Teaching Proof

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LAB 6.3

Name(s) ______________________

Making Quadrilaterals from the Inside Out

Equipment: Dot or graph paper, straightedge

Definition: To bisect means to cut *exactly* in half.

Instructions: For the following problems,

a. On dot or graph paper, draw two intersecting line segments that satisfy the given conditions. These will be the diagonals.

b. Join the endpoints to make a quadrilateral.

c. Name the quadrilateral.

Whenever possible, try to make a quadrilateral that is not a parallelogram, a trapezoid, or a kite.

1. Perpendicular, equal segments that bisect each other
2. Perpendicular, equal segments that do not bisect each other
3. Perpendicular, unequal segments that bisect each other
4. Perpendicular, unequal segments that do not bisect each other
5. Nonperpendicular, equal segments that bisect each other
6. Nonperpendicular, equal segments that do not bisect each other
7. Nonperpendicular, unequal segments that bisect each other
8. Nonperpendicular, unequal segments that do not bisect each other

Discussion

A. For four of the Problems 1–8, it is possible to get a quadrilateral that is not a parallelogram, a trapezoid, or a kite. What do they all have in common?

B. What requirements do you need to put on the diagonals to get a kite? (It is not one of conditions 1–8.)

C. What requirements do you need to put on the diagonals to get an isosceles trapezoid? (It is not one of conditions 1–8.)

LAB 6.4
Making Quadrilaterals from Triangles

Name(s) ______________________

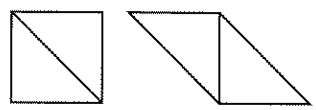
Equipment: Template

Eight types of triangles

Equilateral (EQ)	Acute isosceles (AI)
Right isosceles (RI)	Obtuse isosceles (OI)
Acute scalene (AS)	Right scalene (RS)
Half-equilateral (HE)	Obtuse scalene (OS)

Use a separate sheet or sheets of paper to prepare a well-organized and clearly illustrated report that answers the following questions.

1. Using two congruent triangles placed edge to edge, what types of quadrilaterals can you make? For example, as shown below, with two congruent right isosceles triangles, you can make a square or a parallelogram. Discuss all eight types of triangles.



2. Using three congruent triangles placed edge to edge, what types of quadrilaterals can you make?

Math 2B Name:________________

Making Quadrilateral Conjectures

A conjecture is a mathematical statement that someone thinks might be true.

1. Write five conjectures about quadrilaterals, in the form:

   For a quadrilateral, if [statement 1], then [statement 2]

   For each one, also represent it with an if-then diagram.

   For the statements, choose among the following (you may use the same statement in more than one conjecture):

   ◊ opposite sides are parallel
   ◊ opposite sides are equal
   ◊ all sides are equal
   ◊ one pair of sides are both parallel and equal
   ◊ two sides are parallel but unequal, and the other two are equal, but not parallel
   ◊ two pairs of consecutive sides are equal
   ◊ opposite angles are equal
   ◊ consecutive angles add up to 180°
   ◊ two pairs of consecutive angles are equal
   ◊ all angles are equal
   ◊ the diagonals are equal
   ◊ the diagonals are perpendicular
   ◊ the diagonals bisect each other (meet at each other's middle)

   Example:

   In a quadrilateral, if opposite sides are parallel, then opposite sides are equal.
   [Or: *Opposite sides of a parallelogram are equal,* which means exactly the same thing.]

"If, then" diagram:

2. Write three conjectures that you believe are false. For each one, also represent it with an "if, then" diagram.

3. You have each written eight conjectures. Make a group list, organized as follows:
   a. conjectures that are obviously false
   b. conjectures that are probably false
   c. conjectures that are probably true
   d. conjectures that are obviously true

   Every student will need a copy of the group list.

Math 2B Name:________________

Map of Theorems About Quadrilaterals

What conditions make a quadrilateral be a...

Properties of special quadrilaterals:

Kite
(two pairs of
consecutive,
equal sides)

Trapezoid
(exactly one pair
of parallel sides)

Isosceles trapezoid
(trapezoid with
other pair of
sides equal)

Parallelogram
(two pairs of
parallel sides)

Rectangle
(all angles equal)

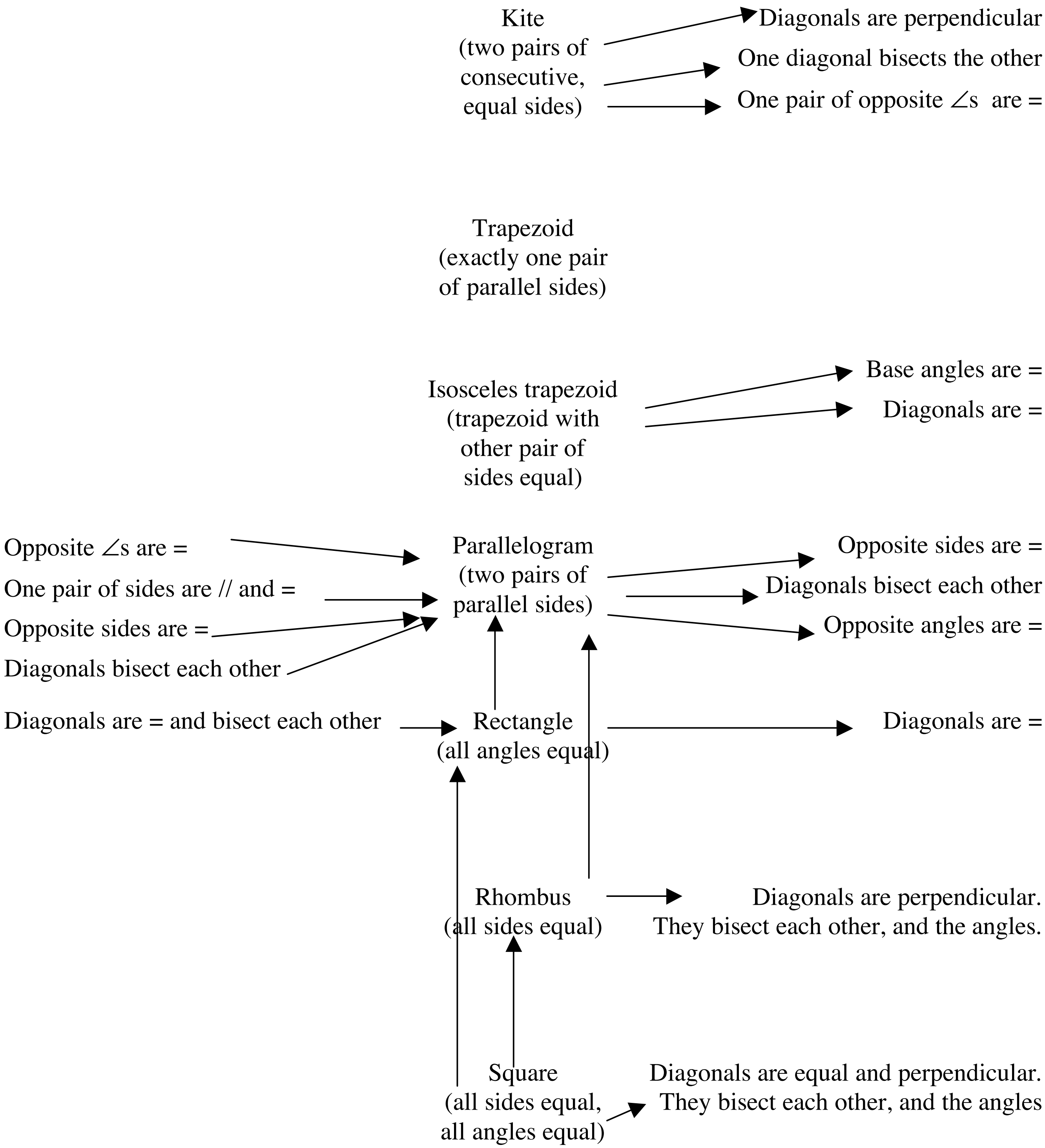
Rhombus
(all sides equal)

Square
(all sides equal,
all angles equal)

Math 2B   Name:________________

Map of Theorems About Quadrilaterals

What conditions make a quadrilateral be a...   Properties of special quadrilaterals:

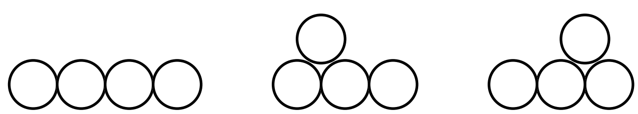


The Strong Law of Small Numbers

Richard K. Guy, a contemporary mathematician in Calgary, Canada, formulated the Strong Law of Small Numbers: *coincidences cause careless conjectures!*

Guy collects examples of patterns that work with small numbers. However, many of them break down sooner or later. Here are a few examples. Some break down, some don't. For each one, find the pattern, and try to figure out if it keeps going or breaks down.

1. What is the greatest number of pieces you can get by making n straight cuts through a circular pizza?
2. What is the greatest number of pieces you can get by joining n points on a circle in every possible way?
3. What is $\frac{n^4 - 6n^3 + 23n^2 - 18n + 24}{24}$ for n = 1, 2, ...
4. How many ways are there to arrange n pennies flat on a table in unbroken rows so that each penny not on the bottom row touches two pennies below it. For example, there are three ways to arrange four pennies following these rules:



5. Is $n^2 + n + 41$ always prime for n = 1, 2, 3, …?
6. Can every odd number greater than 3 be written as a sum of a prime and a power of 2?

Definition: n! (read "n factorial") is the product 1·2·3·…·n. For example, 3! = 6, and 4! = 24.

7. A number is a Niven number if it is divisible by the sum of its digits. For example, 21 is a Niven number, since it's divisible by 3, but 22 is not since it's not divisible by 4. Check whether n! is a Niven number for n = 1, 2, 3, etc.
8. Can every odd number greater than 1 be written as a prime plus two times a square?
9. **Goldbach's Conjecture:** Every even number greater than 2 can be written as the sum of two primes.

∞ Name: _______________

Introduction to Mathematical Induction

Mathematical induction is used to prove that a fact is true for *all* (natural number) values of n.

◊ Proving the fact works for the initial term (or terms) of the sequence is *the anchor*.

◊ Proving that *if it works for n–1, then it works for n* is the *inductive* (or recursive) *step*.

◊ If you prove both, you are done.

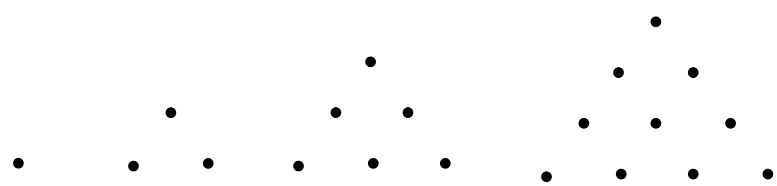
One way to help you prove a formula by mathematical induction uses the understanding you already have of recursive and explicit formulas:

Recursive Formula: equation expressing x_n in terms of x_{n-1} (and sometimes x_{n-2}, and so on.)

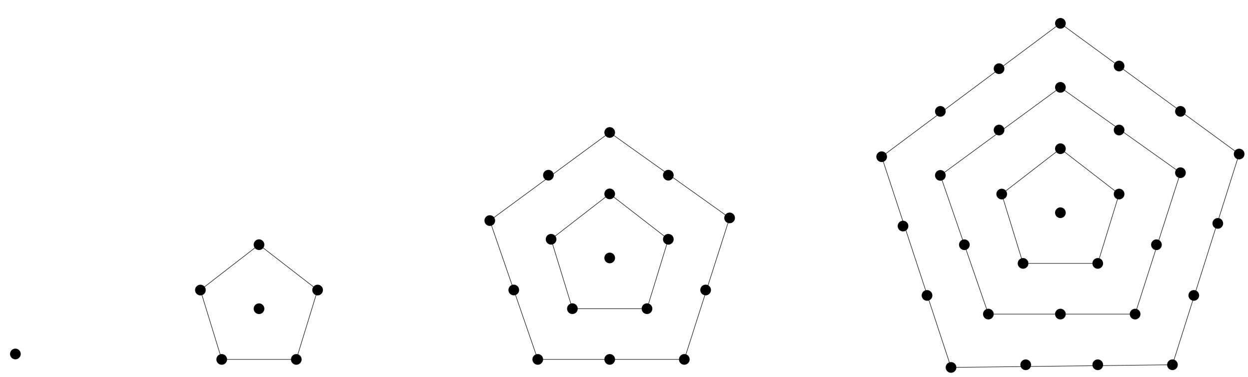
Explicit Formula: equation expressing x_n in terms of n.

For each problem below:

a. List the first eight terms of the sequence.
b. Make a conjecture about the explicit formula for x_n.
c. Write the same formula for x_{n-1}.
d. Find the recursive formula.
e. Use algebra to derive the equation you found in (b) from the ones you wrote in (c) and (d).
f. You are now ready to write a proof by mathematical induction! Do it.



1. The n^{th} triangular number.
2. The n^{th} pentagonal number .



3. The n^{th} odd number.
4. The sum of the first n odd numbers.
5. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n\cdot(n+1)}$
6. $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^n}$
7. The sum of the first n square numbers.
8. The sum of the first n triangular numbers.
9. The sum of the first n cubes.
10. The number of rectangles of any size (including squares) in an n-by-n checkerboard.