Table #2: Open-ended problem (Iron Chef Week #3)
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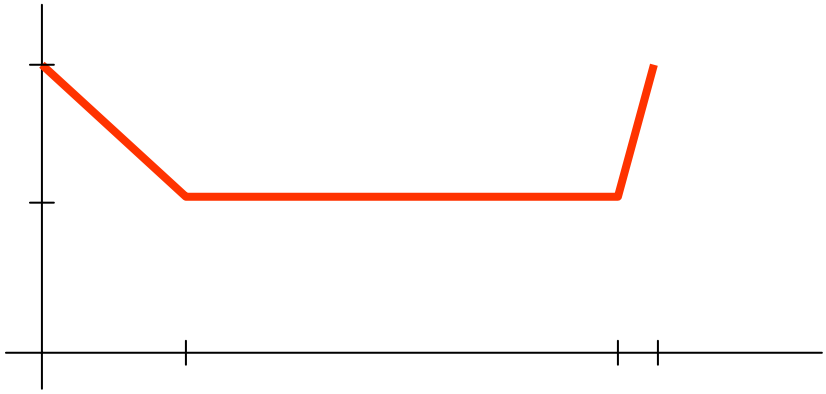
What's the Plot?

Purpose/Goals:

- How does the choice of variable in a given situation affect the interpretation of the graph?
- How does a given graph represent the interdependence of two variables?

Picture:



Problem Directions:

- Write a story that can be represented by the graph above (make sure to include dimensions on the graph and label key points from your story)
- Next, extend your story and exchange your paper with a classmate (their challenge is to complete the graph according to your extension)

Anticipating misconceptions:

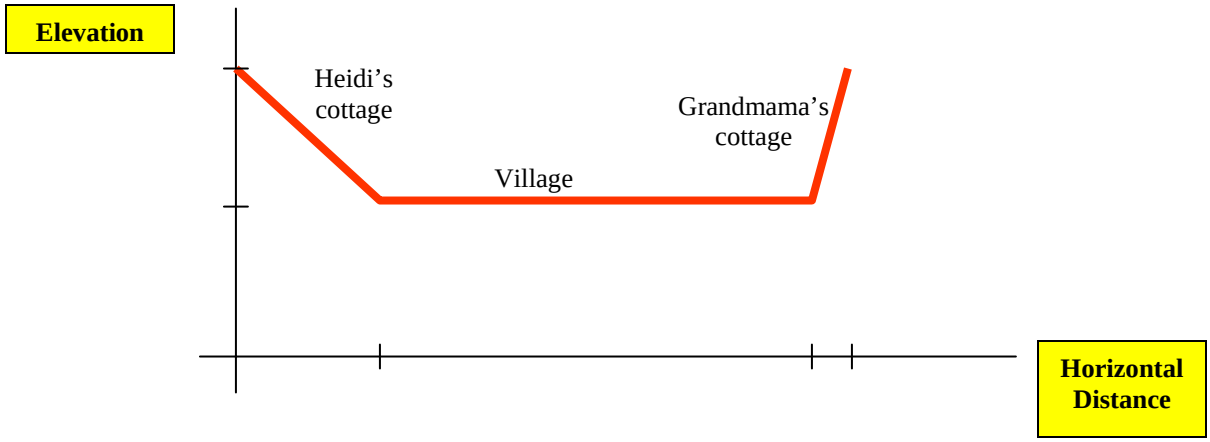
- **Dependent vs. Independent Variable selections:** It is important for students to understand that the x-axis is used to represent the independent variable while the y-axis represents a dependence on the x-axis. Students might graph time vs. elevation and place time on the y-axis. Yet with our graph, this would suggest that time can go down, be held constant, and then reverse while elevation changes.

- **Consistency of scales (proportionality):** It is important for students (and the public) to use graphs to represent stories accurately. By changing the scales of certain portions of the graph, different results occur. This can be seen in the following two interpretations: Distance vs. Time and Elevation vs. Time. In each case, there is an inaccurate representation of either the horizontal or vertical axes.

- **Confusions with instantaneous slope changes:** Students might struggle with the "kinks" (instantaneous slope changes) in the graph and imagine "sudden" change in the story.

- **Understanding slope in context of story:** Students might struggle with the zero slope portion of the graph. They might interpret zero slope as zero velocity when in fact the velocity is simply being held constant.

Anticipated Response: Interpreting graph as **elevation in terms of horizontal distance**



Student-designed story: *Heidi's eyes fluttered open. With a yawn, she snuggled deeper into the warmth of her eiderdown comforter, and thought about the day ahead. Market day; and after that, dinner at Grandmama's cottage.*

She rose, dressed quickly, and hurried outside, scattering chickens and geese as she passed by. What a glorious day! From her mountaintop vantage point, Heidi could clearly see the village spread out below. Brightly colored awnings were already in place over some of the market stalls. Across the valley, in the purple distance, rose Mt. Sturmfels. Squinting, Heidi imagined she could make out Grandmama's cottage perched at its summit.

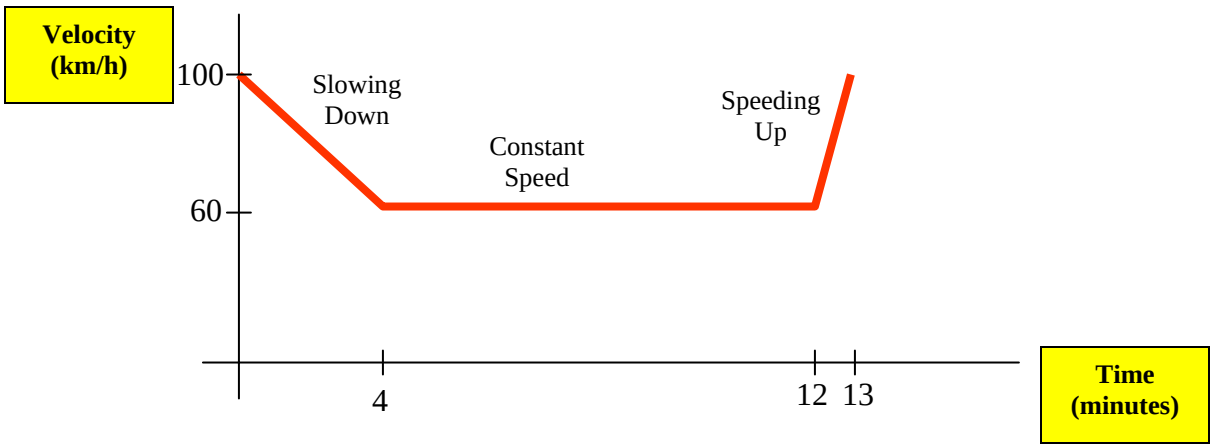
Were she a bird—perhaps one of the doves cooing softly beneath the thatched eaves of the cottage—Heidi could soar directly across the valley to Grandmama's in a matter of minutes without veering from a level path. Instead, she would have to make the long, steep trek down into the valley, and the even steeper climb up the winding path to her grandmother's tiny cottage. Oh well. Heidi was most definitely not a bird! And anyway, there was marketing to be done. Perhaps she would get a chance to see her friend, Peter the goat boy, selling his pails of fresh milk in the marketplace…

Student-designed extension: *…As Heidi approached Grandmama's cottage she saw with some alarm that the front door was standing wide open. Dashing inside, she looked wildly about. The cottage was a shambles, and not a soul was to be found, not even Grandmama's dear old tabby, Heinrich.*

Fearing the worst, Heidi ran back outside and circled around to the rear of the cottage. Cautiously she approached the sheer cliff that dropped away behind the cottage. She knelt and peered over the edge…

Extensions from this response: Graphs such as these are understandable even by very young children. One particularly effective activity is to have a child model a terrain at the sand table. She can then go on to create a cross-section of the terrain by slicing through it vertically with a sheet of transparent acrylic. After clearing sand away from one side, the elevation versus distance information is very visible through the sheet of plastic. It is simple to trace the information onto a sheet of paper, and label key points as shown above. In this way, students develop a sense of how graphs convey information long before they encounter such niceties as variables, functions, coordinates, or axes. Indeed, such experiences provide the foundation for grasping such concepts.

Anticipated Response: Interpreting graph as **velocity in terms of time**



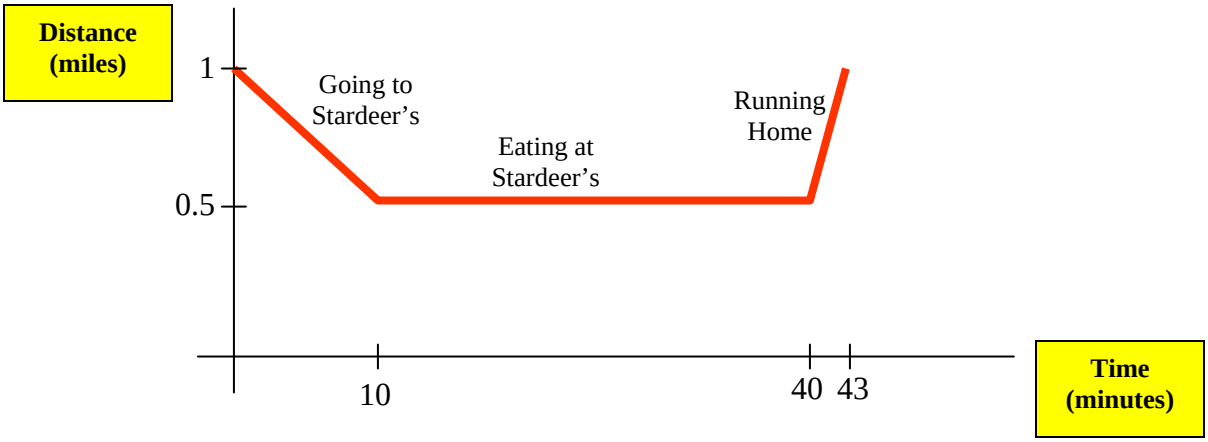
Student-designed story: *Driving my car from Salt Lake City at 100 km/h, I slowly reduce my speed (over a period of 4 minutes) to 60 km/h as I pass through a residential area. After passing through the residential area at same speed, I increase my speed slowly (over a period of 1 minute) until I reach 100 km/h again.*

Student-designed extension: *I drive for another five minutes at 100 km/h before reaching my destination and coming to a stop after an additional 30 seconds.*

Analyzing this response: One difficulty students may have with this graph is thinking that the flat section represents a stopped object, rather than just an object moving at constant speed.

Extensions from this response: Upon completing the task of having another student graph their story's extension, students may extend this situation by finding the total distance traveled by finding the area under the graph. This would be an appropriate extension for a calculus or a pre-calculus course.

Anticipated Response: Interpreting graph as **linear distance traveled in terms of time**



Student-designed story: *Megan woke up and walked down the street to Stardeers for breakfast. She stayed there for 30 minutes and then realized she forgot her scarf. She ran home to get it.*

Student-designed extension: *After spending 2 minutes digging through her closet, she found her scarf. Then she continued on to the farmers' market.*

Analyzing this response: Students who choose to interpret the graph as "linear distance in terms of time" may choose to do so with the following seemingly different representation:

- The graph does not clearly indicate that the person ended up where they started. It could be that they started at one point and ended up at a different point that is the same distance away from whatever the horizontal axis represents.

The student's initial response would be a likely response for a middle or high school student. The student could have included some more detail in the story that is represented on the graph, i.e. it took her 10 minutes to walk there and 3 minutes to walk home, Stardeer's was half a mile from her home and half a mile from the market. Notice that this student made a classic mistake of not labeling what the distance measures. The one mile mark obviously refers to her home, but we do not know what the horizontal axis represents.

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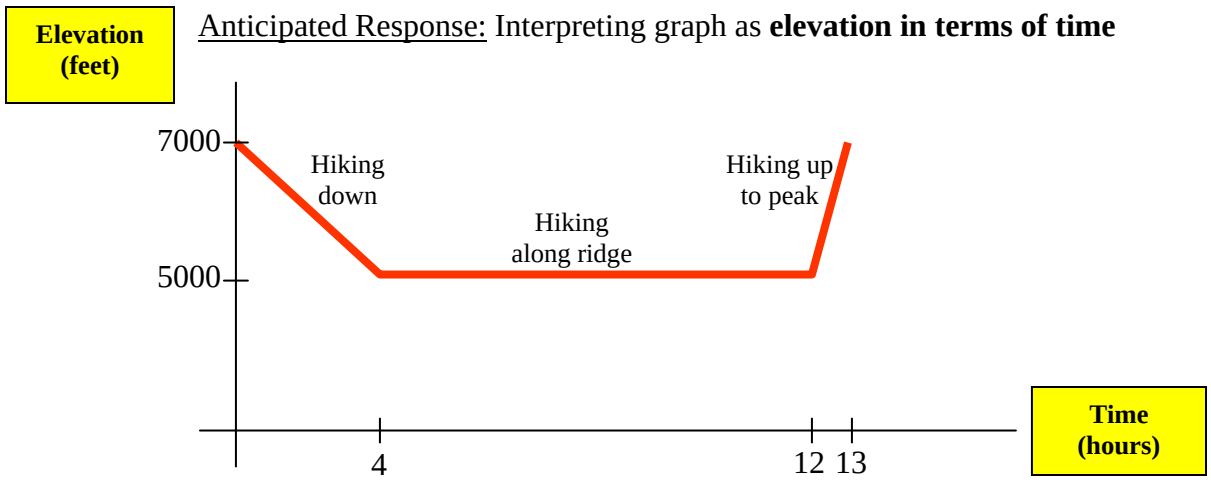
Extensions from this response: Upon completing the task of having another student graph their story's extension, students may extend this situation in the following way:

- What was the person's average speed? (Only while they were moving, or the average speed over the entire time period?)

Anticipated Response: Interpreting graph as **elevation in terms of time**



Student-designed story: *Aki began his hike from an elevation of 7000'. After 4 hours, he had hiked down to an elevation of 5000'. For the next 8 hours he hiked along a ridge at an elevation of 5000'. Finally, he spent the last hour of his journey hiking up to a peak of height 7000'.*

Student-designed extension: *Upon reaching the 7000' peak, Aki decided he wanted to hike all the way down to sea level. It took him 6 hours to reach 4000' and then another 3 hours to complete his hike to sea level. Show an extension of the above graph that will represent this continued story.*

Analyzing this response: Students who choose to interpret the graph as "elevation in terms of time" may choose to do so with the following seemingly different representations:

- Lowering in elevation, continuing on a steady elevation, and then rising in elevation to a different ending point
- Lowering in elevation, resting for a given amount of time, and then rising in elevation to a different ending point
- Lowering in elevation, resting for a given amount of time, and then rising to return to the starting point

Yet all of these representations reflect the same choice of representing elevation versus time. This representation would very likely represent the response of a middle or high school student who has had some experience representing "height vs. time" stories on a graph.

Extensions from this response: Upon completing the task of having another student graph their story's extension, students may extend this situation in the following ways:

- Finding total time, total elevation loss/gain
- Calculating elevation per hour average (and discussing reasonableness of result)
- Discussion of constant rate of elevation loss/gain versus real-life situations