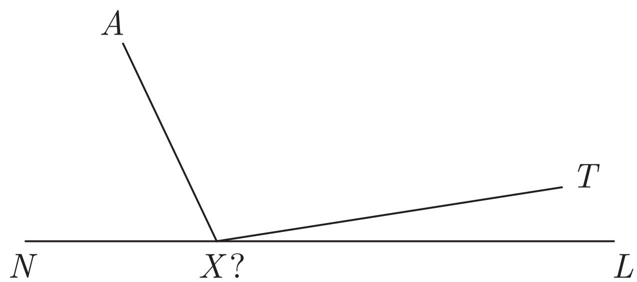
2 *PsyChoMetrIcs*

PROBLEM

Art (at point A) has to take a high-stakes test at point T. Since he is in Park City, he is extremely thirsty, and needs to take a big gulp from nearby river NL.



Where should he run toward (point X) to minimize the total distance $AX + XT$?

Drink water! Be sure your table synergies efficiently. If you figure out where point X should be, start thinking about how you could *construct* point X more directly, how you could *prove* that point X must be the right one, or how to change the problem in interesting ways.

What river is Art drinking from? deNiaL!

Okay, now it's time to think outside of the (rectangular) box.

Important Stuff.

1. (a) What's $(11 + \sqrt{2}) + (11 - \sqrt{2})$?
(b) What's $(11 + \sqrt{2})(11 - \sqrt{2})$?
(c) What's $(11 + x)(11 - x)$?

2. (a) A rectangle has perimeter 44 and area 117. What are its length and width?
(b) A rectangle has perimeter 44 and area 119. What are its length and width?
(c) A rectangle has perimeter 44 and area 100. What are its length and width?
(d) A rectangle has perimeter 44 and area 130. What are its length and width?

We mean a real rectangle here, not something as imaginary as a belief that Vanilla Sky is more popular than IMCePtion.

3. Find all solutions to each quadratic equation.

(a) $w^2 - 22w + 121 = 0$
(b) $w^2 - 22w + 120 = 0$
(c) $w^2 - 22w + 117 = 0$
(d) $w^2 - 22w + 119 = 0$
(e) $w^2 - 22w + 100 = 0$
(f) $w^2 - 22w + 2 = 0$
(g) $w^2 - 22w - 408 = 0$
(h) $w^2 - 22w + 130 = 0$

We like shortcuts, but we *especially* like shortcuts that do not involve the phrase "$4ac$".

4. Two numbers add up to 200 and their product is 9,991. What are the numbers?

5. Find three rectangles that have the same numeric value for their perimeter and area.

6. Blanca says she has found two rectangular boxes with different dimensions, but the same surface area and volume. Either find your own pair of boxes with this property, or prove that no such pair of boxes can exist.

These boxes show multiple personality disorder.

7. Triangle SAM has points $S(2, 1)$, $A(4, 1)$, and $M(4, 6)$.

(a) Draw triangle SAM in the plane.

(b) New points are created from the points in triangle SAM according to the rule

$$(x, y) \mapsto (-y, x)$$

Draw the new triangle created in this way in the same plane, and describe how this triangle is related to the original.

This Sam is not wearing a hat.

Check with others to verify the new points and feel validated.

8. Transform triangle SAM according to the rule

$$(x, y) \mapsto (-x, -y)$$

Draw the new triangle and describe how this triangle is related to the original.

We need you to leverage your enterprise. It's a paradigm shift.

9. (a) Show, beyond a reasonable doubt, that $(14, -4)$ is *not* equidistant from $(-2, -5)$ and $(4, 11)$.

(b) Again for $(-10, 7)$.

(c) Again for (x, y). Hmm!

Neat Stuff.

10. Cuong repeats the transformation in Problem 7 a whole bunch of times.

(a) What happens after the transformation is applied twice?

(b) What happens after the transformation is applied three times?

(c) . . . four times?

(d) . . . five times?

(e) . . . thirteen times?

(f) . . . 101 times?

These triangles won't self-actualize until you find their meaning and purpose.

11. For any rectangle you can assign a point (l, w) in a coordinate plane, defined by the length and width of the rectangle.

(a) Plot four points that all correspond to rectangles with area 20.

(b) How many rectangles are there with area 20? Plot them all.

(c) Plot all the rectangles with perimeter 20.

(d) Is there a rectangle with perimeter 20 *and* area 20?

12. (a) Put a point Q on the number line. Define $f(P)$ to be the distance from point P on the number line to point Q. What does the graph of $f(P)$ look like?

(b) Put point R on the number line, and now define $f(P)$ as the total distance from any point P to the two given points. What does the graph of $f(P)$ look like?

(c) Put point S on the number line, and do all that stuff we said again.

Sketchpad can be helpful here, but it's a challenge. You can pick specific values for the points, or be general.

These problems take a holistic approach in parabolizing the . . . yeah, we got nothin'.

13. Felipe says he has found two triangles with different side lengths, but the same perimeter and area. Either find your own pair of triangles with this property, or prove that no such pair of triangles can exist.

Careful, he might be delusional!

14. Find three rectangular boxes where the numeric value of their surface area equals the numeric value of their volume.

One of them is gleaming, like a 1989 skateboard movie.

15. Find three triangles that have the same numeric value for their perimeter and area.

16. Let $f(x) = x^3 + 3x^2 - 8x - 80$. Use long division to find the remainder when $f(x)$ is divided by each of these.

(a) $(x - 1)$

(b) $(x - 2)$

(c) $(x - 3)$

(d) $(x - 4)$

(e) $(x - 5)$

Feel free to uh, **divide** the work on this one among one another, there is no need to do all five of these yourself. But make sure you record all five answers and verify that the values are correct.

17. Complete this table for $f(x) = x^3 + 3x^2 - 8x - 80$.

x	$f(x)$
0	
1	
2	
3	
4	
5	

18. Find the remainder when $f(x) = x^{12} + 3x - 1$ is divided by each of these.
(a) $(x - 1)$
(b) $(x - 2)$
(c) $(x - 10)$
(d) $(x + 1)$

A codependent relationship with synthetic division will not help here.

19. Jessica takes triangle SAM from Problem 7 and applies a wacky transformation:

$$(x, y) \mapsto (x + y, -3x + 7y)$$

(a) Draw this new triangle JES. Is it even a triangle anymore?
(b) What is the area of this new shape? How does JES compare (in area) to SAM?

Jessica and Sam are equally-valued partners in our shared holism, and any comparison is purely for educational purposes.

20. Find all the ways to write $\frac{1}{10}$ as the sum of two unit fractions. Here's one for free:

$$\frac{1}{10} = \frac{1}{20} + \frac{1}{20}$$

Tough Stuff.

21. Given positive integer n, the unit fraction $\frac{1}{n}$ can be written as the sum of two other unit fractions:

$$\frac{1}{n} = \frac{1}{a} + \frac{1}{b}$$

Find a rule for the number of ways to write $\frac{1}{n}$ as the sum of two unit fractions.

22. There's a point inside most triangles that forms three 120° angles with segments to the three vertices. A *Matsuura triangle* is a triangle whose side lengths are all integers, *and* whose three interior segment lengths from the 120° point are also integers. Find some Matsuura triangles, or prove they do not exist.

23. Find a rule for the number of ways to write $\frac{1}{n}$ as the sum of *three* unit fractions.