Institute for Advanced Study/Park City Mathematics Institute
International Seminar: Bridging Policy and Practice in the Context of Reasoning and Proof
3-8 July 2006

Statements on:

- Problem-Solving in the School Mathematics Curriculum
- Preparation of Teachers for Teaching Problem Solving, Reasoning and Proof
- Conditions for Teachers to Engage in Problem Solving and Reasoning in Their Classrooms

Bridging Policy and Practice in the Context of Problem Solving, Reasoning and Proof

Introduction

To explore common issues and concerns in mathematics education, the Park City Mathematics Institute International Seminar: Bridging Policy and Practice brought together groups of mathematics educators from across the world during the summers of 2001, 2002, 2003, 2005, and 2006. These seminars, sponsored by the Institute for Advanced Study and funded by the Wolfenson Family Foundation and the Bristol-Myers Squibb Foundation, allow teams of two educators—a university mathematics educator or policy-maker and a secondary teacher—from up to eight nations to engage in a stimulating five-day discussion about issues in mathematics education with respect to policy and practice in their respective countries. The goals for the international seminars are to:

- promote open discussion of the goals, content, and delivery of pre-service and in-service education for mathematics teachers, as well as the policies that govern these in each nation,
- identify common issues faced across national contexts, and
- identify teacher preparation and development programs and practices that work well in a particular nation and that may suggest ideas that can be adapted by others.

Reports and proceedings from the international seminars are available online at http://mathforum.org/pcmi/.

The 2006 seminar, led by Michéle Artigue, University of Paris, and Gail Burrill, Michigan State University, was organized to stimulate conversation and productive exchange of information that could serve as a basis for continued efforts to address issues in mathematics teacher preparation and development related to the teaching and learning of problem solving, reasoning and proof. The nations represented in this seminar were Cameroon, Germany, Mexico, Pakistan, Poland, Singapore, Uganda, and the United States (See Appendix A for a list of participants). Participants used the framework of problem solving, reasoning and proof to shape the discussion about the mathematical knowledge teachers need to teach well and how they can obtain this knowledge (See Appendix B for the agenda). The discussion centered on the promises and challenges in the answers to the following questions: How is problem solving integrated into the secondary mathematics curriculum and how are teachers prepared to teach problem solving in Cameroon and Singapore? What mathematical content knowledge is necessary for secondary teachers to teach problem solving in Mexico? Has the vision of problem solving changed in the United States from the past? If the vision has changed, how has it changed? How are reasoning and proof integrated into the secondary curriculum and how are teachers prepared to teach reasoning and proof in Ugandan and German schools How are reasoning and proof integrated in secondary schools and how are these topics approached in secondary teacher preparation in Pakistani schools? What research findings or current practices would be helpful in teaching reasoning and proof in Poland?

The three policy briefs contained in this report emerged during the discussion about how to make problem solving, reasoning and proof integral to mathematics education across the international community. The briefs are designed to be useful to those involved in mathematics education as starting points for their own work to bring changes in both policy and practice that will result in improved understanding of quality mathematics education for all students in any country. The nature and features of each nation's policies and practices were filtered through the experiences of the individual members of the two-person teams. Because the team members were not official representatives of their nations, the views expressed by the members of these teams and the statements that were produced are not intended to reflect the status of mathematics teacher preparation and professional development in any nation, nor the views of the Institute of Advanced Study/Park City Mathematics Institute.

Special thanks go to Herbert Clemens for his vision in pursuing the creation and continuation of the International Seminar, to Catherine Giesbrecht for her support and efforts to integrate the Seminar into the larger PCMI operations, to Johnny Lott for his work in contacting and facilitating travel for the participants and helping organize the seminar, to Glenda Breaux for her note taking and support in producing this document, and to Abbi Stewart for making the necessary arrangements to run the Seminar, for her extraordinary care and concern for each participant, and for making sure the Seminar ran smoothly.

Problem-Solving in the School Mathematics Curriculum

Why use problem solving?

Many reasons for including problem solving in the school mathematics curriculum have been discussed in many places (e.g., Cockcroft, 1982; NCTM, 2000). Essentially, problem solving is a natural process in the development and application of mathematical knowledge, which stimulates and develops critical thinking and, in particular, higher order thinking skills such as analysis, synthesis, evaluation and judgement.

Problem solving can be used to motivate students by providing them with interesting situations that represent a challenge and that often relate mathematics to real-life situations. It offers the opportunity to "use knowledge meaningfully" (Marzano and Pickering, 1997, pp. 1-6). Problem-solving can also be used effectively as a vehicle for introducing new mathematical knowledge to students by creating a need for that knowledge. Its use in this context can allow students to acquire a deeper conceptual understanding of the topic through their active participation in building their understanding.

The use of problem-solving can place both teachers and students in a position of uncertainty, often accompanied by a degree of anxiety, which if appropriately handled can stimulate effective learning and foster creative thinking.

Problem solving breaks the paradigm of thinking about mathematics as a "magic box of tricks" to be applied in a mechanical fashion. Students should be encouraged to reflect on the strategies and tools that could be used before embarking on the process; it is important to avoid the danger of replacing one set of tricks with another. Problem solving forces the student to become an active participant in the learning process and can make mathematics more accessible to students by allowing them to take part in the construction of their own knowledge.

What is problem solving? (in the context of mathematics education)

> The resolution of any task for which the student does not have an immediate method available and which may lead to the development of new mathematical content or processes through a non-linear approach.

This definition is clearly dependent on context. Depending on the experience and mathematical knowledge of individual students, the same problem may represent different challenges for each student.

The purpose of problem solving

- To develop a new piece of mathematics

In this case the teacher retains a high degree of control over the process in order to ensure that students reach the desired end point. These problems may be simple in nature but must be very carefully selected in order to ensure that the fundamental mathematical idea emerges.

- To apply mathematics in a previously unknown way

In this case the teacher can relinquish much more control to the student, giving them flexibility and ownership of the process.

  - Focus on process

    In some problems the end result in itself is not as important as the process required to obtain it. This does not mean that incorrect solutions are acceptable but rather that the purpose of the problem is primarily to encourage reflection on the strategies used.

  - Focus on result

    It is important for students to experience the application of mathematics to the real world, and in this case it is necessary to evaluate the solution in the real context to see whether it is viable.

- To foster the social, emotional, personal and intellectual development of students

The nature of problem solving

Problem solving implies some or all of the following: exploration, experimentation, collaboration, communication and perseverance, within a systematic framework (plan) that is subject to constant revision and modification. The role of previous knowledge and experience is important, and the use of problems that are similar in nature to previously seen work allows students to draw critically and selectively on their mathematical knowledge, thus establishing connections and relationships.

Characteristics inherent to the nature of problem solving, which contrast with passive learning, are the non-linearity and unpredictability of the process. These might cause some students and teachers to resist the introduction of problem solving in the mathematics classroom. Further contrasts are found in the variety of approaches that students can employ (Burton, 2002) and the multiplicity of methods of solution that are often available for a given problem.

The experiences of seminar participants suggest that the uncertainty implicit in problem solving is an obstacle to its widespread use. Vygotsky's theory of the zone of proximal development (Vygotsky, 1962) suggests that appropriate scaffolding is necessary for students to feel able to approach the problem but that in excess it can destroy the problem by taking the challenge of looking for a suitable strategy away from the student. Since this zone depends on the individual student, there is clearly an enormous challenge for the teacher in providing the right amount of support for each student. The challenge of providing support for students in the learning process without overly scaffolding has emerged in recent studies as a barrier to real student learning (Stein et al, 2000; Stigler & Hiebert, 2004). Fragile teacher knowledge is also recognised as a major obstacle (compounding the uncertainty element in the teacher). This issue is discussed later in this report (see the brief titles *Conditions for Problem-solving, Reasoning and Proof*.

The process of problem solving

The process of solving mathematical problems involves:

- Understanding and defining the problem
- Exploring the problem
- Hypothesising
- Testing
- Formalising
- Reflecting/evaluating

It is important to note that these stages are not hierarchical in nature but rather form part of a complex cyclic process. In many instances of general practice the stages involving understanding the problem and reflection on the process tend to be weak, with a trivialising of these stages being common. Ways of strengthening and deepening the reflection process could be the use of problems with more than one solution and an emphasis on reflecting on the process, as well as on the result, with a view to generalising the learning that has taken place. Teacher scaffolding often cuts exploration short, and hypothesising was also considered to be a weakness in some countries.

Recommendations

- Look for ways to use problems, where possible and appropriate, to introduce new mathematics.
- Encourage the problem-solving approach as a way of approaching mathematics and not as a topic to be covered in isolation.
- Recognise and provide for well-designed and appropriate resources for the introduction of new subject knowledge through problem solving.

References

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Preparation of Teachers for Teaching Problem Solving, Reasoning and Proof

Mathematicians and mathematics educators generally agree that the mathematical processes of problem solving, reasoning, and proof are integral to the teaching and learning of mathematics. As such, the preparation required of teachers to teach these processes is not trivial. Most countries have some type of teacher preparation program; however, most of these programs do not explicitly prepare teachers to teach problem solving, reasoning, or proof. Furthermore, practicing teachers may not possess the skills, knowledge, or attitudes to effectively teach these processes.

In an attempt to bridge practice and policy, participants of the Park City Mathematics Institute's International Seminar offer characteristics as well as knowledge teachers must have so they are prepared to plan and teach students how to problem solve, reason, and prove in a mathematics classroom.

A. Necessary Components of Teacher Education

1) Knowledge

To help illustrate each knowledge type identified below, an example has been included at the end of this section.

i) Mathematical Content Knowledge

Recognizing that grade levels vary around the world, the divisions are made for discussion purposes only. The content courses reflect what may be the minimum requirement in some countries and the maximum content in others. The content knowledge presented is meant to frame general guidelines to help identify what mathematical content knowledge is required for teachers to be best prepared to teach problem solving, reasoning, and proof in the mathematics classroom.

Primary (grades 1-5) Content Knowledge
Teachers of primary students should possess a profound understanding of school mathematics up to but not necessarily including calculus from a higher point of view, with an emphasis on foundations of mathematics, arithmetic, geometry, elementary probability, and statistics.

Lower Secondary (grades 6-8) Content Knowledge
Teachers of lower secondary students should possess all the primary content knowledge indicated above, plus algebra, calculus in one dimension, advanced geometry, and trigonometry.

Upper Secondary (grades 9-12) Content Knowledge
Teachers of upper secondary students should possess all the lower secondary content knowledge indicated above, plus calculus in several dimensions, differential equations, further knowledge of probability and statistics, linear algebra, and matrices.

ii) Pedagogical Knowledge

Teachers at all levels should have knowledge of child development, general knowledge about teaching methods, educational theories, knowledge of diversity (e.g. language, gender, ethnicity), and theories of learning. Knowledge of child development theories is necessary across all educational levels.

iii) Didactical Knowledge (Mathematical Pedagogical Knowledge)

To teach problem solving, reasoning, and proof, teachers must possess subject related didactical knowledge (e.g., knowledge about teaching of geometry). An understanding of psychological theories about teaching mathematics (e.g., constructivism, social cultural and social critical theories as they relate to the teaching of mathematics) and knowledge about the processes involved in how students think about problem solving, reasoning, and proof are also required. Teachers must also have the knowledge of teaching methods to be able to facilitate the teaching of problem solving, for example, be able to apply Polya's principles for problem solving (1957). To effectively teach proof, it is critical that teachers know what constitutes a valid proof at appropriate grade levels for their students. Furthermore, teachers must also be aware and knowledgeable about the various levels of proof, ranging from pre-formal to formal, and when and how each level can be used adequately. Lastly, teachers must recognize and honor the fact that students interact in different ways with different mathematical representations (visual, symbolic, numeric).

Technology plays a crucial role in the learning of mathematics, especially as it relates to problem solving. Teachers should have proficiency with various kinds of mathematical software (e.g., dynamical geometry software, computer algebra systems, spreadsheets), and they should be able to use the technology in a variety of mathematical contexts. For example, technology could be used as a tool for visualization, modeling, or as means to support discovery learning.

Following is an example to help clarify the knowledge types identified above.

Example:

"Explain why the sum of two even numbers is even. "

<u>Mathematical Knowledge</u>: One component of the mathematical knowledge required for this problem is an understanding of foundations of number properties and the knowledge of what constitutes proof for the given example. This knowledge would also include recognition that different definitions of even are possible (pairing without a remainder, have a factor of 2, 2n for any whole number n).

Pedagogical Knowledge: Teachers would need to know effective classroom management, learning methods, and ways of thinking. When a teacher presents students with the problem, a pedagogically knowledgeable teacher will not merely demonstrate a solution but might instead design an activity using cubes and guided questions to help students "discover" why the sum of two even numbers is even.

Didactical Knowledge: Teaching might involve the decision to use concrete manipulatives to help students understand the problem or to use the number line as a tool. Teachers should recognize and connect multiple solution methods and representations. Teachers also need to know what constitutes a valid proof at a level appropriate for their students in order to be able to react adequately to different proposals developed by the students or in order to facilitate different proposals by the students. A teacher's didactical knowledge would also be shown in her ability to connect an explanation of pairing cubes to the demonstration that the sum of two even numbers is even with a more abstract explanation of $2n+2n=4n$.

2) Beliefs, Attitudes, and Values

Teachers hold beliefs about both the subject of mathematics and the teaching of mathematics. Some teachers believe mathematics is about discovering and experimenting. Others believe that mathematics is about proving and deducing mathematical truths. Both perspectives together comprise the whole of mathematics, and teachers need to be aware of both faces and not reduce mathematics to just one or the other.

When it comes to beliefs about the teaching of mathematics, teachers must have an open mind about both the teaching and learning of mathematics. That is, learning and teaching of mathematics does not happen at only an abstract level. Learning is a multi-dimensional process. It is important for teachers to hold the belief that teachers and students learn from each other. Furthermore, teachers must recognize the inherent value of problem solving, reasoning, and proof as part of the learning process. Finally, to teach these processes, a teacher must recognize that modeling and discovery learning are important approaches in mathematics teaching and that mathematics should be taught with an emphasis on using activity-oriented approaches.

Teachers should be able to reflect critically on both the mathematics they teach as well as the teaching of that mathematics.

3) Volition

The effective instruction of problem solving, reasoning, and proof requires that teachers possess a willingness and desire to put into practice the above beliefs and knowledge. In order to do that, teachers must be able to connect the different kinds of knowledge (mathematical knowledge, pedagogical knowledge, and didactical knowledge) by using higher order skills, such as critical thinking, creative thinking, and self-regulated thinking (sometimes referred to as meta-cognitive competencies or proto-competencies). They must also be willing to change their beliefs and practice to reflect best practice as revealed by current research. That is, they must be dedicated to the concept of themselves as life long learners. For the sake of student learning, it is imperative that teachers encourage and acknowledge different student solutions as well as different approaches to proving mathematical claims.

B. How do we prepare teachers to teach problem solving, and reasoning and proof?

The preparation of teachers of mathematics is always of significant concern. However, to teach problem solving, reasoning, and proof, particular attention must be paid to teacher training. Teachers of mathematics are typically given opportunities to learn the mathematics but not necessarily in a problem-solving environment. Therefore, it is important that teachers have the opportunity to experience problem solving, reasoning, and proof first hand from the perspective of a learner.

i) Pre-service Teachers

Future teachers need the opportunity to experience problem solving, reasoning, and proof with close connection to school practice. Ample opportunities for work in actual classrooms are required to help pre-service teachers gain experience in transferring mathematical knowledge about problem solving into the classroom setting. Additionally, mathematical knowledge, pedagogical knowledge, and didactical knowledge should be a part of teacher education.

ii) In-service Teachers

Reflection is critical to the professional growth of a teacher. Therefore, practicing teachers should be provided the opportunity to reflect on their own teaching, preferably with others, concerning problem solving, reasoning, and proof.

Specifically, teachers need training and experience in problem development, reasoning, and proof according to their educational level and mathematical content knowledge. One effective way to help practicing teachers develop these skills is to create teacher collectives. These are groups of teachers, either in the same school or in nearby areas, that regularly come together to solve problems, share ideas, discuss pedagogy, plan lessons, and reflect collaboratively about teaching and learning. Other ways to help develop knowledge and skills in problem solving, reasoning, and proof could be the use of peer observation and team teaching. Mentoring and other support measures should be established on a broad basis. Lastly, school-based working groups can be used to provide in-service training for teachers related to these areas in the mathematics curriculum.

We have attempted to provide a framework for preparing teachers to teach problem solving, reasoning, and proof. We recognize that there are many constraints and issues that differ from country to country, but the beliefs, knowledge, and training we have identified are critical to properly prepare teachers to be confident and successful in the classroom.

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Conditions for Teachers to Engage in Problem Solving and Reasoning in Their Classrooms

Problem solving and reasoning are crucial in developing mathematics skills and competencies. Conditions affecting the teaching and learning of problem solving and reasoning include the following:

Assessments — Since problem solving and reasoning are central to mathematics learning, we recommend they be assessed in both internal and external examinations. Tests may be expanded to include problems that are non-routine, open-ended and/or from real life situations. While tests on formal proofs may be limited to the advanced courses, tests on pre-formal and content-related reasoning should be tested at all levels. Rubrics should be developed and used for grading problem solving, reasoning and proof to ensure reliability.

Class size — Problem solving and reasoning should be taught regardless of class size. In the case of large classes,

- Students could be divided into smaller groups with group leaders helping their peers. The teacher should ensure that leaders are well motivated and trained in how to manage mathematical content. Student leaders should also have some techniques for working with groups. The teacher should be prepared to monitor what happens within all the groups.
- Giving long-term assignments will allow students to explore and reflect on possible solutions. This may also help with time issues mentioned later.

Teacher training — Pedagogical training must include the strategies involved in problem solving and reasoning. The use of technology, techniques of assessment in the classroom, some theoretical background, and sharing and learning from each other are important. A mentoring program, in-service training, and participation in seminars and conferences should also be part of helping teachers learn to use problem solving in their classrooms.

Cultural norms — Different cultures and countries may have different approaches to problem solving, reasoning and proof. These approaches could affect how teachers and students study and learn mathematics. Examples include the following:

- Peer pressure may discourage certain groups of students (females or very bright students) from engaging in these mathematical activities.
- Less motivated teachers may discourage other teachers from the creativity and inventiveness needed to try different approaches and methods to engage in problem solving, reasoning and proof.
- Traditions of memorization, copying notes and exercises may work against the inclusion of more open-ended problem solving and reasoning.
- In-grained teacher or cultural beliefs that all students cannot engage in problem solving, reasoning and proof activities may be a detriment to student learning.

Despite the potential barriers, we strongly recommend that learning through problem solving, reasoning and proof be part of the curriculum for all students. The recommendation is made knowing that change does not occur rapidly, and new ideas and approaches may have to be introduced gradually. One source of advice regarding effective approaches to introducing new ideas is seen in the *Adapting/Adopting Best Practices*, a brief produced during the PCMI International Seminar meeting in 2005. Another possibility is visiting and learning from other countries.

Support for change — Changes can take place and take place rapidly if there is general support for the change. Hence it is essential for teachers to work to ensure that authorities in schools, parents and others concerned about education understand the need for problem solving, reasoning and proof in mathematics.

Syllabus — Problems solving must be an explicit part of the national curriculum or standards if it is to be valued and taught in the classroom. Problem solving must also be valued at the college and university levels, and creativity and problem solving must be incorporated in the entrance examinations to reflect that importance.

Time — Teachers should be aware that problem solving and reasoning are time consuming. Therefore, we recommend that some topics in the mathematics curriculum be reduced in order to give space for student exploration ,and that efforts are made to design the development of mathematical concepts in ways that involve problem solving, reasoning and proof. This will affect teachers' planning and preparation for problem solving, reasoning and proof. A classroom teacher needs some flexibility in the allocation of time to teach within a classroom depending on student needs in learning problem solving, reasoning and proof. While teachers are accountable to the system in which they work, they have some control of the syllabus in their own classrooms.

Technology — Technology can enhance the teaching and learning of problem solving, reasoning and proof. A variety of non-routine problems may be approached with the use of technology. When possible, technology should be used to enable students to experience problem solving, reasoning and proof.

Language — Teaching problem solving, reasoning and proof demands effective communication among students and teachers. Contexts and brainstorming ideas for problem solutions along with the reasoning involved place increased demands on teachers to work to overcome any language difficulties. Teachers and researchers need to examine issues locally to see what needs to be done with language to enhance learning.

To communicate mathematical ideas to others, it is eventually necessary for students to use precise mathematical language.

Fear of change — A teacher with minimal knowledge of mathematics content and pedagogy may not have the confidence to incorporate problem solving and reasoning in the classroom in more than a peripheral way. Such a teacher may not be confident enough to accept different student answers, approaches and reasoning when non-routine problems are introduced. A fear of repercussions from authorities and parents for introducing new problems and content may impede implementation. Mentoring may help eliminate such fear.

Lack of resources —Teachers and students should have access to a variety of resources including textbooks, technology (hardware and software), materials for problems, and people with whom to discuss problems, including everyday life problems. Teachers may need to share existing resources. Teachers have to be innovative and improvise with existing resources. Teachers are encouraged to look for possibilities of sharing resources among schools in their locality and from external sources.

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Appendix B

Park City Mathematics Institute
International Seminar
Mathematics Education Around the World: Bridging Policy and Practice
Monday, 3 July - Saturday, 8 July 2006

Prospector Square Lodge and Conference Center

Saturday-Sunday 1-9 July 2006

Saturday, 1 July - Optional 6:30 p.m.; if people arrive on Saturday and are interested in joining a group to go to dinner, please meet Johnny Lott in the lobby of the Prospector Square Conference Center.

Sunday, 2 July - 5:30 p.m. Meet in the Moose and Miner Room at the Grub Steak Restaurant (across street from the conference center) for dinner.

Monday, 3 July

8:15-9:00 Review of Past PCMI International Seminar Work: 2001-2004 – *Gail Burrill*

9:30 – 10:00 Overview of seminar goals – *Michèle Artigue*

Work Sessions: 20 minute presentation by country, discussion on the promises and challenges for other countries and implications for policy and practice

<u>Work Session I</u>

10:15–10:45 How is problem solving integrated into the secondary mathematics curriculum and how are teachers prepared to teach problem solving in Cameroon? – *Lawrence Diffo Lambo and Maurice Kianpi*

10:45-12:00 Promises and challenges for other countries in the Cameroon approach

<u>Work Session II</u>

1:00 – 1:30 How is problem solving integrated into the secondary mathematics curriculum and how are teachers prepared to teach problem solving in Singapore? – *Berinderjeet Kaur and Avarami Tang*

1:30 – 2:45 Promises and challenges for other countries in the Singapore approach

3:15 – 4:00 Reflections on the day and introduction to International Seminar Record – *Gail Burrill*

Tuesday, 4 July: This is a holiday in the United States; thus there will be no International Seminar sessions on this day. Recreational activities related to the holiday are offered in Park City.

Wednesday, 5 July

8:20 – 10:30 Experience PCMI:
Secondary School Teachers Program- Silver King 2-4 (8:20)
Graduate Summer School - Grand Theater (8:30)
(Mathematics) Research Program Seminar - Grand Theater (9:40)
Undergraduate Summer School – Coalition 1-2 (9:40)

<u>Work Session III</u>

10:45 – 11:15 What mathematical content knowledge is necessary for secondary teachers to teach problem solving in Mexico? – *Alan Downie and Maria Dolores Lozano Suarez*

11:15-12:00 Promises and challenges for other countries in the Mexican approach.

<u>Work Session IV</u>

1:00 – 1:30 Has the vision of problem solving changed in the United States from the past? If the vision has changed, how has it changed? – *Gwendolyn Zimmermann and Johnny W. Lott*

1:30 – 2:45 Promises and challenges for other countries in the U.S. approach

2:45 – 3:15 A mathematician's reflection on problem solving, its role in the secondary curriculum and implications for preparing teachers – *Hyman Bass*

3:15 – 4:15 Clay Senior Scholar-in-Residence Lecture

4:30 – 5:00 Discussion of possible topics for the International Seminar Record

Thursday, 6 July

<u>Work Session V</u>

8:15 – 8:45 How are reasoning and proof integrated into the secondary curriculum and how are teachers prepared to teach reasoning and proof in Ugandan schools? – *George Ekol and Richard Awichi Opaka*

8:45 – 10:00 Promises and challenges for other countries in the Ugandan approach

<u>Work Session VI</u>

10:15 – 10:45 How are reasoning and proof integrated into the secondary curriculum and how are teachers prepared to teach reasoning and proof in German schools? – *Gabriele Kaiser and Jens Weitendorf*

10:45 – 12:00 Possible Observation of Teaching Lab with Deborah Ball

1:00 – 2:00 Experience PCMI
Secondary School Teachers Program – Silver King 2-4, Coalition 3
Graduate Summer School, Problem Session - Grand Theater
Undergraduate Summer School – Coalition 1-2

2:00 – 3:15 Promises and challenges for other countries in the German approach

3:15 – 4:15 Cross-Program Activity: "PD³" – Grand Theater

4:15 – 4:45 Reflect on the Teaching Lab

Friday, 7 July

Work Session VII

8:15 – 8:45 How are reasoning and proof integrated in secondary schools and how are these topics approached in secondary teacher preparation in Pakistani schools? – *Alamgir Khan and Khalid Mahmood Khan*

8:45-10:00 Promises and challenges for other countries in the Pakistani approach

Work Session VIII

10:50 – 10:45 What research findings or current practices would be helpful in teaching reasoning and proof in Poland? – *Aleksandra Gebura and Krzysztof (Kris) Nowakowski*

10:45 – 12:00 Promises and challenges for other countries in the Polish approach

1:00 – 1:15 Reflections by a mathematician on reasoning and proof, their role in the secondary curriculum and implications for preparing teachers — *Hyman Bass*

1:15 – 1:45 *PCMI International Seminar Record*

1:45 – 4:15 Working Groups on *PCMI International Seminar Record*

4:15 – 5:00 Share Working Group Progress

Saturday, 8 July

8:15 – 10:15 a.m. Working Groups on *PCMI International Seminar Record*

10:30 – 12:00 Working Groups on *PCMI International Seminar Record*

1:00 – 2:00 Sharing drafts of Working Groups

2:00 – 4:00 Refine drafts of International Seminar Record

4:00 Sharing International Seminar Record/Recommendations

Closing remarks

6:00 Closing dinner (location to be announced)

Sunday, 9 July
Guests depart for home from Salt Lake City Airport.

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* Indicates attendance is optional

Daily break schedule:

Morning	Lunch	Afternoon
10:00 – 10:15 a.m.	12:00 – 1:00 p.m.	3:00 – 3:15 p.m.