11 *Rainbow Connection*

PROBLEM

Here is David's data.

x	y
1	1
3	8
5	3
7	8

Decide, without using anything other than your brain and perhaps some paper and pencil, on an equation for a line that "best" represents his data table. We ♢ an equation! Be prepared to defend your choice: why did you pick this line above others?

Kermit: "It's okay, Peter, you can be yourself." Peter Sellers: "There is no me. I do not exist. There used to be a me but I had it surgically removed."

Important Stuff.

1. One way to test the "badness" of a line toward data is to compute the *sum of squared errors*. Use the data from the box above.

POINTS. . . IN. . . SPAAAAAACE!

(a) Trevor used the line $y = x$ and wants you to confirm that the sum of squared errors is 30.

(b) Marakina used $y = x - 1$ and Olimpia used $y = x + 1$. How'd they do?

(c) Barb says there must be a rule, in terms of b, for the sum of squared errors of $y = x + b$. Go find it.

(d) Kate screams "OMG QUADRATIC" and finds the best possible b. Join her!

Movin' right along. . .

2. So, given a slope, it's possible to find one best line of that slope for the data (according to the sum of squared errors, anywho). Hop to it, and find the best line for each slope.

Divvy up the work however you like. However you like.

(a) slope 2
(b) slope 3
(c) slope 4
(d) slope 0
(e) slope -1

3. Graph the data and all six of the lines from the last two problems on the same axes. What!!

Wow, that is just gonzo. Or maybe more like Lew Zealand and the boomerang fish.

4. Marcelle hands you this new data set.

x	y
-3	-4
-1	3
1	-2
3	3

In terms of m, find the sum of squared errors of $y = mx$. Then determine the Best m Ever!

Fozzie once did a tribute to Marcel Marceau by impersonating a man standing on one leg, then a man standing on no legs. Waldorf said his act didn't have a leg to stand on. OH ho ho ho.

5. Type the original four points into a piece of technology and tell it to compute a linear regression.

(a) What is the slope of the linear regression?

(b) What important point does the graph of the linear regression pass through?

(c) Compute the badness of the linear regression. Is it simply the "best"?

Don't use Sketchpad, since it doesn't know how to compute linear regressions automatically.

6. A second way to test the "badness" of a line toward data is to compute the *sum of absolute errors*. Use the data from the box not above.

(a) Lauren used the line $y = x$ and wants you to confirm that the sum of absolute errors is 8.

(b) James used $y = x - 1$ and Timon used $y = x + 1$. How'd they do?

(c) Allison got fancy and used $y = x + \frac{1}{2}$. How'd she do?

(d) Becky says there must be a rule, in terms of b, for the sum of absolute errors of $y = x + b$. Go find it.

(e) Sketch the graph of Becky's rule as a function of b. What do you notice?

This is different from the *sum of absolut errors*, calculated by drinking vodka inappropriately. Wocka wocka!

We interrupt this problem for a Muppet News Flash: There is no news tonight.

Neat Stuff.

7. In yesterday's problem in the box, Shirley used Geometer's Sketchpad to visualize the locus of all points $v^2 + v$ when the complex number v has magnitude 2. Now, check out this awesome file:

`http://tinyurl.com/comppolygsp`

Explain, using the file, that $z = 2$ is a solution to the equation

$$z^2 + z = 6$$

Then find *all* of the solutions to $z^2 + z = 6$, resizing the circle as necessary to track them down.

Sorry, we meant *Randall* here, but he keeps telling us to call him Shirley.

This sketch is barely-controlled electric mayhem. Gonzo: "It's a question of mind over matter." Waldorf: "Well, we don't mind, and you don't matter! OH ho ho ho!"

8. Use the file to find at least one solution to each of these equations.
(a) $z^2 - z = 6$
(b) $z^2 - z = 12$
(c) $z^2 - z = -9$ (do a quick approximation)
(d) $z^2 - z = 20$

Start by using the sliders to change the function, then set the green circle's radius to 3.

"We're going to need a bigger circle."

9. Take a number z in the complex plane. How can you construct the conjugate $\overline{z}$? What's the conjugate of $z^2 + z$?

Take a number? Is this a deli?

10. Zee deestunce-a frum (x, y) tu zee pueent $(25, 0)$ is 30 mure-a thun its deestunce-a tu zee pueent $(-25, 0)$. Vhet pueents (x, y) meke-a thet stetement trooe-a? Bork, bork, bork!

Iff yuoor unsver gues thruoogh $(15, 0)$, yuoo ere-a oofer cuukeeng sumetheeng.

11. The distance from (x, y) to the point $(25, 0)$ is $\frac{5}{3}$ of its distance to the line $x = 9$. Test some points, then find an equation!

12. The *eccentricity of a hyperbola* is the ratio $\frac{c}{a}$ of two distances: the distance from the center of the hyperbola to a focus (called c), and the distance from the center of the hyperbola to a vertex (called a). For any hyperbola discovered in Important Stuff, compute its eccentricity.

The eccentricity of the Muppets is Sam the Eagle, since everyone else is pretty much a wacko.

13. For the data in the box problem find the unique line with the smallest sum of absolute errors.

14. Brian hands you this list of points:

t	h
1	3
1	7
3	5
5	3
5	7

Find the line that has the smallest sum of absolute errors. But wait, there's more! Find them all.

This data is 5 observations: t is time in seconds, and h is the number of times Animal banged his head against a cymbal in that many seconds.

15. Let $z = 34 + i$ and $w = 55 + i$. The product zw can be written as a scalar multiple of another complex number in the form (something) $+ i$. What's the something? (Oh, what about $\frac{z}{w}$?)

16. Let $a = 2 + i$, $b = 5 + i$, $c = 13 + i$, $d = 21 + i$. Find the direction of the product $abcd$. Can this be generalized?

17. Find the three solutions to $x^3 = i$ using Sketchpad or some other means.

34 and 55, you say? I'd be fibbing if I said I'd never seen *those* numbers before. Perhaps the answer is 89. This problem is obviously inspired by that bunny episode of Veterinarian's Hospital. . . the continuing story of a quack who's gone to the dogs. It's a real rabbit and Costello routine.

18. (a) Multiply out $(a+bi)^3$.
(b) If z is a complex number with magnitude 1 and direction θ, what are its coordinates?
(c) Write rules for $\cos 3\theta$ and $\sin 3\theta$ based on the first two parts of this problem.
(d) The rule for $\cos 3\theta$ is $4\cos^3\theta - 3\cos\theta$. What up with that?

If you're not a trig fan, just move along, nothing to see here.

19. (a) Expand the expression $(x+2)^3$.
(b) How many faces are there on a cube? How many edges? How many vertices? Wacky.
(c) Does this pattern continue at all, in lower or higher dimensions?

What the hexahedron is going on here?

20. Suppose $\tan A = \frac{1}{x}$ and $\tan B = \frac{1}{y}$, where x and y are integers. Is it possible for $\tan(A+B)$ to also be in the form $\frac{1}{z}$ for some other integer z? If so, how?

21. Let z be a complex number with magnitude 1. If $z = x+yi$ and $y > 0$, explain why $y = \sqrt{1-x^2}$.

22. Let $z = x + (\sqrt{1-x^2})i$ as in the last problem.
(a) Expand z^2 and look at the real part. Zeros?
(b) Expand z^3 and look at the real part. Zeros? Connections?
(c) Expand z^4 and look at the real part. Hmmmmm!
(d) Graph the real part of z^2, z^3, z^4 as a function of x.
(e) What might z^n look like?

The TI-Nspire can handle this, believe it or not.

Pafnuty says hi. Who's that? Look it up!

Tough Stuff.

23. So, you managed to find two noncongruent boxes with the same surface area and volume? Good, but can you find *three* noncongruent boxes with the same surface area and volume? What now!!

"Hey, these problems aren't half bad." "Nope, they're *all* bad! OH ho ho ho!"

24. How does the Law of Cosines translate into the hyperbolic plane?

25. Let $f_n(x) = \cos x \cos 2x \ldots \cos nx$. For which integers $1 \le n \le 10$ is the integral from 0 to 2π of $f_n(x)$ non-zero?

26. Meep. Meep. Meep meep. Meep meep meep. Meep meep meep meep meep. Meep meep meep meep meep meep meep meep?

Just when you think this class is terrible something wonderful happens. . . it ends! OH ho ho ho!