10 *New Directions*

PROBLEM

Download this file for today's sketch:
`http://tinyurl.com/complexgsp`

Let v be a complex number with magnitude 2.

- Draw where v could lie in the complex plane.
- Pick a value of v and cube it: where does it go? Describe all the possible places where v^3 could lie.
- Where does $v + i$ go and how does it move as v changes? What about $v + 3$? $2iv$? $v^2 + v$? $v^2 - v$?

If you "Plot Point," GSP will keep the point's coordinates when you translate the origin, zoom in, or zoom out. Use a plotted point to fix the radius of your circle to the coordinate plane. This helps if you need to change your scale. Plot points also advance the storyline!

You might be interested in experimenting with the "Trace" or "Locus" functions in GSP. Sue: "You might want to put us all out of our misery and shave off that Chia Pet."

Important Stuff.

1. Here are two Pythagorean triples: 5-12-13 and 85-132-157. Use them to produce more than one Heronian triangle. Then find its area and incircle radius using any method you like.

You can do it however you like, however you like...

2. Solve these quadratic equations.
(a) $(x - 3)(x - 17) = 0$
(b) $(2x - 3)(2x - 17) = 0$
(c) $(10x - 3)(10x - 17) = 0$
(d) $(100x - 3)(100x - 17) = 0$

FACT0R'D!!!!!

3. Solve these quadratic equations. We're not going to do your dirty work *this* time. Please, no formulas.
(a) $x^2 - 14x + 45 = 0$
(b) $4x^2 - 28x + 45 = 0$
(c) $100x^2 - 140x + 45 = 0$
(d) $10000x^2 - 1400x + 45 = 0$

Dude, use the first one to help you do the second one!! Dude!! Dude.

4. (a) Write a quadratic equation whose solutions are $x = \frac{2}{3}$ and $x = -\frac{4}{7}$.
(b) Oh, we meant one with no fractions, and completely multiplied out, and with 0 by itself on one side.
(c) Did you know that $\frac{2}{3} - \frac{4}{7} = \frac{2}{21}$? Interesting. Discuss.
(d) In the quadratic equation $ax^2 + bx + c = 0$, what is the sum of the two solutions? What is the *average* of the two solutions?

Sue: "You know what they say? Those who can't, teach. Turns out, maybe you actually can."

5. Let $f(x) = ax^2 + bx + c$, and use the quadratic you used in the previous problem.
(a) Graph $f(x)$ using your favorite technology.
(b) What are the x-intercepts of the graph?
(c) What is the x-coordinate of the vertex?

The pencil was invented in 1795. Papyrus was first manufactured by the Egyptians around 3000 BCE. Beat that.

The vertex is the turny spot.

6. What is the x-coordinate of the vertex of $f(x) = ax^2+bx+c$ in terms of a, b, and maybe c?

7. Consider points $P(x, 0)$, $Q(3, 0)$, $R(17, 0)$, $S(1, 0)$, and $T(11, 0)$.
(a) Define $a(P)$ to be the distance PQ. What does the graph of $a(P)$ look like, and where is its minimum?
(b) Redefine $a(P)$ to be the sum of the distances PQ and PR. What does the graph of $a(P)$ look like, and where is its minimum achieved?
(c) Redefine $a(P)$ to be the sum of the distances PQ, PR, and PS.
(d) Redefine $a(P)$ to be the sum of the distances PQ, PR, PS, and PT. What does the graph of $a(P)$ look like, and where is its minimum achieved?

Again, again! Hooray for repetition!

8. Consider points $P(x, 0)$, $Q(3, 0)$, $R(17, 0)$, $S(1, 0)$, and $T(11, 0)$.
(a) Define $b(P)$ to be the *square* of the distance PQ. What does the graph of $b(P)$ look like, and where is its minimum achieved?
(b) Redefine $b(P)$ to be the *sum of the squares* of the distances PQ and PR. What does the graph of $b(P)$ look like, and where is its minimum achieved?
(c) Redefine $b(P)$ to be the sum of the squares of the distances PQ, PR, and PS.
(d) Redefine $b(P)$ to be the sum of the squares of the distances PQ, PR, PS, and PT. What does the graph of $b(P)$ look like, and where is its minimum achieved?

Again, again! Hooray for repetition!

Wink wink, nudge nudge, know what I mean?

9. Which of the following points is a total of 30 units away from $(-9, 0)$ and $(9, 0)$?

(a) $(-15, 0)$ (c) $(6, 11)$ (e) $(12, b)$
(b) $(0, 12)$ (d) $(9, -\frac{48}{5})$ (f) (x, y)

10. The distance from (x, y) to the point $(9, 0)$ is $\frac{3}{5}$ of its distance to the line $x = 25$. Which of the following points makes that statement true?

(a) $(15, 0)$ (c) $(13, 6)$ (e) $(-12, b)$
(b) $(0, -12)$ (d) $(9, \frac{48}{5})$ (f) (x, y)

Neat Stuff.

11. The *eccentricity of an ellipse* is the ratio $\frac{c}{a}$ of two distances: the distance from the center of the ellipse to a focus (called c), and the distance from the center of the ellipse to a vertex (called a). For any ellipse discovered in Important Stuff, compute its eccentricity.

The eccentricity of Glee is Emma Pillsbury.

12. The distance from (x, y) to the point $(9, 0)$ is $\frac{4}{5}$ of its distance to the line $y = x$. Which of the following points makes that statement true?

(a) $(7, 2)$	(c) $(11, -14)$	(e) $(27, -18)$
(b) $(23, -2)$	(d) $(19, -19)$	(f) (x, y)

Don't stop believing that you can do this! Sue: "If I hear one song from that classic rock outfit Journey, I will start pulling catheters."

13. Show that this is true:

$$(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$$

Squares, squares, everywhere. Sue: "I will kick you square in the taco."

14. Suppose m can be written as the sum of two squares, and n can also be written as the sum of two squares. Prove that mn can *also* be written as the sum of two squares.

15. Use the result in problem 13 to prove that when two complex numbers are multiplied, their magnitudes are multiplied.

16. Suppose the sum and product of two complex numbers z and w are both real. Prove that either z and w are both real numbers, or $w = \overline{z}$.

17. Find the eccentricity of the shape in problem 12.

Maybe Rachel is the eccentricity. Actually, the real eccentricity is that the school has a Slurpee machine. Sweet!

18. Use multiplication of complex numbers to find a formula for $\tan(\alpha + \beta)$ in terms of $\tan\alpha$ and $\tan\beta$.

19. Find two values of $\tan\alpha$ and $\tan\beta$ so that $\tan(\alpha+\beta) = 5$.

20. Calculate this:

$$\frac{(5 + i)^4}{(239 + i)}$$

Any guesses on what this might be useful for?

Useful?! Math is supposed to be useful? Sue: "Asking someone to believe in a fantasy, no matter how comforting, is cruel."

Tough Stuff.

21. Let S be the set of complex numbers z with magnitude 2. Find a function (such as $z \mapsto z^2$) that produces an ellipse (and *not* a circle) as the output when S is used as the input.

Sue: "You are the honey X, nature's most ferocious animal. Look it up on Youtube."

22. The complex number w is a member of set M if the rule $z \mapsto z^2 + w$ with starting point 0 never has $|z| > 2$.

(a) Find some numbers w that are in set M, and some that aren't.

(b) Find a maximum bound for the *area* of the shape made by set M in the complex plane.

(c) What does set M look like if it is graphed in the complex plane?

We mean here that the rule is executed repeatedly. For example, if $w = i$ then the sequence of z values is $0, i, -1 + i, -i \ldots$ and the hope is that none of those has magnitude more than 2.

23. What shapes tesselate in the hyperbolic plane?

24. (a) Evaluate $\tan 89°$ to two decimal places.

(b) How many degrees are in 1 radian? Give your answer to three decimal places.

(c) What is going on here? Is this a coincidence?

Coincidence? I think not!