PROBLEM

Here is David’s data.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Decide, without using anything other than your brain and perhaps some paper and pencil, on an equation for a line that “best” represents his data table. We ♦ an equation! Be prepared to defend your choice: why did you pick this line above others?

Important Stuff.

1. One way to test the “badness” of a line toward data is to compute the sum of squared errors. Use the data from the box above.
   (a) Trevor used the line $y = x$ and wants you to confirm that the sum of squared errors is 30.
   (b) Marakina used $y = x - 1$ and Olimpia used $y = x + 1$. How’d they do?
   (c) Barb says there must be a rule, in terms of $b$, for the sum of squared errors of $y = x + b$. Go find it.
   (d) Kate screams “OMG QUADRATIC” and finds the best possible $b$. Join her!

2. So, given a slope, it’s possible to find one best line of that slope for the data (according to the sum of squared errors, anywho). Hop to it, and find the best line for each slope.
   (a) slope 2    (d) slope 0
   (b) slope 3    (e) slope −1
   (c) slope 4

3. Graph the data and all six of the lines from the last two problems on the same axes. What!!

Kermit: “It’s okay, Peter, you can be yourself.” Peter Sellers: “There is no me. I do not exist. There used to be a me but I had it surgically removed.”

POINTS... IN... SPAAAAACE!
4. Marcelle hands you this new data set.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$3$</td>
</tr>
<tr>
<td>$1$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$3$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

In terms of $m$, find the sum of squared errors of $y = mx$. Then determine the Best $m$ Ever!

5. Type the original four points into a piece of technology and tell it to compute a linear regression.
   (a) What is the slope of the linear regression?
   (b) What important point does the graph of the linear regression pass through?
   (c) Compute the badness of the linear regression. Is it simply the “best”?

6. A second way to test the “badness” of a line toward data is to compute the sum of absolute errors. Use the data from the box not above.
   (a) Lauren used the line $y = x$ and wants you to confirm that the sum of absolute errors is 8.
   (b) James used $y = x - 1$ and Timon used $y = x + 1$. How’d they do?
   (c) Allison got fancy and used $y = x + \frac{1}{2}$. How’d she do?
   (d) Becky says there must be a rule, in terms of $b$, for the sum of absolute errors of $y = x + b$. Go find it.
   (e) Sketch the graph of Becky’s rule as a function of $b$. What do you notice?

Neat Stuff.

7. In yesterday’s problem in the box, Shirley used Geometer’s Sketchpad to visualize the locus of all points $v^2 + v$ when the complex number $v$ has magnitude 2. Now, check out this awesome file:

   http://tinyurl.com/comppolygsp

Explain, using the file, that $z = 2$ is a solution to the equation

$$z^2 + z = 6$$

Then find all of the solutions to $z^2 + z = 6$, resizing the circle as necessary to track them down.

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Fozzie once did a tribute to Marcel Marceau by impersonating a man standing on one leg, then a man standing on no legs. Waldorf said his act didn’t have a leg to stand on. OH ho ho ho.

Don’t use Sketchpad, since it doesn’t know how to compute linear regressions automatically.

This is different from the sum of absolute errors, calculated by drinking vodka inappropriately. Wocka wocka!

We interrupt this problem for a Muppet News Flash: There is no news tonight.

Sorry, we meant Randall here, but he keeps telling us to call him Shirley.

This sketch is barely-controlled electric mayhem. Gonzo: “It’s a question of mind over matter.” Waldorf: “Well, we don’t mind, and you don’t matter! OH ho ho ho!”
8. Use the file to find at least one solution to each of these equations.
   (a) \( z^2 - z = 6 \)
   (b) \( z^2 - z = 12 \)
   (c) \( z^2 - z = -9 \) (do a quick approximation)
   (d) \( z^2 - z = 20 \)

9. Take a number \( z \) in the complex plane. How can you construct the conjugate \( \bar{z} \)? What’s the conjugate of \( z^2 + z \)?

10. Zee deestunce-a frum \((x, y)\) tu zee pueent \((25, 0)\) is 30 mure-a thun its deestunce-a tu zee pueent \((-25, 0)\). Vhet pueents \((x, y)\) meke-a thet stetement trooe-a? Bork, bork, bork!

11. The distance from \((x, y)\) to the point \((25, 0)\) is \(\frac{5}{3}\) of its distance to the line \(x = 9\). Test some points, then find an equation!

12. The eccentricity of a hyperbola is the ratio \(\frac{c}{a}\) of two distances: the distance from the center of the hyperbola to a focus (called \(c\)), and the distance from the center of the hyperbola to a vertex (called \(a\)). For any hyperbola discovered in Important Stuff, compute its eccentricity.

13. For the data in the box problem find the unique line with the smallest sum of absolute errors.

14. Brian hands you this list of points:
   \[
   \begin{array}{c|c}
   t & h \\
   \hline
   1 & 3 \\
   1 & 7 \\
   3 & 5 \\
   5 & 3 \\
   5 & 7 \\
   \end{array}
   \]
   Find the line that has the smallest sum of absolute errors. But wait, there’s more! Find them all.

15. Let \( z = 34 + i \) and \( w = 55 + i \). The product \( zw \) can be written as a scalar multiple of another complex number in the form \((\text{something}) + i\). What’s the something? (Oh, what about \(\frac{z}{w}\)?)

16. Let \( a = 2 + i \), \( b = 5 + i \), \( c = 13 + i \), \( d = 21 + i \). Find the direction of the product \(abcd\). Can this be generalized?

17. Find the three solutions to \(x^3 = i\) using Sketchpad or some other means.
18. (a) Multiply out \((a + bi)^3\).
(b) If \(z\) is a complex number with magnitude 1 and direction \(\theta\), what are its coordinates?
(c) Write rules for \(\cos 3\theta\) and \(\sin 3\theta\) based on the first two parts of this problem.
(d) The rule for \(\cos 3\theta\) is \(4 \cos^3 \theta - 3 \cos \theta\). What up with that?

19. (a) Expand the expression \((x + 2)^3\).
(b) How many faces are there on a cube? How many edges? How many vertices? Wacky.
(c) Does this pattern continue at all, in lower or higher dimensions?

20. Suppose \(\tan A = \frac{1}{x}\) and \(\tan B = \frac{1}{y}\), where \(x\) and \(y\) are integers. Is it possible for \(\tan(A + B)\) to also be in the form \(\frac{1}{z}\) for some other integer \(z\)? If so, how?

21. Let \(z\) be a complex number with magnitude 1. If \(z = x + yi\) and \(y > 0\), explain why \(y = \sqrt{1 - x^2}\).

22. Let \(z = x + (\sqrt{1 - x^2})i\) as in the last problem.
(a) Expand \(z^2\) and look at the real part. Zeros?
(b) Expand \(z^3\) and look at the real part. Zeros? Connections?
(c) Expand \(z^4\) and look at the real part. Hmmm!
(d) Graph the real part of \(z^2, z^3, z^4\) as a function of \(x\).
(e) What might \(z^n\) look like?

Tough Stuff.

23. So, you managed to find two noncongruent boxes with the same surface area and volume? Good, but can you find three noncongruent boxes with the same surface area and volume? What now!!

24. How does the Law of Cosines translate into the hyperbolic plane?

25. Let \(f_n(x) = \cos x \cos 2x \ldots \cos nx\). For which integers \(1 \leq n \leq 10\) is the integral from 0 to \(2\pi\) of \(f_n(x)\) non-zero?