Iron Chef Week 3 Table 1
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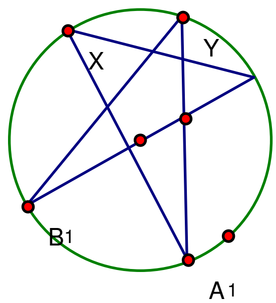
Problem:
What is the sum of the angles in a five-pointed star inscribed in a circle? Six-pointed? Seven-pointed? N-pointed? Is there a pattern? Describe your method in constructing your stars.



Mathematical Content and Purpose:
- Find an algebraic generalization for patterns created
- Apply the proportionality between inscribed angles and arc angles
- Compare the patterns found when stars are created by not lifting the pencil as well as those created with the pencil lifted off the paper.

Further Mathematical Development:
- Study of cyclic groups
- Lead students to using Sketchpad to test their conjectures

Possible Student Responses:
1. Placing points on the circles and connecting alternate points with line segments, generates pointed stars whose angle sums result in a pattern. Students may generalize to 180n – 720, where n is the number of points on the circle.

Five-pointed

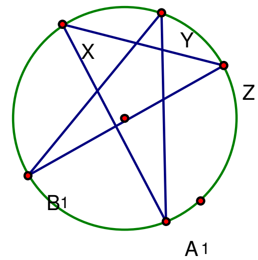
$m\angle A_1XZ = 47.98^\circ$

$m\angle B_1YA_1 = 40.55^\circ$

$m\angle XZB_1 = 44.07^\circ$

$m\angle YA_1X = 26.13^\circ$

$m\angle YB_1Z = 21.28^\circ$



$m\angle A_1XZ + m\angle B_1YA_1 + m\angle XZB_1 + m\angle YA_1X + m\angle YB_1Z = 180.00^\circ$

Six-pointed

$m\angle H_1D_1F_1 = 68.09^\circ$

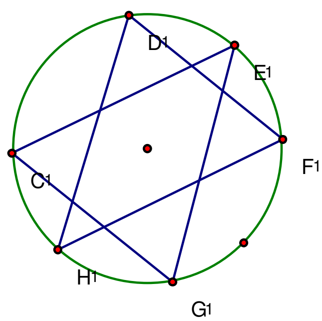
$m\angle C_1E_1G_1 = 49.71^\circ$

$m\angle D_1F_1H_1 = 65.02^\circ$

$m\angle E_1G_1C_1 = 66.13^\circ$

$m\angle F_1H_1D_1 = 46.89^\circ$

$m\angle G_1C_1E_1 = 64.16^\circ$



$m\angle H_1D_1F_1 + m\angle C_1E_1G_1 + m\angle D_1F_1H_1 + m\angle E_1G_1C_1 + m\angle F_1H_1D_1 + m\angle G_1C_1E_1 = 360.00^\circ$

Seven-pointed

$m\angle ABC = 82.48^\circ$

$m\angle DEF = 76.62^\circ$

$m\angle BCG = 91.96^\circ$

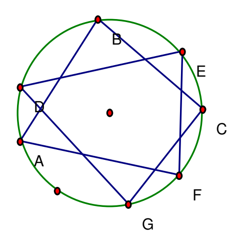
$m\angle EFA = 78.79^\circ$

$m\angle CGD = 81.06^\circ$

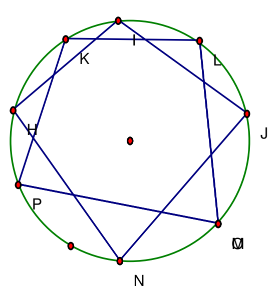
$m\angle FAB = 69.69^\circ$

$m\angle GDE = 59.41^\circ$

$m\angle ABC + m\angle DEF + m\angle BCG + m\angle EFA + m\angle CGD + m\angle FAB + m\angle GDE = 540.00^\circ$



Eight-pointed



$m\angle HIJ = 104.00^\circ$

$m\angle KLM = 97.54^\circ$

$m\angle IJN = 84.82^\circ$

$m\angle LOP = 72.97^\circ$

$m\angle JNH = 76.00^\circ$

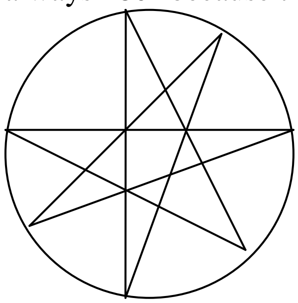
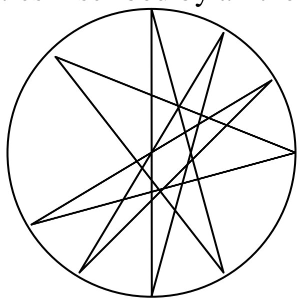
$m\angle MPK = 82.46^\circ$

$m\angle NHI = 95.18^\circ$

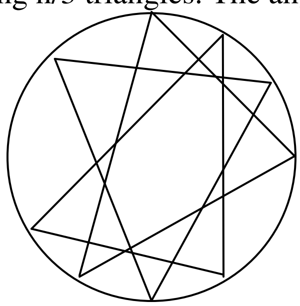
$m\angle PKL = 107.03^\circ$

$m\angle HIJ + m\angle KLM + m\angle IJN + m\angle LOP + m\angle JNH + m\angle MPK + m\angle NHI + m\angle PKL = 720.00^\circ$

2. For odd number-pointed stars, by connecting every (n-1)/2-th point, the angle sum is always 180° because the arcs inscribed by all the angles sum up to 360° (a circle).

3. For odd number-pointed stars that are divisible by 3, by connecting every (n/3)-th point you can make the star using n/3 triangles. The angle sums in these cases are (n/3)×180°.



4. For even number-pointed stars that are divisible by 4, by connecting every (n/4)-th point you can make the star using n/4 quadrilaterals. The angle sums in these cases are (n/4)×360°.

5. By connecting every third point, the sum of inscribed arc angles is (n-6)×360°. Therefore, the angle sum is (n-6)×180°.