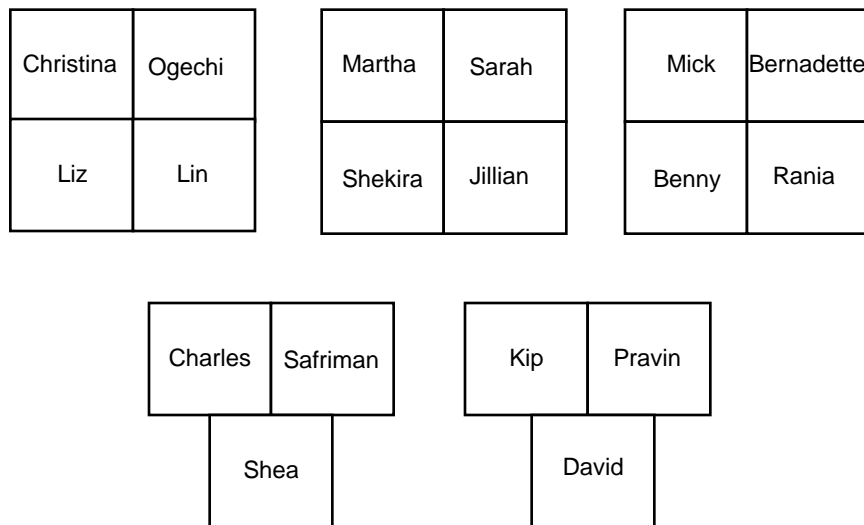


# The Three-Coin Problem in the Third Grade

September 18, 1989  
Spartan Village Elementary School  
Teacher: Mrs. Ball

## Seating Chart



This was the fourth mathematics class of the school year. Since the first day, the class had been working on a variety of combination and permutation problems.

On the blackboard was written: I have pennies, nickels, and dimes in my pocket. If I pull three coins out, what amounts of money could I have?

[00:00:11.00]            **Ball:** Can I ask somebody to read the problem on the board?

[00:00:15.00]            David could you read it?

[00:00:18.03]           **David:** I have pennies... nickels... and dimes... in... my pocket.

[00:00:24.08]           If... I... pull three... coins... out... what... amount of money  
(pause) would?

[00:00:41.27]           **Ball:** Put a C. Could.

[00:00:44.29]           **David:** - Could I have.

[00:00:47.12]           **Ball:** What problem is this similar to? (pause) Charles?

[00:00:53.22]           **Charles:** The one with the the coins.

[00:00:55.17]	<b>Ball:</b>	What number of coins did we work with last week?
[00:00:58.00]	<b>Charles:</b>	Um, two.
[00:00:59.07]	<b>Ball:</b>	Two coins.
[00:01:05.18]		Can somebody give one example of amount of money that you could have?
[00:01:10.11]		Shea?
[00:01:12.14]	<b>Shea:</b>	Like you, like, you could have um...
[00:01:15.21]	<b>Ball:</b>	Could you speak up just a little? I'm not sure that Bernadette can hear you.
[00:01:18.21]	<b>Shea:</b>	You could have like, five- you could like, pull out one of each of them and you could like, you could get um, sixteen.
[00:01:30.28]	<b>Ball:</b>	Okay, one penny, one nickel, and one dime.
[00:01:36.11]		Is that three coins?
[00:01:38.00]	<b>Shea:</b>	Uh huh.
[00:01:40.02]	<b>Ball:</b>	And he says that's sixteen cents all together. What do other people think about that?
[00:01:47.29]		Lin?
[00:01:49.06]	<b>Lin:</b>	I agree with him.
[00:01:50.01]	<b>Ball:</b>	How would you get sixteen cents?
[00:01:51.23]	<b>Lin:</b>	Um, one dime would be ten cents, a nickel, a nickel, ten cent-
[00:01:59.09]		Wait, a dime is ten cents and then a nickel is five cents and it's fifteen cents and if it's a penny it will be sixteen cents.
[00:02:08.09]	<b>Ball:</b>	Okay.
[00:02:11.09]		I would like you to work on this for a few minutes and see what different combinations you can figure out.
[00:02:16.00]		And use the money if you want, to to help you.
[00:02:19.05]		And then after a little bit we will stop and talk about it together.
[00:02:24.00]		First I would like you to work on it a little bit alone and see what you can come up with by yourself.
[00:02:29.05]		Make sure you have the whole problem copied.
[00:02:33.19]		And then write down whatever you need to write down in your notebook to help you remember what you're figuring out.

The students worked on the problem alone or with the people seated near them .

After they had been working for about 12 minutes, Mrs. Ball called the class together

for a brief discussion.

[00:03:11.04] **Ball:** I have a question to ask.

[00:03:14.01] Does anybody have a prediction of how many solutions they think they will find for this problem?

[00:03:18.18] **Lin:** I'm not that...

[00:03:19.19] **Ball:** Shea?

[00:03:20.21] **Shea:** How about around ten.

[00:03:22.10] **Ball:** Excuse me just a second. Ogechi and Lin can you hear Shea?

[00:03:25.05] **Lin:** Yeah.

[00:03:26.03] **Shea:** Around ten.

[00:03:27.03] **Ball:** Around ten. Is there a reason why you predicted ten?

[00:03:30.02] **Shea:** Because um, I'm not sure.

[00:03:34.13] **Ball:** Any, any different predictions? Lin?

[00:03:37.28] **Lin:** Nine.

[00:03:39.03] **Ball:** Anybody else? Mick?

[00:03:41.08] **Mick:** I've found nine.

[00:03:42.18] **Ball:** You come up with nine already? Mick has already found nine.

[00:03:48.03] How are you going to know when you have all the solutions?

[00:03:54.13] Any ideas? How would you know if you had found them all?

[00:03:60.08] Liz?

[00:04:01.08] **Liz:** You would start um...

[00:04:02.08] **Ball:** Excuse me just a second, Kip and Pravin and Safriman-

[00:04:06.08] Right now Liz is talking and I would like you to be able to hear her.

[00:04:09.08] **Liz:** You would start doing the ones that you've already done over.

[00:04:15.13] **Ball:** Pravin could you hear Liz?

[00:04:16.28] **Pravin:** Yes.

[00:04:19.03] **Ball:** What did she say?

[00:04:20.18] **Pravin:** (shakes head)

[00:04:21.13] **Ball:** Excuse me? Can you say it one more time I'm sorry. Pravin listen to Liz, okay? Speak a little louder.

[00:04:27.03] **Liz:** Okay. Um, you would start-you would use the, the same ones over again.

[00:04:34.18] **Ball:** You would use the same one over again?

[00:04:35.22] Anybody have any other ideas of how you would know if you had gotten them all?

[00:04:41.28] Is there any other way to tell?

[00:04:47.18] Shekira?

[00:04:48.23] **Shekira:** When we confer with somebody and if they have the same answers as you.

[00:04:53.14] If they don't, then you don't have all the answers and you need to write it down, then you have all of them.

[00:04:60.06] **Ball:** I have a small concern, right now.

[00:05:02.13] I would like everybody to put their coins down and their pens down.

[00:05:07.16] I would like everybody to put their coins down and their pens down for a moment.

[00:05:12.11] One of the things that's very important is that if one person in the class is talking other people need to listen-

[00:05:18.01] Because people are saying things that can help you think about the problems.

[00:05:21.25] Shekira said some interesting things, and so did Liz, but lots of people were not listening to them.

[00:05:27.25] I know it's because you're finding more solutions yourself.

[00:05:31.03] But, one of the things that I would really like you to, to see you doing is to listening-

[00:05:34.27] -to be listening really hard when someone else has an idea, because it might help you with your thinking.

[00:05:40.11] I'm sure a lot of people didn't hear Shekira.

[00:05:42.24] And I had to ask Liz to say, what she said three times.

[00:05:46.27] Stop for a moment now, and listen hard to Shekira and see what you think about what she is saying.

[00:05:51.04] Could you say it one more time?

[00:05:52.29] **Shekira:** Well, when you think you're done, confer with somebody else at your table and if they have the same answers as you do then...

[00:06:02.20] Then, you know you have all of them and if you don't then you write it down on your paper.

[00:06:07.10] Or they write the ones they have that you have.

[00:06:10.20] **Ball:** So Liz and Shekira gave us two different ideas.

[00:06:13.02] Liz said when you start repeating yourself in the ideas you come up with, you think you probably have them all.

[00:06:20.24] Shekira said-Shekira added something to it. Shekira said

when you think you have them all-

[00:06:25.15] You could confer with somebody else at your table and see if they found any that you haven't found.

[00:06:30.21] Are there any other ways to know if you have all the answers?

[00:06:34.01] Mick?

[00:06:35.22] **Mick:** It's not about the answers.

[00:06:37.05] **Ball:** So, it's another comment?

[00:06:38.26] **Mick:** Yeah.

[00:06:39.08] **Ball:** What?

[00:06:40.03] **Mick:** Um, I think it's ten because I just came up with one more.

[00:06:43.12] **Ball:** Okay, well, Mick just came up with one more. He's up to ten possible answers.

[00:06:48.10] **Ball:** Take a few more minutes to work on it and then I'd like to spend a few minutes talking about what you've come up with.

The children returned to working on the problem individually and in their small groups.

After a few minutes, Mrs. Ball called them together again.

[00:07:02.12] **Ball:** Let's stop for a moment. Put your coins down.

[00:07:10.09] It's easy to tell when people have stopped with the coins, because the coins make a lot of noise.

[00:07:14.13] Put the coins down and put the pens down for a moment. You're going to want your pen though, because while we have-

[00:07:20.10] While we discuss the problem, if somebody brings up an answer that you didn't find, you might like to record it in your notebook.

[00:07:27.14] We have one answer recorded on the board, one solution. Who would like to share another solution that they came up with?

[00:07:37.17] Safriman?

[00:07:39.17] **Safriman:** Twelve.

[00:07:40.27] **Ball:** Okay, twelve cents. Could somebody tell how Safriman got twelve cents?

[00:07:48.07] What coins did Safriman use to get twelve cents? Who

thinks they know?

[00:07:57.30] David?

[00:07:60.11] **David:** Um

[00:08:07.22] Safriman used dime um, and two pennies.

[00:08:18.03] **Ball:** What do other people think about what David said. David said he thinks Safriman used two pennies and one dime.

[00:08:23.25] What do other people think about that?

[00:08:26.18] Sarah?

[00:08:27.19] **Sarah:** I agree.

[00:08:28.13] **Ball:** Can you prove that, that's right?

[00:08:30.04] **Sarah:** Yeah, because it's three, three coins.

[00:08:34.19] **Ball:** Three coins. How can you prove that that's twelve cents?

[00:08:38.01] **Sarah:** Because... ten and two is twelve and that's three coins.

[00:08:47.13] **Ball:** Any comments? Liz?

[00:08:50.08] **Liz:** I agree with that.

[00:08:51.04] **Ball:** Anybody disagree with it?

[00:08:54.11] Okay, another solution? Another possible way to do this problem? No, I just asked you.

[00:09:04.04] Somebody different? Bernadette?

[00:09:08.09] **Bernadette:** I got seven cents.

[00:09:09.27] **Ball:** Who thinks they know how Bernadette got seven cents?

[00:09:17.28] Kip?

[00:09:18.29] **Kip:** One nickel and two pennies.

[00:09:22.18] **Ball:** Christina? What do you think about what Kip said?

[00:09:26.09] **Christina:** I agree with him.

[00:09:27.12] **Ball:** Can you prove that, that's seven cents?

[00:09:29.27] **Christina:** Yeah. Because um, a nickel and two pennies is-a nickel is five cents and two pennies will add to seven cents.

[00:09:39.22] **Ball:** Comments from anybody else?

[00:09:42.15] Anybody disagree with this?

[00:09:47.04] Okay, do we have another solution?

[00:09:53.16] How about Ogechi, do you have something different in your notebook?

[00:10:09.16] **Ogechi:** Thirty cents.

[00:10:10.24] **Ball:** How much?

[00:10:11.29] **Ogechi:** Thirty

[00:10:14.07] **Ball:** Ogechi, do you want to tell us how you got it or should we ask other people to figure it out?

[00:10:18.12] **Ogechi:** Three dimes.

[00:10:19.02] **Ball:** Okay, three dimes. Jillian, what do you think about what Ogechi said?

[00:10:24.25] **Jillian:** I think she's right.

[00:10:26.14] **Ball:** Why do you think she's right?

[00:10:29.06] **Jillian:** Because... ten plus ten is twenty and plus another ten is thirty.

[00:10:36.19] **Ball:** Any comments from anybody else? David?

[00:10:39.17] **David:** Twenty five.

[00:10:40.16] **Ball:** Oh, you're already giving another one?

[00:10:41.27] **David:** Uh huh.

[00:10:42.17] **Ball:** Did you agree with Jillian?

[00:10:44.03] **David:** Um, yeah.

[00:10:44.23] **Ball:** Okay, what is your solution?

[00:10:46.24] **David:** Fifteen.

[00:10:47.20] **Ball:** Fifteen. Who thinks they know how David got fifteen cents?

[00:10:53.27] Sarah?

[00:10:54.29] **Sarah:** Um, three nickels.

[00:11:07.05] **Ball:** Shea, what do you think about that?

[00:11:17.29] **Shea:** I'm not sure.

[00:11:18.23] **Ball:** Why are you not sure?

[00:11:38.16] What are you figuring out?

[00:11:40.20] **Shea:** Oh.

[00:11:48.08] **Ball:** Shea, how much is one nickel?

[00:11:50.07] **Shea:** Uh, five cents. And two nickels are ten cents and three nickels are fifteen.

[00:11:59.18] **Ball:** So-Do you agree with this then or disagree?

[00:12:03.29] **Shea:** Um, agree.

[00:12:06.30] **Ball:** We're going to stop. I would like everyone to look over this way for a moment.

[00:12:12.11] When we start math tomorrow, we're going to continue with this problem a little bit longer.

[00:12:17.25] I have a question to ask before we stop and a comment to make. I'll make my comment first and then ask my question.

[00:12:24.05] My comment was, I thought people did a better job just now listening to each others' solutions and giving each other time.

[00:12:31.14] Did you notice that when somebody was figuring something out people weren't going uh,uh,uh-

[00:12:35.30] Or interrupting, people were listening and thinking about

whether it made sense.

[00:12:41.26] Did you notice that? And did you also notice that people were explaining why it made sense.

[00:12:44.28] Like people would say it's thirty cents because, Jillian said, because ten plus ten is twenty and ten more is thirty.

[00:12:51.27] Was that a good explanation?

[00:12:55.16] It was a good explanation because it helped us understand why that answer made sense.

Mrs. Ball asked the students about a task she had posed to them the day before. After hearing brief reports from a couple of students, she had them put their things away and ended class.

Class list as of September 1989

NAME	GENDER	RACE	COUNTRY	ENGLISH PROFICIENCY	HOW LONG AT THIS SCHOOL <sup>1</sup>
Benny	M	White	Ethiopia	fluent	3 years
Bernadette	F	White	Canada	native speaker	just started
Charles	M	Asian	Taiwan/Canada	fluent	3 years
Christina	F	African-American	U.S.A.	native speaker	12 months
David	M	Asian	Indonesia	developing	3 years
Ira	F	White	Indonesia	developing	5 months
Jillian	F	White	U.S.A.	native speaker	3 years
Kip	M	African Black	Kenya	fluent	3 years
Lin	F	Asian	Taiwan	fluent	2 years
Liz	F	White	U.S.A.	native speaker	3 years
Marta	F	Latina	Nicaragua	beginning	just started
Mick	M	White	U.S.A.	native speaker	2 years
Ogechi	F	African Black	Nigeria	fair	3 years
Pravin	M	White	Nepal	beginning	5 months
Rania	F	White	Egypt	good	3 years
Safriman	M	Asian	Indonesia	developing	12 months
Sarah	F	White	U.S.A.	native speaker	2 years
Shea	M	White	U.S.A.	native speaker	2 years
Shekira	F	African-American	U.S.A.	native speaker	just started

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<sup>1</sup>NOTE: This column reflects the length of time the child had been in this school as of 9/89. No one had been in this class longer than a few days.

*Calendar of "problems of the day" — September 1989*

<b>MONDAY</b>	<b>TUESDAY</b>	<b>WEDNESDAY</b>	<b>THURSDAY</b>	<b>FRIDAY</b>
<p align="center"><b>September 11</b></p> <p>I have pennies, nickels, and dimes in my pocket. Suppose I pull out two coins. How much money would I have?</p>	<p align="center"><b>September 12</b></p> <p>Today's date is 9/12. Take the 9, the 1, and the 2 and make as many 3-digit numbers as you can</p>	<p align="center"><b>September 13</b></p> <p>(problems involving 4-digit permutations)</p>	<b>September 14</b>	<b>September 15</b>
<p align="center"><b>September 18</b></p> <p>I have pennies, nickels, and dimes in my pocket. If I pull three coins out, what amounts of money could I have?</p>	<p align="center"><b>September 19</b></p> <p>Write numbers sentences that equal 10</p>	<p align="center"><b>September 20</b></p> <p>Mrs. Rundquist had 100 pencils. Miss Latham borrowed 90 pencils. How many pencils did Mrs. Rundquist have then?</p>	<p align="center"><b>September 21</b></p> <p>Mrs. Rundquist has 300 stickers. Miss Latham has 190 stickers. How many more stickers does Mrs. R. have than Miss L.?</p>	<b>September 22</b>
<p align="center"><b>September 25</b></p> <p>(Continuation of) write number sentences that equal 10.</p>	<p align="center"><b>September 26</b></p> <p>(Begin elevator problems exploring the addition and subtraction of integers. . .)</p>			<b>September 29</b>