Day 10 (July 18, 2011)

Trace: Causes a point to leave a trail behind it as it moves. To turn it on, select the point you want to trace, click on "Display"… "Trace", then cause the point to move. You might enjoy "Animating" things too...

Locus: The set of all points satisfying a certain property. Select the point whose path you want to view, then select the point that causes that first point to move, then click on "Construct" … "Locus".

The distance from (x, y) to $(4, 0)$ is twice its distance to $(7, 0)$. Which of these points makes this statement true?

(a) $(6, 0)$ yes, because dist to $(4, 0)$ is 2 and dist to $(7, 0)$ is 1.

(b) $(8, 2)$ distance to $(4,0)$ is $\sqrt{(8-4)^2+(2-0)^2} = \sqrt{20}$
distance to $(7,0)$ is $\sqrt{(8-7)^2+(2-0)^2} = \sqrt{5}$
Yes! $\sqrt{20} = 2\sqrt{5}$

(c) *try one of your own* : $(8, -2)$
distance to $(4,0)$ is $\sqrt{20}$
distance to $(7,0)$ is $\sqrt{5}$
yes!

To find all (x,y) that satisfies the statement, repeat what we did in (b) but replace $(8,2)$ with (x,y).

$$\sqrt{(x-4)^2+(y-0)^2} = 2\sqrt{(x-7)^2+(y-0)^2}$$

square both sides

$$(x-4)^2 + y^2 = 4\left[(x-7)^2 + y^2\right]$$

$$x^2-8x+16+y^2 = 4x^2-56x+196+4y^2$$

$$-180 = 3x^2-48x+3y^2$$

complete square

$$64 - 60 = x^2-16x+64+y^2$$

$$4 = (x-8)^2 + y^2$$

This is a circle centered at $(8,0)$ with radius 2.