is similar to that of commercially made tiles. The side of the $x^2$ tile is equal to the length of the $x$ tile. The length of the $x$ tile is not an integral multiple of the side length of the unit tile.

The width of the $x$ tile is equal to the side length of the unit tile.

Commercially made plastic algebra tiles are constructed with one side of one color and the other side of another color. Algebra Lab Gear (Picciotto 1990), for example, also contains $x$-$y$ blocks. The homemade tiles are not plastic and consist of only two colors: one color to show positive values and another to show negative values.

Use an enlarged version of the master shown in figure 1 to run copies on two colors of card stock. We use the conventional red color for negative numbers and green for positive numbers. Give each student two sheets of card stock—one red and one green—and the take-home assignment of cutting the tiles; putting them in a resealable plastic bag.
such as a Ziploc bag; and returning with them to school the next day. In fact, students should bring the tiles to class every day for use on homework and tests, just as they would bring a calculator, protractor, or compass.

Another material that can be used to make the tiles is Foamies. This foamlike material, which is about 1/8-inch thick, is available in many craft stores and can easily be cut with scissors or a paper cutter.

Create a set of tiles to use on the overhead projector by cutting up colored plastic report covers. Alternatively, create a set of demonstration-sized tiles for a magnetic board by cutting very large pieces from the card stock or the Foamies and placing small magnets, available at most craft stores, on the back of each piece.

WHY USE ALGEBRA TILES?
Manipulating algebra tiles combines an algebraic and a geometric approach to algebraic concepts using an array-multiplication model similar to that employed in many elementary school classrooms. Our experience leads us to believe that students benefit from seeing algebra concepts developed from such a geometric perspective.

Furthermore, we believe that we reach a broader group of students by sequencing instruction from the concrete level, through the pictorial level, and finally to the abstract—or symbolic—level. Such sequencing gives students several modes, in addition to just abstract manipulations, that help them understand and solve algebraic problems. The algebra tiles give a frame of reference to students who are not abstract thinkers.

LAYING THE FOUNDATION
Before using the algebra tiles, make clear to students what your expectations are regarding the appropriate use of the tiles. You should also build in some exploration time so that students can make creative designs, satisfy their curiosities about the tiles, and get the “play out of their systems.” To send a message that manipulating the tiles is an important part of the teaching-learning process, model the problems on the overhead projector or on the magnetic board.

When most algebraic concepts are introduced, neither the teacher nor students should do abstract work but should rely on the tiles to solve problems and answer questions. Students should just manipulate the tiles and write the answers to the problems. At the next stage, students should manipulate the tiles and draw a brief sketch. The sketch should not be tedious. Eventually, the sketches will become mental visual representations that enable students to understand paper-and-pencil manipulations of algebraic symbols.

The following prealgebra and algebra concepts are among those that can be introduced and developed using the tiles:
- Adding, subtracting, and multiplying integers
- Modeling algebraic expressions and combining like terms
- Using the distributive property
- Solving linear equations using addition, subtraction, multiplication, or division
- Solving general linear equations involving two or more steps
- Multiplying a monomial by a monomial, a binomial by a monomial, or a binomial by a binomial
- Factoring quadratic trinomials or the difference of two squares
- Completing the square

We next share the introductory portion of two lessons: solving linear equations using addition or subtraction and factoring quadratic trinomials.

SOLVING LINEAR EQUATIONS USING ADDITION OR SUBTRACTION
Before this lesson, students should be able to add and subtract integers and should understand the concept of zero pairs of tiles. A zero pair of $x$ tiles is a negative $x$ tile and a positive $x$ tile, which together form a sum of zero. Likewise, a negative and a positive unit tile or a negative and a positive $x^2$ tile form a zero pair. Ask students to draw a long vertical line in the middle of a piece of paper. Have them model the equation $x + 7 = 10$ by placing one positive $x$ tile and seven positive unit tiles on the left side of the line and ten positive unit tiles on the right side. See figure 2. Explain to students that to maintain equality of the “sides,” each action must be performed on both sides.

Some teachers prefer to isolate the variable by using the addition property of equality, adding seven negative unit tiles to each side, removing the zero pairs of tiles, drawing a sketch showing the manipulations, and then writing their work symbolically, that is, $x + 7 + \ -7 = 10 + \ -7 \Rightarrow x = 3$. See figure 3. Other teachers prefer to isolate the variable by using the subtraction property of equality.
to remove, that is, subtract, positive tiles from each side until all seven tiles on the left have been removed. See figure 4. Your technique should be consistent with the paper-and-pencil symbolic solution that you require of students.

Ask students to model the equation \( x + 8 = 6 \) by placing one positive \( x \) tile and eight negative unit tiles on the left side and six positive unit tiles on the right side. See figure 5. To maintain equality, all additions or subtractions on one side must also be made on the other side.

Some teachers prefer that their students isolate the variable by adding eight positive unit tiles to both sides, removing zero pairs of unit tiles, drawing a sketch showing the manipulations made, and then writing the symbolic version of their work. Other teachers have students isolate the variable by subtracting, that is, by removing eight red tiles from both sides. Before this approach can be used, an alternative model for 6 must be found. Four models for 6 are shown in figure 6; the model must include at least eight negative unit tiles, as shown in figures 6c and 6d.

Although both models show 6, students soon recognize that the model in figure 6c is less work, that is, more efficient. Students can subtract by removing eight negative unit tiles from both sides, drawing a sketch showing the manipulations, and then writing the symbolic version of their work. See figure 7. Students should be encouraged to discover that this procedure has the same effect as adding eight positive tiles to both sides, as shown in figure 5.

**Factoring Quadratic Trinomials**

Students should have used the tiles to multiply monomials and binomials before beginning this lesson. Use the following activity to review the concept of factors. Distribute centimeter grid paper, and ask students to find all possible rectangles that have integral dimensions and that enclose twelve square units, draw the rectangles on their grid paper, and label dimensions. This activity helps students find the factors of such a number as 12 by forming rectangles that enclose twelve unit squares; the dimensions of the rectangle are the factors of the number. See figure 8.

After ascertaining that students possess the prerequisite knowledge of multiplying binomials and factoring, give students any trinomial that factors into the form \((x + \_)(x + \_),\) for example, \(x^2 + 3x + 2.\) Ask students to find the algebra tiles that represent \(x^2 + 3x + 2\) and to place these pieces on a workmat to form a rectangle, as shown in figure 9a or 9b. Remind the students that a square is a rectangle.

Students next need to determine the dimensions of the rectangle. What quantities must be multiplied to get \(x? \) \? \? \(x? \) \? \? \(x? \) \? that is, \(+1x^2, +2x,\)
+1x, +2. Answering these questions gives the dimensions \((x + 1)\) and \((x + 2)\). Therefore, the expression \(x^2 + 3x + 2\) factors into \((x + 1)(x + 2)\). This factorization can be shown with algebra tiles by placing \(x\) tiles and unit tiles outside the bars on the workmat. See Figure 10.

A complete solution to this problem should include not only the factorization \((x + 1)(x + 2)\) but also the sketch of the final tile arrangement, as shown in Figure 10, and an explanation. Require students to draw the sketch to help them build mental representations of the tiles as they progress toward using only symbolic techniques. Drawing a sketch helps students solve problems when they do not have tiles, and the explanation helps you determine whether they understand the process.

After students have discovered that subtracting \(6\) is equivalent to adding \(-6\), have them form a rectangle with the tile pieces representing \(x^2 - x - 6\). Students cannot form a rectangle using only these pieces. Additional zero pairs of \(x\) tiles must be added to the collection, requiring an alternative way to model \(x^2 - x - 6\). Figure 11 shows three possibilities. A rectangle cannot be formed using the pieces from Figures 11a and 11b. Have students arrange the pieces in Figure 11c into a rectangle on their workmat. They should find the factors of \(x^2 - x - 6\) by placing \(x\) tiles and unit tiles outside the bars on the workmat. Caution students that pieces placed outside the bars must multiply to form \(a + 1x^2, -3x, +2x, \text{ and } -6\), as shown in Figure 11c.
Using the box method to factor quadratic expressions from textbook problems. These problems should be selected carefully so that the assignment includes only problems with integral factors and so that students have enough tiles to complete the operation. Problems involving more interesting factorizations can come later, after conceptual understanding has been built using the tiles.

Manipulating algebra tiles and drawing sketches can be time-consuming. After students become proficient with the concepts involved in using the tiles, it may be easier for them to draw a different type of sketch, called the box method, which works very well for factoring trinomials, factoring the difference of two squares, and factoring by grouping.

Consider the problem of factoring $x^2 + 4x - 12$. Have students write the terms $x^2$ and $-12$ in the boxes, as shown in Figure 12a. The $x^2$ placed in the upper-left box represents the product of the factors $1x$ and $1x$, which are placed outside the $x^2$ box. Next, students must consider the factors of $-12$. Remind students that regardless of the factors of $-12$ that they choose, products formed in the upper-right and lower-left boxes must sum to $+4x$. Some students will probably choose such incorrect factors as those shown in Figures 12b and 12c. They should then complete the multiplication by filling in products in the upper-right and lower-left boxes, as shown in Figures 12e and 12f. Each of these two solution attempts presents a difficulty: the sum of $\pm 3x$ and $\pm 4x$, or the sum of $\pm 12x$ and $\pm 1x$, can never be the desired $+4x$.

A third, and correct, factorization is shown in Figures 12d and 12g. Note that the sum of $\pm 6x$ and $\pm 2x$ is $+4x$ only if the addends are $+6x$ and $-2x$, which in turn implies that the correct factors of the $-12$ shown in the lower-right box are $-2$ and $+6$. Therefore, the factorization of $x^2 + 4x - 12$ is $(1x - 2)(1x + 6)$.

**FINAL THOUGHTS**

Teaching is a challenge, and most teachers would agree that we would have it no other way. No teaching manipulatives, tools, or techniques will totally improve our instruction, improve students’ learning, and reach all students—even the ones who

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seem to fall between the cracks of the educational system. We agree with Patterson’s (1997, p. 240) claim that “hands-on activities that begin where previous mathematics courses left off afford us the opportunity to bridge the gap to algebraic thinking.” We believe that using algebra tiles and the box method bridges this gap and brings algebra for all closer to reality.

BIBLIOGRAPHY


