Origins of Sexagesimal Mathematics and Circle Sub-division:

An Hypothesis for the Original Design Principles

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ABSTRACT

The specific sexagesimal subdivision associated with circle geometry is an ancient piece of mathematics that has never been satisfactorily explained with regard to its design logic. This study provides an assessment of the features of circle geometry in combination with observed characteristics of some ancient units of length measurement, in order to discern some logic associated with the origins of the sexagesimal system and circle mathematics.

The main conclusion reached is that circle theory appears to be defined by electromagnetic principles, with subdivision characteristics related to the equation ‘wavelength = speed of light / frequency’. The system allows recognition of an electromagnetic ‘clock’ as the standard reference upon which the sexagesimal arrangement was possibly anchored, which suggests that at some remote historical date this capability may have existed in society. It also identifies the source of the reference circle alluded to by scholars circa 200BC during mathematical discoveries relating to trigonometry.

The clock frequency and wavelength characteristics appear to have also been applied to the development of a standard length unit, around which subsidiary length units were defined. In this regard the inch and the ancient Royal Cubit appear to be significant standards based on a well-defined electromagnetic phenomenon.

The electromagnetic explanation for the features of the sexagesimal system appears to stem from the creation of a specific length unit that arises naturally during subdivision of a circle according to the electromagnetic equation. This unit has a length of 1.667 Royal Cubits, or 34.38 inches, and provides a path of logic around which the development of the sexagesimal system can be understood.

An electromagnetic source with a frequency of $2.778 \times 10^{11}$ Hz is presented as the determining factor in development of circle theory and the sexagesimal system, and as the wavelength reference used to define a standard unit of length in ancient society. This electromagnetic source was furthermore specifically used to define the duration of ‘1 second’ of time, thereby fixing circle geometry to time measurement and resulting in the development of astronomy.
1.0 Introduction

The mathematics of circle theory and its associated sexagesimal substructure is perhaps the oldest piece of mathematics known to modern society. Dilke (1987) has reported that Sumerian numbers were already sexagesimal as early as 3000 BC. It is also a structure whose design principles have presented an insurmountable problem to scholars over many centuries. Part of the difficulty lies with the fact that a decimal number system was also in use for normal mathematical applications, which in itself carries some evidence suggesting that circle theory required a mathematical approach that could not be supplied by a decimal system. The other major difficulty is one of judgement, whereby scholars automatically take the simplistic view that such an early society could not have had advanced knowledge that may account for the development of the mathematics. This article presents an interpretation of circle theory with the aim of dispelling simplistic historical explanations for its origins.

The assumption that a decimal number system is the simplest one to use for all mathematical applications in any society is obviously only true to a certain extent. Such an observation is borne out by the fact that both the sexagesimal and decimal systems were in use simultaneously in Sumerian society, and that metrication of the older units of measure following the French Revolution failed where circle theory was involved. Modern society maintains the dual sexagesimal and decimal mathematics system as a result.

Certain features of ancient measurement units appear to contain evidence of a design based on circle geometry and much of the discussion centres around these features, in order to discern any pattern that might explain the choices made. There is also a reasonable amount of evidence to suggest that the Earth’s dimensions were well known in the remote past. In addition it appears possible that the measurement of longitude, to an appreciable degree of accuracy, was carried out at some stage. This process directly applied circle geometry to Earth measurement at some unknown date in history, and almost certainly prior to 300 BC.

Although a significant amount of historical information supports the view that a well-structured system of measurement units had been in use for a long time in human society, a missing piece of information resulted in the move to set up a modern unit of measure. The culmination of this process was the adoption of the metre as the standard length unit. The missing information was that a specific standard reference unit could not be identified in the archaeological evidence, a standard that should anchor all other measurement units and which
could be accurately reproduced when required. This situation had allowed a large number of measurement units to proliferate around the world, some obviously very similar and derived from a single original source, but all rapidly becoming a source of confusion and conflict with increasing international trade and commerce.

The process of providing a new length standard was set in motion during the French Revolution of the late 18th century, and consisted of measuring the surface distance across a quadrant of the Earth, then dividing this distance into ten million equal parts. Each part would be termed a metre. Suffice to say that this method suffered from inherent inaccuracy because of the need to still account for the true curvature of the Earth. The metre was later redefined as the distance travelled by light during one oscillation of the caesium source used to define one second of time. Oddly enough, the process of metrication retained the ancient sexagesimal time units while rejecting the other units of measure.

Following the modern establishment of accurate Earth dimensions the polar radius has been found to be 6357km, and a ten millionth part of this distance is equivalent to 25.0265 inches which is the length of the ancient Sacred Cubit. Is this evidence that some ancient units of measure were based on the same theoretical approach of defining a unit of length based on Earth dimensions, or is it evidence that scholars during the 18th century used ancient documents as their source of inspiration for determining a standard length unit?

An answer to this question would still not prove however that this was indeed the method applied by ancient society when defining their length units. The reason for this statement will hopefully become clear during the ensuing discussion.

2.0 Historical Aspects of Sexagesimal Mathematics and Circle Theory

The earliest historically recorded description for the 360 - degree sub-division of circles is associated with the work of Hipparchus of Nicaea, a Greek astronomer of the Alexandrian Age between approximately 180 – 125 BC. According to Boyer (1991), “It is not known just when the systematic use of the 360˚ circle came into mathematics, but it seems to be due largely to Hipparchus in connection with his table of chords. It is possible that he took over from Hypsicles, who earlier had divided the day into 360 parts, a sub-division that may have been suggested by Babylonian astronomy. Hipparchus was a transitional figure between Babylonian astronomy and the work of Ptolemy.
Moreover, since the Babylonian positional system for fractions was so obviously superior to the Egyptian unit fractions and the Greek common fractions, it was natural for Ptolemy to subdivide his degrees into sixty ‘partes minutae primae’, each of these latter into sixty ‘partes minutae secunde’, and so on. It undoubtedly was the sexagesimal system that led Ptolemy to subdivide the diameter of his trigonometric circle into 120 parts; each of these he further subdivided into 60 minutes and each minute of length into 60 seconds.”

Many scholars, including Toomer and Neugebauer amongst others, have also credited Hipparchus with the founding of trigonometry, although this aspect of the mathematics is not really under question here. The principles of trigonometry are founded upon circle geometry, and it is this aspect of the historical record that requires explanation. Toomer has argued that Hipparchus was responsible for the direct transmission of both Babylonian observations and procedures and for the successful synthesis of Babylonian and Greek astronomy. (Toomer, G.J. "Hipparchus" and "Trigonometry", in Oxford Classical Dictionary, 3rd ed. Oxford 1996.)

Closer inspection of the available commentaries on the Greek contribution to a sudden advance in trigonometric mathematics and sub-division of the circle into 360 degrees, and further sub-division of each degree according to sexagesimal principles, leads to the conclusion that a Babylonian influence was the dominating factor. The inferences made in some of the references quoted here tend to suggest that the Greek mathematicians and astronomers were the first to make the 360 – degree sub-division of both the zodiac and the circle, but Neugebauer’s work puts the correct perspective in place. The Sumerian / Babylonian civilisation contains the earliest known records of the precursor work leading to the Greek discoveries.

All the above information is essentially academic in terms of recorded history however, and does nothing to solve the problem under consideration. There is no evidence provided by both the Babylonian and Greek mathematicians and astronomers to explain the developmental steps involved with the sub-division of the circle into 360 – degrees, and $1.296 \times 10^6$ seconds. Furthermore, there appears to be no serious hypothesis available in modern times to alleviate the problem.

For those who regard the choice of the sexagesimal method of circle subdivision as a random process, and are critical of scholars who have expended time and effort studying the mathematics, it should be pointed out that some illustrious names in mathematics have deemed it a worthwhile pursuit.
3.0 New Hypothesis for Origins of Circle Geometry and Sexagesimal Theory

3.1 The Speed of Light

A central tenet of the hypothesis advanced in this article is that the ancient society that developed the sexagesimal system, and subdivided the circle, was capable of accurately measuring the time of 86400 seconds for one day of Earth rotation. They would therefore also be able to measure velocities. Lines of evidence presented in this work suggest that the speed of light and electromagnetic relationships were known to the people who developed circle theory, and established some of the ancient units of length. As a result, a first step is taken here to calculate the speed of light in terms of the ancient Royal Cubit length unit.

An important conversion must therefore be made from the modern metre length to an ancient equivalent, and for the major part of this discussion it is the ancient Royal Cubit unit that appears to provide a solid base from which to work. In this regard one has to make a choice out of a number of alternative conversion ratios between the metre and Royal Cubit that have been proposed by scholars of ancient metrology, even though the ratios are very similar. The most common average value for conversion of the Royal Cubit to metre units, is 0.524, meaning that there are approximately 524mm to 1 Royal Cubit. The Royal Cubit is therefore slightly longer than a half metre.

For the remainder of this discussion however, a value of 0.5236 m per Royal Cubit is used as the conversion factor. This factor was proposed by the French archaeologist, and mathematician, Charles Funk-Hellet. Using this value, and the modern value for the speed of light (2.9979 x 10^8 m.s^-1), it is possible to convert the speed of light to Royal Cubit’s per second units as shown in Table 1 below. A conversion is also made for the speed of light expressed in inches per second.

**Table 1: - Speed of Light Values**

<table>
<thead>
<tr>
<th>Length Unit</th>
<th>Speed of light</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal Cubit</td>
<td>5.73 x 10^8 RC.s⁻¹</td>
</tr>
<tr>
<td>Inches</td>
<td>11.82 x 10^9 inches.s⁻¹</td>
</tr>
<tr>
<td>metre</td>
<td>2.998 x 10^8 m.s⁻¹</td>
</tr>
</tbody>
</table>
Since modern expression of the speed of light is fixed to the definition of the metre length, it means that we can convert this value to a Royal Cubit equivalent without being concerned about the actual duration of 1 second of time. In other words we need not concern ourselves with whether 1 second of time (possibly) measured in ancient society has an equivalent duration to the modern 1 second. We can answer this question immediately however, since we know that the Earth’s rotation rate has been slowing down gradually over an unknown period of time. If an earlier society had managed to accurately measure 86400 seconds during one day of rotation, then each of those seconds would have had a shorter duration than the modern one because the Earth would have been rotating more quickly then.

The information presented here will now be used to examine some of the historical evidence available, to see if it can explain features of the sexagesimal system and circle sub-division. It can be said here that the Royal Cubit unit of length is used because it appeared to immediately offer some direct links to circle theory, when the early stages of this study began.

The relationships between the speed of light, frequency and wavelength of an electromagnetic wave are furthermore of relevance to the ensuing discussion, and the relevant equation is as follows:

\[ \text{Wavelength} = \frac{\text{speed of light}}{\text{frequency}} \]

### 3.2 Circle Relationships

During this study certain aspects of circle geometry revealed potential relationships to some of the ancient measurement units. The evidence allowed speculation that some measurement units may have been strictly defined within the context of circle geometry. Certain dimensions also appeared to be defined in a fixed ratio that suggested an understanding of the electromagnetic equation mentioned earlier. Some of these aspects are listed as follows:

- The ‘1 Radian’ angle in circle geometry is 57.297°, comparable in value with the speed of light.

- When the ‘1 Radian’ sector is subdivided into 20.63 equal parts, each part is subtended by the angle of 2.778°. In comparison, the length unit of 1 Royal Cubit can be subdivided into 28 fingers or 20.63 inches.
When the electromagnetic equation is applied to the above data it produces the following result:

\[
57.3^\circ / 2.778^\circ = 20.63.
\]

A full circle contains 129.6 sectors of 2.778°, bearing reference to the number of arc-second subdivisions in the 360° circle, of \(1.296 \times 10^6\).

There exist \(2.063 \times 10^5\) seconds of arc in the ‘1 Radian’ angle of 57.3°.

The above information does not present all features that have been discovered during this study. Others will be discussed in other sections of the article. What does seem obvious is that if circle subdivision has been made in accordance with electromagnetic principles, then we should find a significant number of features within the circle context that supports this possibility. Furthermore, if an ancient length unit was in fact based on an electromagnetic standard, then we should also be able to discern this by applying the electromagnetic equation while examining the unit.

Based on a process of trial and error, and some elimination, a framework was established within which circle geometry could be tested. This framework consists very simply of providing a possible value for the electromagnetic frequency that appears to be defined by the 2.778° sector in circle geometry, and making the assumption that an electromagnetic standard operating at this frequency was involved with developing circle theory and sexagesimal mathematics. The chosen frequency has a value of \(2.778 \times 10^{11}\) Hz, which produces a wavelength of \(2.063 \times 10^{-3}\) Royal Cubit’s when the speed of light is expressed as \(5.73 \times 10^8\) RC/second.

The assumption that an electromagnetic standard was potentially associated with the development of circle geometry has the added implication that a time definition must have been involved, otherwise the speed of light could not be known. In this regard the only logical assumption to make is that \(2.778 \times 10^{11}\) oscillations of the electromagnetic source defined the duration of 1 second. For the electromagnetic source oscillating at \(2.778 \times 10^{11}\) Hz, we can calculate the time duration associated with each oscillation, the reciprocal of the frequency, and we obtain the value of \(3.6 \times 10^{-12}\) s. Each wavelength of radiation produced by this source occupies this time interval. The above assumptions were then used to assess circle geometry as set out in the following sections.
3.3 The ‘1 Second’ of Light Circle

When the time interval of 1 second is considered, an electromagnetic wave will travel the radial distance of $5.73 \times 10^8$ Royal Cubit’s. Since electromagnetic waves propagate outwards from their source as a spherical shell, the ‘1 second’ condition results in the creation of a circle with a radius of $5.73 \times 10^8$ Royal Cubit’s, and a circumference of $3.60 \times 10^9$ Royal Cubit’s. There may be sufficient evidence to associate creation of circle theory and sexagesimal mathematics with a circle of these dimensions.

Firstly, the circumference of $360 \times 10^7$ Royal Cubit’s is easily sub-divisible into 360 equal sectors, each of length $1 \times 10^7$ Royal Cubit’s. These sectors can effectively enable definition of $1^\circ$ of angle, by assuming that the ‘1 second’ time interval is the standard by which all circle features are described. Secondly, the 1 Radian sector of $57.3^\circ$ central angle subtends the arc length of $5.73 \times 10^8$ Royal Cubit’s. Further subdivision of this 1 Radian arc into 20.63 sectors each of $2.778^\circ$ produces sector arc lengths of $2.778 \times 10^7$ Royal Cubit’s, in proportion to the clock frequency. Such a step infers that a specific wavelength is associated with measuring the arc length of a circle.

Another interesting aspect of this circle is that the 1 Radian arc length of $5.73 \times 10^8$ RC can accommodate $3.438 \times 10^8$ parts of a 1.667RC length. Furthermore, the 1.667 RC length is equivalent to 34.38 inches, and one can immediately observe that in the ‘1 second’ circle with $3.438 \times 10^8$ parts of 1.667 RC, there exist $3.438 \times 10^7 \times 34.38$ inches = $11.82 \times 10^9$ inches in the 1 radian arc. This relationship appears to suggest that the speed of light expressed in inches.s$^{-1}$ was a possible determinant for the choice of circle dimension as discovered by the Alexandrian Greeks when trigonometry was under development, or rather re-discovery. A constant ratio of 12.38 is associated with the conversion between inches and the Royal Cubit. The significance of the above information relates primarily to the fact that the earliest historical records showing mathematical developments in trigonometry all refer to a reference circle whose radius is 3438 units in length. The inference is not being made here that the ancient Greek mathematicians themselves knew about the speed of light.

It is useful to point out that for the above circle with dimensions expressed in Royal Cubit’s and inches, both produce circumference lengths that are relevant within the sexagesimal framework. Furthermore, both can be considered to represent potential original reference circles used to subdivide the circle into $360^\circ$ and the smaller subdivisions.
The discovery of a 1.667 RC length within the system has significance towards possibly understanding the real origin of sexagesimal mathematics. Further discussion of this aspect will be made later, but there appears to be reason to believe that this length represents a fundamental subdivision stage in circle geometry. Due to its non-integer value however, it potentially presented the critical level beyond which further practical subdivision could not be made. In order to deal with this length in a practical manner, the only recourse is to multiply by 6 in order to obtain the length of 10RC. From this point onwards subdivision of the circle could be achieved as convenient multiples of 6 and 10, removing the influence of the 1.667RC unit. This procedure automatically results in a sexagesimal structure.

4.0 Measurement Units Examined

4.1 The Royal Cubit and Circle Geometry

In order to discover the possible design origins for circle theory it is necessary to draw upon all forms of ancient evidence that might offer clues. The mathematics itself comprises archaeological evidence, and requires appropriate interpretation. It is also obvious that all the advantages of modern scientific knowledge should be used in this task, primarily because earlier simplistic approaches have not been successful. As the first step in this study, the features of an ancient length standard are placed under scrutiny with reference to circle theory.

The Royal and Sacred Cubit Measurement Ruler

The existence of 20.63 inches to one Royal Cubit is the first piece of evidence that is considered relevant to this study, for the simple reason that it offers some information regarding possible definition of the Royal Cubit and the inch within a circle context. There are superficial indications that the Royal Cubit has origins associated with the geometric sub-divisions of a circle, when the 28 fingers and 20.63 inches are interpreted.
within a 1 Radian sector. In this regard, the 20.63 divisions termed *inches* can be produced by subdividing the 1 Radian arc into 20.63 sectors each subtended by the angle of 2.778°.

To begin the discussion regarding the Royal Cubit, Diagram 1 is presented and discussed below.

**Diagram 1: -The Royal Cubit and the Circle**

![Diagram 1](image)

Notes to Diagram 1:
A) 20.63 parts of 2.778° exist in the 1 Radian arc. This is open to the interpretation that the 1 inch length was defined by sub-division of the arc length into 2.778° sectors.
B) Another consequence of this sub-division is that the full 360° circle will contain 129.59 parts of the 2.778° sectors. This appears to have some significance towards discovering the origin of $1.296 \times 10^6$ seconds of arc in the circle.

In the above diagram a circle of radius 1 Royal Cubit is presented. This circle therefore has a 1 Radian arc length equivalent to the radius, and subdividing the 1 Radian arc into 20.63 equal parts produces the *inch* unit length. Each “1 inch” unit corresponds to the subtended angle of 2.778°. An interesting observation at this stage is that “1 inch” is equivalent to 0.36 Royal Cubit’s, reinforcing the background influence of the number 36 within the system. Furthermore, the full circle contains 129.6 sectors of 2.778°, carrying potential relevance towards understanding the subdivision of the circle into $1.296 \times 10^6$ seconds of arc.

A significant aspect of the relationships shown in Diagram 1 can be immediately seen, and is directly associated with the 2.778° or 27.78° sectors. This is the fact that in circle theory, a 1 Radian arc contains $2.063 \times 10^5$ sub-divisions into seconds of arc. It would appear that such sub-division is connected to recognition of the 2.778° angle. For a circle of radius 1 Royal Cubit however as shown in Diagram 1, it is obvious that subdivision of the 1 Radian arc length into the normal $2.063 \times 10^5$ seconds of arc would result in very small lengths that would be inconvenient for practical use. This fact suggests that a larger circle must be involved
with definition of the Royal Cubit, if indeed this unit of measure was defined by such methods originally. The initial evidence presented above is reasonably superficial at this stage, but the coincidences with numbers, units of length and angles are enough to proceed with the main aim of this study. An obvious point to make as well though, is that the basic information relating speed of light, wavelength and frequency are inherent in the ‘1 Radian’ sector of a circle when it is subdivided into 20.63 parts of 2.778˚ sectors.

A second situation is therefore presented below in Diagram 2, which offers a more likely condition by which the Royal Cubit may have obtained its definition.

**Diagram 2: -The Royal Cubit and the Circle**

Diagram 2 offers the view in which the Royal Cubit might potentially be defined in terms of only 1˚ of arc length for the circle whose radius is 57.3 Royal Cubit’s, and involving a circumference of 360 Royal Cubit’s. The unusual aspects of this circle relates to the coincidence now between the 1 Radian angle of 57.3˚ and the radius of 57.3 Royal Cubit’s, and the fact that circle circumference is 360 Royal Cubit’s. It is obvious that 57.3˚ multiplied by 2 pi provides the 360˚ circle, and that a radius using the same numbers as that associated with the ‘1 Radian’ angle will therefore produce a circumference value equal to 360 units.

In the circle of Diagram 2, calculation of the arc length associated with one second of arc results in a very small length unit of $2.778 \times 10^{-4}$ RC. This result is obtained by dividing the 1 Radian arc length into $2.063 \times 10^5$ seconds of arc. Again, such a small length would have no practical value for everyday use, and tends to support the option that a circle of greater radius was used as the main reference circle for development of circle geometry, and definition of the Royal Cubit and its subsidiary components.
However, a question can be raised by the hypothetical system in Diagram 2, if we accept the possibility that the Royal Cubit length was defined in such a manner. The question is; “why choose a circle whose radius is 57.3 Royal Cubit’s to potentially define the length unit”? This question simultaneously raises a second one; “is the ‘1 Radian’ angle of 57.3˚ the end result of allocating 360˚ to the circle sub-division, or is the 1 Radian angle a determinant that results in 360˚”? One of these aspects must be the driving factor in the development of circle geometry.

Weak points can be found for the Royal Cubit definition as presented in both Diagram’s 1 and 2, and does help with eliminating some proposals regarding the circle dimension that may have been associated with this unit of length, and possibly also the primary reference circle. These are outlined as follows:

- As seen in Diagram 1, the ‘1 Radian’ arc of 1 Royal Cubit can be subdivided into 20.63 inches, associated with 2.778˚ sectors, but further consideration of the required 2.063 x 10^5 seconds of arc sub-division produces unacceptably small length units. The basic structure of Diagram 1 does however carry a lot of evidence to support the proposal that the Royal Cubit length was defined in circle terms.

- Diagram 2 carries the argument to the level where only 1˚ of arc of a circle defines the 1 Royal Cubit length. This situation carries some potentially significant information that will be examined in a following section, but does not offer much improvement on Diagram 1 for the structure of the Royal Cubit. It would appear that in order to suitably define the Royal Cubit within the confines of existing circle geometry, the following aspects should apply:

  - When normal arcsecond sub-division of the circle takes place, it should presumably result in 1 finger being equal in length to the second of arc, if we assume that this was the smallest practical unit of measure under the circumstances. As a result 1 minute of arc would have the length of 2.16 RC, and 1˚ the length of 129.6 RC. Circle radius and circumference would then be 7.425 x 10^3 RC and 4.6656 x 10^4 RC respectively.

Under these circumstances the 1 Royal Cubit unit would be associated with 27.78 seconds of arc, suggesting that the 28 fingers originated from arcsecond sub-division of a circle with the above radius. The historical record indicates that there are 28 fingers to 1 Royal Cubit, but a proposal is made here that the 27.78 fraction may be more accurate. If the ideas presented in this study gain acceptance then it would simply mean the
27.78 fingers were conveniently rounded-up to 28 for general purposes, or archaeological measurements have not had good enough source materials from which more accurate results can be obtained.

To summarise this section, the following points are raised, assuming that the Royal Cubit unit was defined in a circular reference frame:

- A circle of radius 1 Royal Cubit would be too small.
- A circle of radius 57.3 Royal Cubit’s would also not be suitable.
- A circle of radius $7.425 \times 10^3$ Royal Cubit’s would result in the 1 arcsecond length of $3.6 \times 10^{-2}$ Royal Cubit’s, which would constitute a 1 finger unit, and 27.78 such fingers would form the 1 Royal Cubit length. Since there would be 3600 fingers in 1˚ of arc, this length would be 129.6 Royal Cubit's.

Since the above circle does not have a radius of length 3438 Royal Cubit’s, it is not a reliable candidate for the primary reference circle upon which original circle theory was based. Such a circle still requires some study, although some earlier discussion has already presented strong evidence that the system was possibly based on recognition of a 1.667 RC unit which itself contains 34.38 inches. In the 1 second circle of radius $5.73 \times 10^8$ RC there would be $3.438 \times 10^8$ such 1.667 RC lengths. From this we could simply propose that a smaller circle of radius $5.73 \times 10^3$ RC perhaps, was the primary reference. This will be discussed in more detail in following sections.

Discussion of the Royal Cubit is by no means complete at this stage, but a second source of information will be presented now in order to find a historical description for the reference circle associated with the development of trigonometry.

4.2 Egypt’s Official Ancient and Current Geographic Boundaries

Egypt is perhaps the only country in the world whose official ancient boundaries were predominantly straight lines, defined not by geographic features but by lines of longitude and latitude. The 7˚ 12’ latitudinal length of Egypt (from approximately 23˚ 54’ S to 31˚ 06’ N) is the subject of much discussion in historical literature. Primarily it is recognised that the 7˚ 12’ length means that the people involved measured and marked the Tropic of Cancer, which forms the southern boundary to Egypt. Most relevant literature is more consistent in
the attention paid to Eratosthenes, who is on record as being the first person to accurately measure the Earth circumference by making use of the 7˚ 12’ length of Egypt.

Furthermore, the mathematics of trigonometry was theoretically only historically available following the work of Hypsicles, Hipparchus and Ptolemy (around 180 - 125 BC), and yet Eratosthenes (c. 200BC, an earlier date) appears to have made his calculations and measurements, or estimates, prior to the known development of trigonometry. He would also have had to use an angle-measuring device whose design must also be attributed to an earlier source, and be explicitly based on the 360˚ circle subdivision.

If Eratosthenes helped modern historians realise the significance of his work by drawing attention to the 7˚ 12’ latitudinal length of Egypt; he certainly didn’t help explain the significance of the 2.8˚ longitudinal width. Historical scholars have likewise been unable to find any reasonable logic behind this feature, and it remains an unexplained aspect of the ancient Egyptian landscape. This article will present evidence to suggest that, like the 7˚ length, some significance may be attached to this choice of boundary. To begin this process we must consider the information presented in Diagram 3, where an imaginary circle centred on the Great Pyramid intersects the eastern and western boundaries of Egypt at points in the delta. When the intersection is made at the points where the official ancient eastern and western boundaries meet the 31˚ 06’ N boundary, then the radius of the circle has an approximate length of 3.44 x 10^5 Royal Cubit’s, or 1.8 x 10^5m. This distance has been calculated using dimensions of the Clarke geoid, and equations as provided in Appendix A.

**Diagram 3: - The Delta Circle in Northern Egypt**

![Diagram 3: - The Delta Circle in Northern Egypt](image-url)

- Circle circumference is equal to 1˚ of longitude at the equator.
- Circumference = 2.16x10^6 RC
- 2.8˚ longitude width of Egypt
- Distance = 5.34x10^5 RC

Great Pyramid at Giza

Radius of circle = 3.4x10^5 RC
Notes to Diagram 3:

- The circle circumference is $2.16 \times 10^6$ RC.
- The circumference of this circle is almost exactly the $1^\circ$ arc of longitude at the equator.
- A $1^\circ$ arc of this circle is in direct proportion to the $2.8^\circ$ arc of longitude at the equator. I.e. $5.95 \times 10^3$ RC compared to $5.9 \times 10^5$ RC at equator.
- The 1 Radian arc length is $3.438 \times 10^5$ RC, which represents 100 minutes of longitude at the equator.
- 1 second of arc in the Delta circle has length 1.667 RC. (6 arc-seconds therefore equal 10 RC).
- 129.6 such distances would complete the full circumference around the Earth.
- The radial length contains $2.063 \times 10^5$ parts of the 1.667RC unit length. This means that the 1 Radian arc length can also be subdivided into this number of units, and is possibly the origin of the number of seconds of arc in circle theory.

The geometric arrangements created within the circle are not insignificant. Taking into account the minor error that will surely exist in calculating the exact distances involved, the chord created through the Delta circle nevertheless has a length that is constrained by the $2.8^\circ$ width of Egypt’s boundaries. Most obvious however is the fact that the $2.8^\circ$ width of Egypt means that 129.6 such distances would complete the full circumference around the Earth, bearing reference to the $1.296 \times 10^6$ arcsecond subdivisions in circle geometry. In this respect it is proposed that the actual width of ancient Egypt was more accurately $2.778^\circ$ of longitude, and reveals the likelihood that an ancient society had already measured the Earth and marked off accurate points of longitude and latitude according to circle geometry.

Of primary interest to this study however is the close correspondence between the radius of the circle involved with that proposed by scholars such as Toomer and George G. Joseph, to account for the mathematics produced by Hipparchus and other mathematicians. In this regard, the radius of 3438 units recorded by historians could potentially relate to a physical distance measured out across the Earth’s surface in Egypt.

The contents of the above diagram reveal some striking information which suggests that the official $2.8^\circ$ longitudinal width of Egypt, established at an unknown date in ancient history, like most other aspects of its geographical and historical content, also has significance towards understanding a greater picture. The $2.8^\circ$ width effectively constrains the dimension of an imaginary circle that encompasses the Delta region, with the Great Pyramid being the centre of the circle, because it fixes the length for the base of the triangle involved.
All features of this circle contain reference to the dimensions of the Earth’s equatorial circumference, and apparently a direct possibility of being the primary reference circle used by the Alexandrian Greeks to establish trigonometry. The main reason for this proposal is because the circle radius presented in Diagram 3 is considered to be $3.438 \times 10^5$ Royal Cubit’s, taking into account minor error associated with calculating such distances across the Earth surface.

Although there is no modern physical evidence for the existence of such a circle in Egypt, some historical references do mention this circle, presumably based on the contents of older literature sources. This would suggest that mention of this feature could be found in ancient records, but the reasons for its existence unknown. At some stage in history it is considered likely by this author that geodetic markers may have been in place to define the actual circumference of the circle shown in Diagram 3, but most of these have most likely since been destroyed. The significant aspect of the information presented above is that a radius is found which is constantly referred to in historical mathematics texts with regard to the earliest calculation of trigonometric functions. Although historical texts refer to the standard circle radius for which trigonometric tables were constructed as 3438 units, the radius of $3.438 \times 10^5$ Royal Cubit’s effectively means that exactly the same trigonometric results will be found.

The notes attached to Diagram 3 reveal additional features about the Delta circle which all have significance regarding the origins of circle theory and sexagesimal mathematics. Firstly, circle circumference is $2.160 \times 10^6$ Royal Cubit’s. Considered in isolation this number is arbitrary, but when it is remembered that the value of pi makes it quite difficult to construct a circle whose circumference is a convenient round number for further mathematical use, this situation reflects a unique achievement in ancient society. The radius of $3.438 \times 10^5$ Royal Cubit’s apparently reflects a case where the circumference length was a determining factor. George Joseph (1991) states the following in his book, ‘The Crest of the Peacock’, in relation to discoveries made in Chinese mathematical texts from the Tang dynasty. “There are also sine tables at intervals of 3° 45’ for a radius of 3438 units, which are the values given in the Indian astronomical texts Aryabhatiya and Suryasiddhanta*. This is the earliest record of a sine table in any Chinese text. * The choice of a radius of 3438 was determined by the practice of dividing the circumference of a circle into 360 x 60 = 21 600 equal parts. If the length of the arc of each of these equal parts is 1 unit, and the value of pi is taken as 3.1416, then the radius of the circle can easily be established using circumference = $2 \times \pi \times r$. Then radius = 3438”. 

*Mark Musgrave*
An observation can be made if existing sexagesimal theory is used to subdivide the Delta circle into \( 1.296 \times 10^6 \) smaller parts, or arc-seconds. This calculation produces an arc-second length of 1.667 Royal Cubit’s. There is no known use of such a length as some form of standard unit in Egypt or Sumeria, but such an inconvenient length would not be easily adopted by people who were seemingly in the process of developing a metrology system. The 1.667 RC length does however offer a path of reasoning to explain why multiples of 60 were a logical aspect of circle geometry, and ultimately led to the circumference being subdivided into 360°. Such a process obviously begins with the realisation that the circumference length of \( 2.160 \times 10^6 \) RC is divisible by quite a wide range of numbers, but that the end result also needed to be subdivisible by the same number, or numbers. In the end, the creation of \( 1.296 \times 10^6 \) seconds of arc presents a number which is \( 6 \times 60^3 \), while the circumference is \( 6 \times 60^2 \). This condition leads to the further observation that \( 1.667 \times 6 = 10 \), which possibly means that the original 1.667 RC unit was ‘bundled’ into six parts to create a 10 Royal Cubit length unit as the first basic building block for circle sub-division. This step would remove the 1.667 RC length from being a noticeable element in the geometry that followed, and allowed more convenient sub-division in multiples of 6 and 10.

It is interesting to note Joseph’s comment, that ‘dividing the circumference of a circle into \( 360 \times 60 = 21\,600 \) equal parts’ was involved, since there is no mention here of further subdividing the circle into \( 1.296 \times 10^6 \) parts for the arc-second level of angular measure. It may be that such a step was carried out but just not reported by Joseph. Alternatively, even if the original texts show no signs of this step, it still carries some significance that can be quantified. By applying the 1.667 unit length to the above situation, one can propose that 21600 parts of 1.667 units were actually involved, which results in a circle whose circumference is 3.6 \( \times 10^4 \) units. The radius is then 5.73 \( \times 10^3 \) units, and there are 3.438 \( \times 10^3 \) parts of the 1.667 unit length in this radius.

Another factor needs to be considered in the broader scheme of things as well. When subdividing a circle of specific circumference according to sexagesimal principles, the circumference length limits the practical ease of subdividing the circle into smaller and smaller sectors. Using the circle of radius 3438 units, applying the \( 360 \times 60 \) part sub-division means that 21600 ‘minutes’ of angle were created, each with the reasonably convenient length of 1 unit, or 1 Royal Cubit for example if this length measure is used. When the next level of sexagesimal subdivision is carried out however, the arcsecond length becomes \( 1.667 \times 10^2 \) units.
Depending on the actual length of the “unit” in this case, the arcsecond length may have been too small for practical applications.

The point of this comment is to reveal the fact that convenient successive sub-division of a circle by multiples of 60 depends on the actual dimension of the circle. If the circle is too small, then it might only be practical to subdivide it into 6 parts. When a circle of larger radius than 3438 units is involved, then sub-division by 360 x 60 x 60 = 1.296 x 10^6 parts can be made. The sexagesimal arrangement is progressive in the way it can be applied to circles of increasingly greater dimension. The Delta circle is an example of a circle that can be suitably divided up into 1.296 x 10^6 ‘seconds’ of measure. It is also obvious that the Delta circle radius is 100 times larger than the circle mentioned in most historical texts regarding the development of trigonometry.

Furthermore, the appearance of the 1.667 RC length in the Delta circle, as the ‘1 arc-second’, length raises another issue. If this length is considered to represent a ‘fundamental length unit’ for hypothetical purposes, then it allows the 21600 unit circumference to contain 1.296 x 10^4 such lengths. The larger Delta circle contains 1.296 x 10^6 such lengths. This situation means that we can add a new question to the list in our study of the origins to the sexagesimal system; “is recognition of this length unit possibly the major determinant in all circle sub-division theory”? If so it would imply that for any circle, the ‘arc-second’ length is fixed at 1.667 units, and the subsequent number of arc-seconds in the full circumference is reliant on this fact. Only when the circle circumference has a length as a factor of the number 216 units will the number of arc-second, or arc-minute, or degree sub-divisions be in true multiples of 6 and 36 according to sexagesimal theory. However, the number of 1.667 unit lengths that fit into any intermediate circle can be calculated, but may simply not be convenient integer numbers to work with. An example of this system at work will be given later.

The information presented in this section has only superficially discussed some aspects of potential evidence, from the geographical layout of Egypt, which appear to have some relevance towards explaining the origin of a specific reference circle adopted by ancient mathematicians during the creation of trigonometry. Although a case has been presented here which suggests that Egypt’s boundaries may have been specifically designed to permanently record the presence of such a circle, this author has not been able to research historical texts which might offer solid support for the proposal. Some historical scholars have however made extensive comment on the geometrical layout of Egypt. Stechini (In Tompkins, 1973) established that an advanced science of geography, based on accurate astronomical tables that were kept up-to-date all the way down to the
beginning of the 1st millennium BC, existed in ancient Egypt and Babylon. The later Babylonians still had excellent maps for their area of the world between the 30th and 36th degree latitudes, made in segments of 6˚ latitude by 7˚12’ longitude, because these dimensions produced perfect squares. Stecchini makes the comment that "Egypt is the country built according to a geometric plan."

Notwithstanding the possibility that the imaginary circle proposed in Diagram 3 may be considered just conjecture, the 2.8˚ degree longitudinal width still appears to carry a message regarding the early application of circle theory to measurement of the Earth’s surface. The main reason for this is because the two lines of longitude are Great Circles, and split the Earth’s circumference into 129.6 equal sectors. Secondly, there will be 20.63 such sectors in a 1 Radian sector of the Earth circumference, a similar relationship as that proposed to account for the characteristics of the Royal Cubit. An approximate calculation also shows that the length of 2.8˚ of longitude at the Earth’s equator will be virtually 6 x 10^4 Royal Cubit’s. An important fact should be recognised here, that a seemingly accurate positioning of two lines of longitude was completed at an unknown stage of history. In modern terms this implicitly requires the ability to accurately measure time.

In essence, the information presented above has only provided a possible source for the circle used by Alexandrian Greeks such as Hipparchus and Ptolemy. Since the boundaries of Egypt have apparently been in place from early dynastic times however, it would support the view that mathematical developments by Hipparchus and Ptolemy were based on existing knowledge in Egypt. However, the discussion has not yet given some plausible reason for the initial choice of a circle whose radial dimension is 3438 units (perhaps to a greater exponent), with a corresponding circumference of a factor of 21600 units. It is this condition that appears to determine the final application of sexagesimal mathematics to result in the 360˚ of sub-division, and is therefore the fundamental question to answer in explaining circle theory and the sexagesimal system.

4.3 The Royal Cubit Revisited

The unusual appearance of the number 20.63 in both the number of inches to 1 Royal Cubit and in the proposed electromagnetic clock standard, is a potential source of initial confusion. Although there is obviously not a direct relationship between these units of length, the fact that 1 Royal Cubit is subdivided into 20.63 parts (whose subsequent length is termed the inch) is a feature that can be studied within the broader picture of circle geometry and the sexagesimal system.
An immediate thought was that the choice of electromagnetic frequency immediately fixes the number of sub-divisions applicable to a 1 Radian sector, and the number of resulting sub-divisions produce a number similar to the electromagnetic wavelength. In the case of the circle we are all familiar with, a frequency of \( 2.778 \times 10^{11} \) Hz seems to allow the 1 Radian sector to be subdivided into \( 2.778^\circ \) sectors, creating 20.63 sub-divisions.

The logic by which the clock frequency and the speed of light are converted into degrees of angle within the circle context is not understood at this stage, but is considered to be a real possibility. As a result of the tentative evidence presented earlier, the following proposals are made:

- Circle theory is potentially based upon the physics of electromagnetic radiation, and recognises the character of an expanding spherical wave of radiation;

- The frequency of the source being studied, or being used to provide a reference standard for metrology purposes, determines the sub-division structure of the spherical wave. i.e. the 1 Radian angle is subdivided in proportion to the clock frequency.

- The 1 Radian angle is determined by the speed of light.

The ideas put forward here suggest that circle sub-division can in fact be modified to suit any frequency source. In this regard it should be easy to construct a modern circle by choosing a clock frequency already established for standard timekeeping in our atomic clock society, and following the pattern described above. However, it does appear that certain frequencies are more amenable to providing a user friendly end result when the proposed clock frequency is studied in more detail. Two basic pieces of information are immediately significant when the complete sexagesimal structure is considered:

Subdividing a 1 Radian arc into parts of \( 2.778^\circ \) creates 129.6 parts in total for the full circle circumference. This number is exactly sub-divisible by all numbers that are multiples of 6, and is most likely responsible for the existence of \( 1.296 \times 10^6 \) seconds of arc in the circle.

The reciprocal of the clock frequency is \( 3.6 \times 10^{-12} \) seconds. This means that each oscillation of the clock repeats after this interval of time. Alternatively, one can say that the wavelength occupies this time interval. Obviously any consideration of time within this system maintains mathematical simplicity with the numbers of the sexagesimal system, and would be considered a benefit for scientific and everyday purposes.
These electromagnetic features will now be used to study other aspects of evidence potentially available to us from archaeological research. First we will continue to examine the Royal Cubit system in more detail, and then consider other lines of evidence.

By applying the standard wavelength of $2.063 \times 10^{-3}$ RC to any unit of ancient measure we can determine how many wavelengths of the standard are involved, under the assumption that such a process may have accounted for development of measurement units in our historical past. The observed relationships between the proposed wavelength standard and the Royal Cubit, together with its sub-division into fingers and inches are summarised as follows: -

- There are 28 fingers to 1 Royal Cubit, or more likely 27.78 fingers. Metric length is approximately 0.524m.
- 1 finger = $3.6 \times 10^{-2}$ Royal Cubit’s.
- 1 finger of $3.6 \times 10^{-2}$ RC length will contain 17.5 wavelengths of $2.063 \times 10^{-3}$ RC.
- 1 inch has length $4.848 \times 10^{-2} RC$, which contains 23.5 wavelengths of $2.063 \times 10^{-3}$ RC.

Of interest is that 20.63 RC contains $1 \times 10^4$ wavelengths of $2.063 \times 10^{-3}$ RC. This may suggest that a length of 20.63RC was originally a primary standard, based on $1 \times 10^4$ wavelengths being chosen as a suitable multiple for a specific reason. This length is equivalent to 10.80 m. A note is made here that $1 \times 10^4$ seconds of arc exist in the angle of 2.778°.

If we consider a circle to have radius 20.63 inches (1 RC), the circumference will be 129.6 inches. This recalls the number 1296, and indicates some possible connection to the $1.296 \times 10^6$ arc second sub-divisions in the circle. Similarly, a circle of radius 20.63 Royal Cubit’s will have a circumference of 129.6 RC.

Of primary interest is the fact that the Royal Cubit design shown above contains all the components with which to calculate the speed of light. However, since the 28 fingers and 20.63 inches do not openly convey information that clearly shows the speed of light is confined in the whole system, we need to collect some more information to see if it has some logical support. An implication of this calculation is that the 28 fingers are in fact actually 27.78 fingers, but a rounding – off process was possibly applied for simplification, or more accurate measurement of the ancient units is required.
It is possible to gather some time interval data from the system, based on the fact that each wavelength of the standard radiation occupies the proposed interval of $3.6 \times 10^{-12}$ seconds. Each finger of 17.5 wavelengths would then represent the total time duration of $6.282 \times 10^{-11}$ seconds, and 28 fingers will represent $1.745 \times 10^{-9}$ seconds. This allows us to say that light travels the distance of 1 Royal Cubit (20.63 inches) in $1.745 \times 10^{-9}$ s, which provides a speed of $1.182 \times 10^{10}$ inches per second. Converting inches to Royal Cubits gives the normal value for the speed of light as $5.73 \times 10^8$ RC/s ($1.182 \times 10^{10} / 20.63 = 5.73 \times 10^8$). This would appear to support the proposal that the sub-division of the Royal Cubit into 28 fingers is quite possibly directly based on an electromagnetic standard, in a manner that visibly exposes the underlying relationships.

The information presented above significantly maintains constant reference to circle theory, by use of the number 1.745 (which refers to the ratio between circle radius and 1° arc length). This is also indicated by the number of wavelengths associated with each 1 finger of length, and the time light takes to travel the length of 1 Royal Cubit. Furthermore it is relevant to note the value for the reciprocal of the speed of light, $1/5.73 \times 10^8 = 1.745 \times 10^{-9}$s. We need to recall that the modern metre length is now defined as the distance that light will travel in the time of $1/2.998 \times 10^8$ m.s$^{-1} = 3.336 \times 10^{-9}$ seconds. The above evidence would tend to suggest that the 1 Royal Cubit length had a similar definition, albeit more elegantly stated within the context of circle theory. Such a situation supports the odd fact that in ancient Egypt, land surveying involved the use of circles as a primary tool.

The above data also shows that the speed of light can be expressed as approximately $12 \times 10^9$ inches.s$^{-1}$ ($11.82 \times 10^9$ inches.s$^{-1}$), and this value may have been the reason for the use of multiples of 12 in various measurement units. The foot unit is an obvious example. There is also the fact that 1 finger is equal to $3.6 \times 10^{-2}$ RC, reinforcing the presence of the number 36 in the system, and that $6.282 \times 10^{-11}$ seconds of time is associated with this unit ($6.282$ bearing reference to the $2 \times \pi$ ratio in circle theory).

A surprising amount of data appears to be available from the Royal Cubit measure provided that the electromagnetic interpretation is recognised within the structure, and suitable connections made. There is much more that can be gleaned from the system for discussion, but it is considered that enough has been presented in this section to make a point. The next section is potentially more important however with regard to discovering a design principle attached to circle theory.
5.0 Creation of Circle Geometry

All evidence presented in the main article allows a final simplified summary to be made, in which the elements of circle theory are more clearly outlined. This specifically identifies the primary logic associated with creation of $1.296 \times 10^6$ seconds of arc in the circle, and provides an explanation for the origin of 360°.

Diagram 4 presents what is considered to be the original reference circle used to create circle geometry. Reference must also be made again to Diagram 3 presented earlier in order to follow the sequence of logic involved.

Diagram 4: - The Primary Reference Circle; ‘1 second’ of Time

![Diagram 4: The Primary Reference Circle; ‘1 second’ of Time](image)

- Radius = $5.73 \times 10^8$ RC
- $T = 1$ s
- Circumference of the circle = $3.60 \times 10^9$ RC
- 1 Radian arc contains:
  - $2.063 \times 10^5$ arcseconds (each $2.778 \times 10^3$ RC)
  - or $3.438 \times 10^8$ arcseconds (each $1.667$ RC)
- $L = 5.73 \times 10^8$ RC
- 1 arcsecond length = $2.778 \times 10^3$ RC or
- 1 arcsecond length = $1.667$ RC
- = 34.38 inches

Note: The radius of $5.73 \times 10^8$ Royal Cubit’s contains $3.438 \times 10^8$ parts of the $1.667$ RC length unit.

As shown above in Diagram 4, it is the circle produced by an electromagnetic wave after a time interval of ‘1 second’ has elapsed that is proposed as the primary feature in the creation of circle geometry. The ‘1 second’ interval is furthermore defined as $2.778 \times 10^{11}$ oscillations of the associated source of radiation. Dimensions of this circle allow some interpretation to be made concerning the logic attached to arcsecond level sub-division of the circle.
Other features of Diagram 4 can now be discussed with regard to understanding the developmental process involved with circle theory. These are summarised as follows:

- The circumference of $3.6 \times 10^9$ RC is obviously easily subdivided into 360 parts, and one might suspect that this is a likely origin for the creation of $1^\circ$ of angle. However, the logic associated with creation of $2.063 \times 10^5$ seconds of angle in $57.3^\circ$ appears to represent a controlling influence.

- Because of the existence of $2.063 \times 10^5$ seconds of angle in 1 Radian, there exist $1.296 \times 10^6$ seconds in the full circle. ($2.063 \times 10^5 \times 6.282 = 1.296 \times 10^6$). The origin of this number of sub-divisions is therefore apparently tied to conditions recognised in the 1 Radian sector.

- An alternative origin for $1.296 \times 10^6$ seconds of sub-division has been recognised during the course of this study, and is specific to the ‘1 second’ electromagnetic condition, but retains its basic features when smaller circles are considered. This alternative requires that the 1 radian arc of length $5.73 \times 10^8$ RC is subdivided into $3.438 \times 10^8$ smaller divisions instead of $1.296 \times 10^6$, each with a length of $1.667$ RC. The total number of arcsecond divisions in this ‘1 second’ circle would therefore be $2.16 \times 10^9$, instead of the expected $1.296 \times 10^6$. Other lines of evidence lead to the suspicion that the creation of $1.296 \times 10^6$ seconds of arc is directly related to the $1.667$ RC length rather than anything else, and also has a bearing on the fundamental issue of this topic, the origin of numbers to base 60.

- When this ‘1 second’ primary reference circle is reduced to the size of the Delta circle (Diagram 5), the $1.667$ RC length remains as the ‘1 arcsecond’ unit, and this results in a total of $1.296 \times 10^6$ parts of $1.667$ RC in the smaller circle.

- The $2.160 \times 10^9$ seconds of arc now associated with the full circumference of the ‘1 second’ circle also bear relevance to the 21 600 unit circumference associated with the circle used to develop trigonometry.

It is obvious that the number of arcsecond sub-divisions in any circle can increase as the circle dimension increases, mainly because the circumference arc length can accommodate a greater number of sub-divisions. The Delta circle accommodates $2.063 \times 10^5$ parts of the $1.667$ RC unit per ‘1 Radian’ of arc, while the larger ‘1 second’ circle can accommodate $3.438 \times 10^8$ parts of $1.667$ RC per ‘1 Radian’. It is proposed that the $1.667$ RC length unit represents a significant determinant in the whole system, which resulted in circle sub-division as we know it.
Further explanation of the above information requires study of the second reference circle, considered to be that already presented in Diagram 3. It is repeated here without some of the extraneous details of Diagram 3, and compared with Diagram 4.

**Diagram 5: Primary and Secondary Reference Circles**

![Diagram 5: Primary and Secondary Reference Circles](image)

The following aspects of Diagram 5 are considered to be important indicators of the logic associated with developing circle mathematics, and resulting in sexagesimal number theory:

- Based on knowledge of the speed of light and electromagnetic wave properties, a reference circle is defined in terms of a time definition. These conditions demand that the ability to accurately determine a fixed duration of time be available.
The time definition, for 1 second, is a fundamental constant for the system. It is suggested that $2.778 \times 10^{11}$ oscillations of a source with a wavelength of $2.063 \times 10^{-3}$ Royal Cubit’s constituted the ‘1 second’ duration.

This frequency controls sub-division of the 1 Radian sector first into 20.63 parts, each with length $2.778 \times 10^7$ RC. This length presents difficulty with subdivision into smaller units however, that will conveniently be accommodated by the existing dominance of numbers as multiples of 6. The sub-division can only proceed if a unit length of 1.667 RC is involved, since there will then be $1.667 \times 10^6$ such units within the 2.778° sector. This step results in the creation of $3.438 \times 10^8$ parts of the 1.667 RC length within the 1 Radian sector of the ‘1 second’ circle.

The final observation to be made from the information presented in Diagram 5 is that the 1.667 RC length is an inconvenient practical measurement unit itself. It is immediately obvious that multiplication of this unit by 6 will produce a convenient unit of 10 RC. This step is proposed to account for the appearance of multiples of 6 and 10 as the most suitable method for subdividing the circle into angular units that resulted in development of the sexagesimal structure.

The emergence of 60 seconds to 1 minute of arc, and 60 minutes to 1° of arc therefore came into being, primarily as a result of applications in electromagnetic wave propagation. The logic is explained as follows, using the Delta circle reference:

a) In the 1 Radian arc of $3.438 \times 10^5$ RC, there exist $2.063 \times 10^5$ parts of 1.667 RC.

b) Taking 6 multiples of 1.667 RC to form a new length unit of 10 RC, we create $3.438 \times 10^4$ sub-divisions.

c) Now 6 multiples of the 10 RC unit is considered, and $5.73 \times 10^3$ sub-divisions each of 60 RC have been created. This step noticeably reproduces a number in proportion to the speed of light, and there are 36 of the 1.667 RC lengths in 60 RC.

d) The above steps result in a 1° arc containing 100 of the 60 RC length units, since the 1° arc has a length of 6000 RC in the first place. This fact has not been made obvious before. It is also noted here that a decimal application to arc length subdivision may indeed have been made as well.
e) Since there are 36 of the 1.667 RC units in 60 RC, the 1° arc then contains 3600 such units, or 'arcseconds'.

There are many small details that have not been presented to more clearly outline the development of circle theory and the sexagesimal number system, and there are many questions that can be raised as well. However, the author believes that the following points can be made to summarise the whole discussion:

5.1 Summary

Evidence suggests that circle geometry was based on principles of electromagnetic physics, during which process the numbers typified by the sexagesimal system were produced. In this case the chosen clock frequency of $2.778 \times 10^{11}$ Hz results in a reciprocal (time duration) of $3.6 \times 10^{-12}$ s, apparently in order to enable time calculations to merge seamlessly with all other features of the system.

The ‘1 arcsecond’ unit of circle sub-division is defined as the length of 1.667 Royal Cubit’s when a reference circle with a radius of $3.438 \times 10^{8}$ units, each of length 1.667 RC, is considered. This circle has an effective radius of $5.73 \times 10^{8}$ RC, and therefore exists at the ‘1 second’ time duration. One can also observe that if the historical record of 3438 units is taken to mean that the unit itself has a length of 1.667 Royal Cubit’s, then the circle has a radius of $5.73 \times 10^{3}$ RC.

Application of this “fundamental” length unit in multiples of 6 allows the circle circumference to be conveniently measured in lengths of 10 RC, 60 RC or 6 RC, instead of an inconvenient 1.667 RC length unit.

The sexagesimal numbers associated with normal arithmetical applications are proposed to be derived from the sub-division of the circle, not visa versa. The base-60 system is furthermore specifically designed to work with spherical geometry problems within which the measurement of time is also a factor.

6.0 The Origin of 24 Hours in One Day of Earth Rotation

Using an electromagnetic source oscillating at $2.778 \times 10^{11}$ s$^{-1}$, such a clock would have produced $2.40 \times 10^{16}$ oscillations during the course of a full day of rotation. Dividing this number by its own integer value of 24 will provide the 24 intervals, each of duration $1 \times 10^{15}$ oscillations of the clock. As a result, $3.6 \times 10^{3}$ seconds must be associated with $1 \times 10^{15}$ counts of the clock, derived from 86400 seconds divided by 24. From the fact that each oscillation of the clock occupies a time interval of $3.6 \times 10^{-12}$ seconds (the reciprocal of clock
frequency), then $1 \times 10^{15}$ oscillations produce $3.6 \times 10^3$ seconds of time (per 1 - hour interval) as required. These figures show that direct application of the clock frequency proposed to lie at the heart of sexagesimal theory can immediately account for the observed features of historical astronomy and mathematics, both of which continue to be used in modern society, in a format unchanged from the original. The only difference with modern timekeeping is that we use a different source frequency, and we don’t use the same electromagnetic standard for both time measurement and a length standard.

7.0 The Alternative Explanation for Development of Circle Geometry

A relatively superficial examination of literature dedicated to the Great Pyramid will reveal that some scholars have noticed that the height of the pyramid is in proportion to the distance between the Earth and the Sun. The average orbit radius for the Earth is approximately $2.8 \times 10^{11}$ Royal Cubit’s, a factor of $1 \times 10^9$ greater than the height of the Great Pyramid. The odd coincidence is that this value is also almost exactly that proposed for the clock frequency in the main discussion.

Modern measurement of the Earth orbit radius provides a distance of $1.496 \times 10^{11}$ m, which converts to $2.857 \times 10^{11}$ Royal Cubit’s. Since the Earth orbit path may change over time, this author decided to apply some of the evidence already presented in this article, to a picture in which Earth orbit conditions potentially determined circle theory. Such a step will either help reinforce the option that a 360-day orbit once existed for the Earth, leading to the 360˚ circle, or provide further evidence to refute it. In this regard the value of $2.778 \times 10^{11}$ RC was taken to represent a possible Earth orbit radius distance that may have existed at some remote time in history. The rotation time of 86400 seconds is assumed to remain constant, because the actual duration of ‘1 second’ would have been shorter in historical times. Calculations based on these parameters, and assuming an essentially circular orbit for simplicity, reveal the following:

- Orbit circumference = $1.745 \times 10^{12}$ RC.

- Assuming exactly 360 days of orbit are involved, then each day of orbit is associated with a distance of $4.848 \times 10^9$ RC. The Earth must travel at an average velocity of $5.611 \times 10^4$ RC.s$^{-1}$ in order to achieve this. This velocity converts to $2.9379 \times 10^4$ m.s$^{-1}$, slightly faster than modern orbital velocity, and satisfies conservation of angular momentum requirements.
The interesting aspect of the above calculations is that an orbital velocity much closer to an exact 1:1 \( x 10^4 \) relationship to the speed of light is found.

Another interesting observation is that the ‘1 inch’ length can be expressed as \( 4.848 \times 10^{-2} \) RC, while 1 day of Earth orbit (equivalent to 1˚ of orbit in this case) covers the distance of \( 4.848 \times 10^9 \) RC.

The time for an electromagnetic signal to travel the distance between the Earth and the Sun would be 484.817 seconds.

The orbit circumference of \( 1.745 \times 10^{12} \) RC converts to \( 3.6 \times 10^{13} \) inches.

Without presenting any further calculations along these lines, one can already observe that relationships similar to those discussed for development of circle theory are involved. The use of \( 2.778 \times 10^{11} \) RC as an orbit radius, and the assumed 360-day orbit period, create almost identical end results. The question that must therefore be asked is: “can measurement of the Earth orbital parameters be made without first having measurement standards in place to measure the Earth itself, and calculate velocity”?

The answer to this question must surely be no. An observer cannot declare that he knows the distance to the Sun when he has no reference length to refer to. Similarly, measurement of velocity requires the definition of a time unit, and the ability to accurately measure time. These systems must already have been in place before the Solar System characteristics could be measured. The angular measurement system afforded by circle mathematics is also an essential tool in making the above observations and measurements. Furthermore, modern research has revealed that the Earth year consisting of 365.25 days has not changed over millions of years. Since rotation rate has slowed however during this expanse of time, the orbit radius must have increased to accommodate conservation of energy laws. This would allow one to propose that at some earlier time the Earth orbit radius may have been approximately \( 2.8 \times 10^{11} \) RC.

8.0: Author’s Comments on Various Aspects of the Study

8.1 Creating a Metric Circle

It would appear that the evidence presented in this study offers the opportunity to design a metric decimalised circle, by following the steps that have been proposed to explain development of the original circle. However, a single obstacle exists in this respect, and is presented by the appearance of the 1.667 Royal Cubit length in original data. If this unit is directly related to a feature of electromagnetic physics that has not been identified
by this study, then there is no scope for preventing this feature from exerting its influence in any attempt to modify circle theory. As attested by history, all attempts to decimalise the circle have failed, and this should send a strong message to all people involved with mathematics and international standards. Circle theory potentially contains a fundamental basis in electromagnetic physics, and as such represents the simplest possible application of mathematics to work with. Deeper understanding of circle theory could therefore also lead to a better understanding of some aspects of physics.

By following identified design steps with existing circle theory the modern equivalent would have a circumference of $1.131 \times 10^6$ m, (based on a radius of $2.998 \times 10^8$ m for 1 second of light travel) an inconvenient length for decimal sub-division to begin with. Furthermore, the ‘1 Radian’ angle would now be more correctly described as 30˚ as a function of the speed of light, resulting in the effective creation of 188.5˚ of sub-division to the circle. From such considerations there is little chance of manipulating the system to become decimally friendly. This was the obvious conclusion already in ancient society, since a decimal number system existed in tandem with the sexagesimal one. This piece of evidence too should be credited with much more significance in the historical framework.

8.2 Inches, Cubits and Geometry

From the evidence presented in this study it would appear that development of the $\text{metre}$ length unit has driven society away from ancient units of measure that are potentially better defined in a broader framework. The ancient framework appears to have created units of measure within the context of circle geometry and time measurement, all held together by sexagesimal theory. The metric system does not manage to integrate geometry and time measurement with the $\text{metre}$ length unit in a manner that produces a common numerical relationship between all three systems.

For those societies in modern times who still retain use of the $\text{inch}$ unit and its associated lengths, there would appear to be good reason for maintaining the original $\text{inch}$ and $\text{royal cubit}$ as primary standards based on an electromagnetic source. Such a standard should produce a wavelength of $1.08 \times 10^{-3}$ m ($2.063 \times 10^{-3}$ RC), and would place length units in their proper perspective with regard to circle theory.

9.0 Consideration of pi and the Electromagnetic Wave

The physical efforts carried out by mathematicians over a long period of history have provided an extremely accurate value for pi, the ratio between circle circumference and radius. These efforts have apparently not
provided an explanation as to the possible reason for the existence of this specific ratio however. It may seem to be a completely irrelevant question, but there should be a fundamental reason for this condition.

A circle drawn on paper is the common reference we have when dealing with the above question, and all calculations prove that the pi value is 3.14159 to five decimal places. What happens though when we deal with the radius of an electromagnetic wave? In this regard a wavelength of radiation is involved and it has a known physical length, which means that at the instant of measuring a specific radial dimension the circumference of the electromagnetic circle has a defined thickness of one wavelength. Do we calculate the radius of the circle produced by this electromagnetic wave as the distance to the leading edge of the wave, the trailing edge of the wave, or the middle of the wave?

For a single oscillation of an electromagnetic source only a single wavelength has propagated away from the atom. Using the source discussed in this article, the wave will have achieved a radial length of $2.063 \times 10^{-3} \text{RC}$, and at this point we freeze the wave to observe its characteristics. What is found is the following:

- The circumference of the circle defined by the leading edge of the wave is $1.296 \times 10^{-2} \text{RC}$. The number of full wavelengths that can fit into this circumference is 6.283.

- The circle defined by the centre of the wave has a radius of $1.032 \times 10^{-3} \text{RC}$, and a circumference of $6.484 \times 10^{-3} \text{RC}$. The number of full wavelengths that can fit into this circumference is 3.143.

Although the above calculations appear to be telling us something significant concerning the origin of pi, there is a problem that must be taken into account. It is the fact that the circle circumference under consideration should be such that it can only accommodate integral numbers of the full wavelength, while we observe the radial dimension. For this we can simply consider the case where an electromagnetic wave has a circumference that can exactly accommodate two wavelengths of radiation, and then we observe the following:

- The thickness of the circumference ‘wall’ is $2.063 \times 10^{-3} \text{RC}$, and its length is $4.126 \times 10^{-3} \text{RC}$.

- Radius is theoretically defined by the time that light will travel radially during the time in which the above circumference can form, but here we cannot say that two oscillations have taken place. We have to consider the possibilities that the radius is either to the outside edge of the wave, or the middle, or the
trailing edge. For each of these we can then calculate the required time of light travel associated with development of the wave circumference.

- Applying the typical ratio of 6.283 to the above circle, we get $R = 6.5667 \times 10^{-4}$ RC. At this point we do not know whether this value represents the distance to the outside edge, centre, or trailing edge of the system. We can immediately see however that the radius is smaller than a single wavelength of the radiation, which means that only a fraction of one oscillation has been allowed to occur at the atom. It would appear reasonable to infer from this condition that measurement of the system requires us to at least allow the atom to produce a half wavelength or a full wavelength before we attempt to measure it. This tends to suggest that the points made earlier are potentially correct, in that a minimal radial requirement must be met in order to define circle structure. This results in a circumference containing 3.14 wavelengths when measured along the middle of the wave thickness, and 6.283 wavelengths along its leading edge.

- The general observation is that the circumference defined along the middle of the wave is $\pi \times$ wavelength, while the outer edge of the wave has circumference equal to $2 \times \pi \times$ wavelength. This shows that the $\pi$ value of 3.14 is related to the measurement of a half-wavelength of the radiation source, while $2 \times \pi$ (6.283) relates to consideration of the full wavelength.

The above reasoning suggests that circle theory in its general form provides the maximum radius and circumference of a defined circle, and does not take into account the thickness of the line involved. When measuring single wavelengths of electromagnetic radiation however, the mathematics is telling us that $\pi$ is related to measurement of conditions when exactly a half-wavelength is available for measurement, and then only integral multiples of the half-wavelength.
References


5. Stecchini (In Tompkins, 1973)


Appendix A

Equations used to calculate surface distances across the Earth, for the Delta circle.

The Clark geoid is used as the reference frame for basic Earth dimensions.

Earth equatorial radius = 6.378 x 10^6 m or 1.218 x 10^7 Royal Cubit’s

Earth equatorial circumference = 4.0075 x 10^7 m or 7.653 x 10^7 Royal Cubit’s

Arc AB = [(Longitude 1 – Longitude 2) / 360] x 2pi x R

R = r cos Latitude where r = Earth equatorial radius

The above equations were used to calculate the 2.8° of longitude surface distance along the latitude of 31° 06’ N, in Egypt, as follows:

Point A = Longitude 32° 38’ East

Point B = Longitude 29° 51’ East   A –B = 2.778° or 2° 46’ 41”

Cosine of 31° 06’ = 0.856267

R = 1.218 x 10^7 RC x 0.856267 = 1.0429 x 10^7 RC

Arc AB = [2.778° / 360] x 6.283 x 1.0429 x 10^7

= 5.0565 x 10^5 Royal Cubit’s