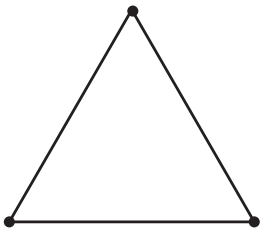
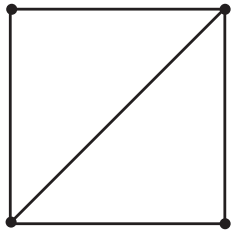
What is a graph?

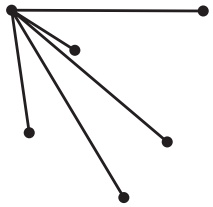
A *graph* is a collection of points called *nodes* that are connected by lines called *edges*. Here are some examples of graphs.



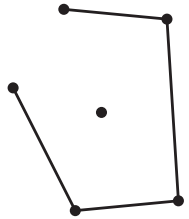
a graph with 3 nodes and 3 edges



a graph with 4 nodes and 5 edges



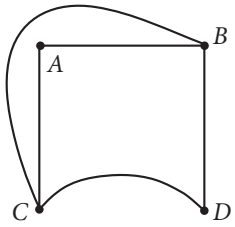
a graph with 6 nodes and 5 edges

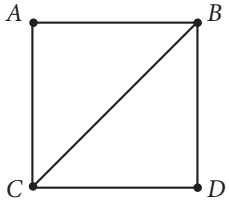


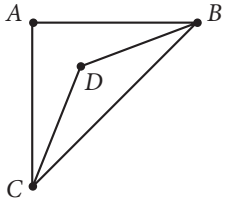
a graph with 6 nodes and 4 edges

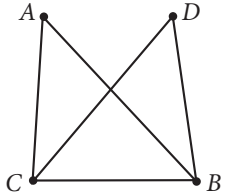
Graph theory is an active area of mathematical research. Graphs have numerous practical applications in computer science, epidemiology, condensed matter physics, genetics, and many other fields of study. Even the links between web sites on the Internet can be thought of as a graph.

It's important to keep in mind that a graph can be drawn in many different ways. For example, even though the following pictures look different, they are all depictions of the same graph.









Application #1: Social networks

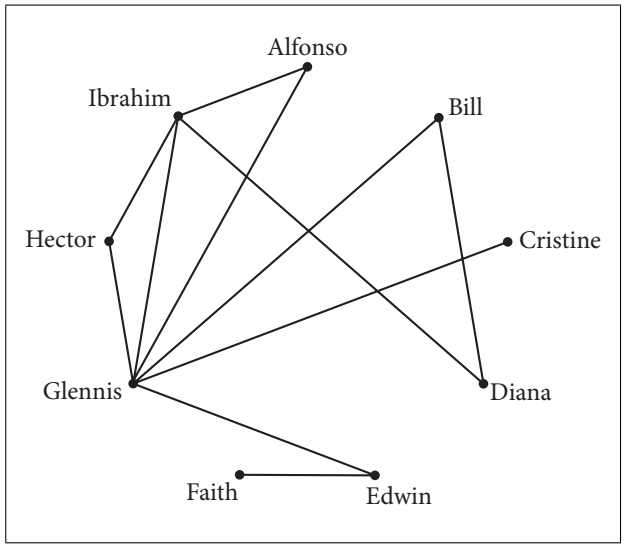
One common use of graphs is to represent social networks. For example, the graph below depicts the relationships between people, represented by nodes, at a party. An edge connecting two nodes means that the two people knew each other before the party.

Discussion questions:

(1) Who do you think organized the party?

(2) If Alfonso wants to meet Hector but Glennis is busy, who can he ask to introduce him to Hector?

(3) If Edwin doesn't attend, Faith won't know anyone at the party. Which other people must be present at the party to make sure that everyone knows at least one person at the party?



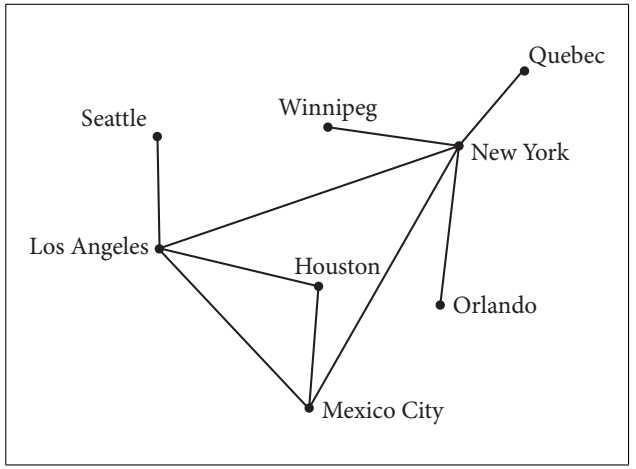
(4) Suppose that you're the organizer of the party. What introductions would you make before the party starts so that everyone has a good time during the party?

Application #2: Airline routes

The graph below is a simplified representation of the routes for BestFares Airlines. Each edge represents a regularly scheduled flight between two cities.

Discussion questions:

(5) Since there is no direct flight from Los Angeles to Quebec, travelers between these two cities must make a connection through what city?



(6) How many different ways are there to get from New York to Houston without revisiting a city?

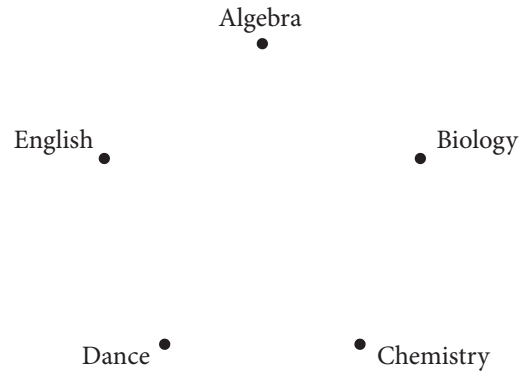
(7) If BestFares Airlines is considering adding another flight among these cities, which two cities you would recommend connecting? Why?

(8) Suppose your boss has asked you, a widget salesperson, to travel to all of the cities shown above (in no particular order) to visit clients. Can you find an itinerary that minimizes the total number of flights?

Application #3: Exam schedules

Here is a list of students and the final exams they must take. Suppose that you are in charge of scheduling the exams so that two exams aren't held at the same time if there is a student taking both exams.

Last Name	Exams
Morton	Algebra, Biology, Dance
Nguyen	Chemistry, English
Ort	Algebra, Biology
Pramanik	Chemistry
Quincy	Dance, English
Repa	Biology, Dance
Sakamoto	Biology, Dance, English
Torres	Algebra, Chemistry, English



Discussion Questions:

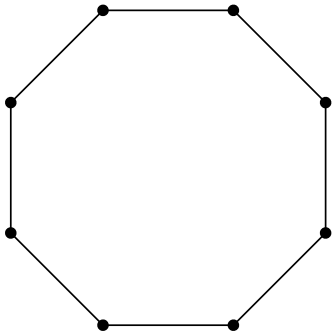
(9) Create a graph in which each node represents a subject. Draw an edge between two subjects if there is a student taking an exam for both subjects. Suppose that there are only two available exam time slots, 8 a.m. and 10 a.m. Is it possible to put each of the five exams in one of the two time slots without creating a conflict? How can you answer this question using the graph you've created?

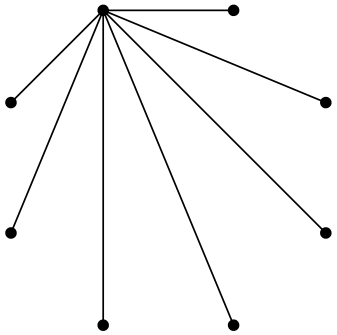
(10) Suppose there are three available time slots: 8 a.m., 10 a.m., and noon. Can you schedule the exams to avoid conflicts? What is the minimum number of exam time slots needed so that there are no scheduling conflicts?

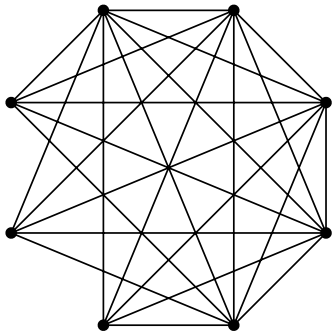
(11) How might the scheduling change if the chemistry exam was cancelled? What if the dance exam was cancelled?

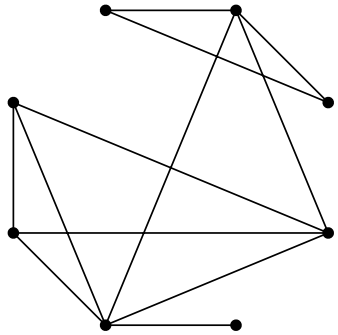
How complex is a graph?

Complexity is in the eye of the beholder.









Which of the graphs above would you say are complex?

Can you articulate why you think one graph is more complex than another?

Can you rank these graphs from most complex to least complex?

To develop your ideas, you might try putting each of these graphs in the contexts of the applications on the previous pages.

Graph Complexity Project

Your task is to define a way of measuring the complexity of a graph and to communicate your ideas to others using a poster, poster, webpage, etc.

To do list:

1. First, discuss in your group what features of a graph you think make it complex.
2. As a team, come up with a definition for the complexity of a graph. In other words, come up with an algorithm, procedure or formula that assigns a nonnegative number to a graph. We will interpret that number as the "complexity number" of the graph. You can even give your "complexity number" a name. For simplicity, let's all agree that higher numbers mean a more complex graph.
3. Test your definition of graph complexity by calculating your "complexity number" on some examples. You may wish to make up some more example graphs and calculate their complexity.
4. Refine your definition until your group is satisfied with it.
5. Make a poster that explains your group's definition of graph complexity.

Questions you may want to consider:

1. Can your definition of "complexity number" be calculated for any graph, or does it only apply to certain graphs?
2. Does your definition of complexity have any special properties? For example, what's the lowest possible "complexity number" that you can get? Is there a highest "complexity number"? Are graphs with more nodes always considered to be more complex according to your definition? Does your complexity definition have the property that if graph H is contained in graph G, then the complexity of H is less than the complexity of G?
3. How does your definition of complexity relate to the applications of graphs that we discussed today?

Group roles

Recorder: someone who will organize and record all of the ideas and suggestions brought up by team members
Skeptic: someone who will question premises and provide counterexamples in a constructive and helpful way
Manager: someone who will make sure that each team member feels welcome to participate and contributes to the group