8 *Pattern Sniffe-NING*

PROBLEM

For each of these complex numbers, plot the number and its square in the same complex plane. You might want to draw segments from the origin to each number. If you need to measure lengths or angles, consider using Sketchpad.

$$\begin{aligned} A &= 3+i \\ S &= 1+2i \\ H &= 3+2i \\ L &= \tfrac{\sqrt{3}}{2}+\tfrac{1}{2}i \\ I &= i \end{aligned}$$

Keep doing examples until you can describe *precisely* where the square is located in relation to the original number.

Reminder: direct teaching is discouraged. All activities here can be completed without formulas.

You can type "3^0.5" to get $\sqrt{3}$ in Sketchpad.

Extension questions: what about cubes? square roots? reciprocals?

Important Stuff.

1. Let $z = \frac{12}{13} + \frac{5}{13}i$. Find and plot all of these on the same complex plane. (Estimate to three decimal places if you like.)

(m) z^0	(n) z^2	(c) z^4
(o) z^1	(i) z^3	(a) z^5

2. Use Sketchpad to find the *angle* that forms when you go from one power of $z = \frac{12}{13} + \frac{5}{13}i$ to the next.

3. What does it look like in the complex plane when you add two complex numbers w and z? Give some examples. What does $w + w$ look like?

4. If a complex number w is multiplied by a real number c (also called a *scalar*), what happens to the magnitude? the direction? What if c is negative?

5. If a complex number w is multiplied by i, what happens to the magnitude? the direction?

6. Let $w = 1 + i$ and $z = \sqrt{3} + i$. Find the magnitude and direction of wz, and compare the results to the magnitude and direction of w and z.

You mean it's the same angle each time? Maybe, maybe not! You'll have to be a pattern sniffer, man.

We have clearance, Clarence. Roger, Roger. What's our vector, Victor?

We heard that on Tuesday one participant multiplied their underwear by i^2.

7. What, in general, happens to magnitude and direction when you multiply two complex numbers?

Well, you know, the numbers, like, *combine*, man...

8. (a) Find the magnitude of $3 + 2i$.
(b) Calculate $(3 + 2i)^2$ and find its magnitude.
(c) What Pythagorean triple has hypotenuse 13?

9. Make some more Pythagorean triples. Totally awesum!!!

Neat Stuff.

10. (a) Three numbers add to 13 and multiply to 36. Find both sets of positive integers.
(b) Find two distinct rectangular boxes with volume 162 and surface area 234.
(c) If each box's dimensions are a by b by c, compute the product

$$(x - a)(x - b)(x - c)$$

and discuss any findings.

11. (a) Point Q has coordinates $(3, 0)$ and point P has coordinates $(x, 0)$. Define $f(P)$ to be the square of the distance from point P to point Q. What does the graph of $f(P)$ look like?

It looks like a big smile, man.

(b) Point R has coordinates $(17, 0)$. Now define $f(P)$ as the sum of the squares of the distances PQ and PR. What does the graph of $f(P)$ look like?
(c) Point S has coordinates $(1, 0)$. You take it from here.
(d) If $f(P)$ is the sum of the squares of the distances PQ, PR and PS, what P produces the minimum of $f(P)$?

12. For each point P, determine if it is twice as far from the point $(-1, 4)$ as it is from the *line* $y = 1$.

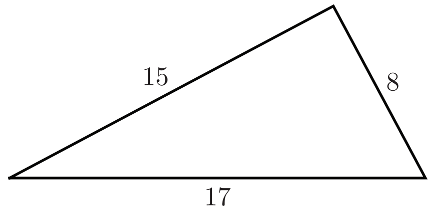
Is $y = 1$ vertical or horizontal? It depends on which way you've decided to graph, man.

(a) $(-1, 2)$
(b) $(-1, -2)$
(c) $(23, 14)$
(d) $(23, -14)$
(e) $(-91, 52)$
(f) (x, y)

13. Find an equation for all the points *three times* as far from $(-1, 4)$ as from the line $y = -1$.

14. Find an equation for all the points *half* as far from $(-1, 4)$ as from the line $y = -1$.

15. Find a triangle similar to the one below that has the same numerical value for its area and perimeter.



16. Find two triangles that have the same area and the same perimeter but that have dissimilar shapes.

17. A right triangle has leg lengths a and b. Find the radius of its incircle.

Try it with numbers first perhaps you will?

18. Use the results from problems 3-5 to prove that when you multiply a complex number by $a + bi$, it . . . oh, crap, we forgot. What does it do?

19. Find a primitive Pythagorean triple whose hypotenuse length is 13^3.

What was that movie where Sean Astin finds a primitive Pythagorean triple in his backyard then Pauly Shore gives it a makeover? Oh, right, *Encino Math*.

20. Show how squaring the complex number $m + ni$ can be used to generate this identity that can be used to generate Pythagorean triples:

$$\left(m^2 - n^2\right)^2 + \left(2mn\right)^2 = \left(m^2 + n^2\right)^2$$

21. Let a be a complex number with magnitude 5, and b be a complex number with magnitude 13. Consider $a+b$: what could its magnitude be? Do you add magnitudes when you add complex numbers, or what?

Vegeta says these magnitudes can be over 9000!!!

22. Suppose x can be written as the sum of two squares, and y can also be written as the sum of two squares. Prove that xy can *also* be written as the sum of two squares.

23. As the powers of $z = \frac{12}{13} + \frac{5}{13}i$ grow, eventually the powers work around the four quadrants, then back around into Quadrant I. Find the first power of z that re-enters Quadrant I.

The powers of z are movin'! They're numbers . . . on the *grow*, man.

24. Suppose a regular n-gon is placed with its center at the origin and one of its vertices at $(1, 0)$.

(a) For what values of n will $(-1, 0)$ be a vertex of the n-gon?

(b) For what values of n will $(0, -1)$ be a vertex of the n-gon?

(c) Find all points that are on the regular 12-gon but aren't on any smaller n-gon.

Darryl says the rectangles in Cracklin' Oat Bran are regular n-gons.

25. Take all the vertices on the regular 12-gon as points in the complex plane. Square them and plot the squares. What happens?

26. You've now seen that squaring complex numbers generates Pythagorean triples. But does this squaring method generate *all* the Pythagorean triples? Explain.

Tough Stuff.

27. Prove that at least one number in every Pythagorean triple must be even. Then prove that *exactly one* number in every primitive Pythagorean triple must be even.

28. Build an identity for generating primitive Pythagorean triples where the hypotenuse is a perfect square.

29. Hey, it's another division problem where we've provided one of the digits, an 8. The other digits are unknown.

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              X X X X X
X X X ) X X X X X X X X
          X X X
          -----
            X X X X
              X X X
            -------
                X X X X
                8 X X X
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30. Sketch the graph of the equation $x^y = y^x$ in the first quadrant. What point is that, and why?

Whaddaya mean, *what point*? THAT one. Duh.