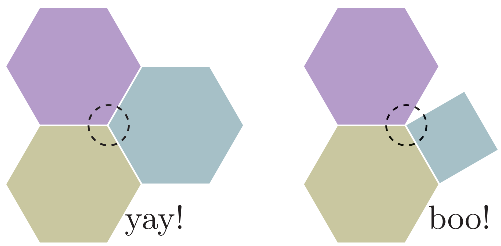
13 *Dining with the STaRs*

PROBLEM

Karen puts 3 regular polygons together so they meet at a point with no overlap and no empty space. One option is to shove together 3 hexagons, but there are other ways.

Find all the ways you can fit 3 regular polygons together at a point. Write each set in increasing order by the number of sides in the polygons.



WHAT!!! WHY!!!

Don't worry about whether or not these polygons will fill the entire plane this way. We're just looking at one vertex.

Three. Three regular polygons. Ah ha ha ha.

Important Stuff.

1. Find all the ways you can fit 4 regular polygons together at a point.

Four! Four regular polygons, ah ha ha ha.

1. Find all the positive integer solutions to this awesome equation, where $G \le A \le B \le E$:

$$\frac{1}{G} + \frac{1}{A} + \frac{1}{B} + \frac{1}{E} = 1$$

3. Find all the ways you can fit 5 regular polygons together at a point. . . 6 regular polygons. . . 7 regular polygons.

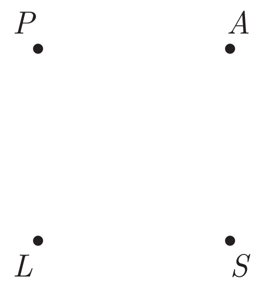
Five!! Five regular polygons, ah ha ha ha!

4. Draw a picture of what it looks like when you do math.

Or, a picture of Marie Osmond fainting. Your choice.

5. Pedro, Arden, Lindsey, and Sandy form a square. They just met at PCMI and now they want to connect themselves. Connect them with paths so the total distance of all the paths is as small as possible. (Two people will be considered *connected* if there is any path from one to the other. Directly connecting is okay, but not required.)

The four people are friends so they will be *PALS*. When they go to the pool, they do *LAPS*. When in Europe, they visit the *ALPS*. If they get in a fight, they *SLAP*. And when Therese joins them for a food fight, they *SPLAT*.



Neat Stuff.

6. Find some right cylinders that have the same numerical value for volume and surface area.

The surface area of these right cylinders includes the top and bottom discs.

7. If Jocelyn tells you a right cylinder's total surface area and total volume, is that enough information to uniquely determine its dimensions? Try it!

These cylinders are just a series of tubes! Not many people know this, but I owned the first radio in Springfield. Not much on the air then, just Edison reciting the alphabet over and over. A, he'd say; then B. C would usually follow.

8. Go back to yesterday's data set and look for a line with the lowest sum of absolute errors. In the m-b plane, where m is the slope and b is the n-intercept, what is the locus of all points (m, b) corresponding to lines with the same "badness"?

9. Open this awesome Geometer's Sketchpad file:

`http://tinyurl.com/comppolygsp`

Investigate how the function $f(z) = z^2 - 2z + 4$ operates on different "magnitude circles" (circles centered at 0). Estimate with very poor accuracy the two solutions to $z^2 - 2z + 4 = 0$. (They are both on the same magnitude circle.)

10. You have 30 seconds. Given $f(x) = x^3 + 6x + 3$, estimate $f(\frac{1}{100})$ and $f(100)$. GO.

29...28...27...

11. Consider the function $f(z) = z^3 + 6z + 3$.
(a) What does the output of $f(z)$ look like when you use a very small magnitude circle?
(b) What does the output of $f(z)$ look like when you use a very large magnitude circle?
(c) Explain how you know that there *must* be a magnitude circle that contains a point z with $f(z) = 0$.

This one has z instead of x. It's *like, totally* different!

If you can't tell, your circle isn't small enough yet.

12. You have 60 seconds. Given $f(x) = x^3 + 12x - 4$. Estimate each of the following. GO.

(a) $f(\frac{1}{100})$
(b) $f(-\frac{1}{100})$
(c) $f(100)$
(d) $f(-100)$

This one probably ended up on the cutting room floor of "Minute to Win It".

13. *Without using Sketchpad*, answer the same questions in problem 11 but for $f(z) = z^3 + 12z - 4$.

14. Convince yourself and everyone else that this is true:

> If $f(z)$ is a polynomial of degree n then there must be at least one complex number z with $f(z) = 0$ in the complex plane.

"Degree n" just means that the polynomial starts $f(z) = az^n + \cdots$

15. Use exterior angles to explain why the stuff with the regular polygons just happened.

16. So the tiling of the two hexagons and the square from the box didn't work out. *Or did it!*
(a) Calculate $\frac{1}{6} + \frac{1}{6} + \frac{1}{4}$. How much more than $\frac{1}{2}$ is it?
(b) How many "missing" degrees are there at the vertex? How many "missing" degrees would there be for 24 vertices?
(c) Is there a tiling with two hexagons and a square at each vertex? What does it look like?

As the Beastie Boys might say, "Another dimension another dimension another dimension another dimension".

17. There's another option: tilings that overlap. For example, a tiling of three heptagons (7-sided figures). What might that look like, and is there any relationship to the fractions?

18. Use Geometer's Sketchpad to construct the locus of points (x, y), whose distance to the point $(1, 0)$ is α times its distance to the line $x = -1$.

It's a plague of locusts! I haven't seen this many locusts since nineteen-dickety-two. We had to say dickety because the Kaiser had stolen our word "twenty".

19. Consider the function $f(z) = z^2 - z$.
(a) Find a solution to $z^2 - z = 1$ using Sketchpad.
(b) Find a way to use this sketch to build a golden rectangle.

20. Use Sketchpad to approximate the five solutions to $x^5 + 2x^2 + 10 = 0$. How many of the roots are real numbers?

21. Use Sketchpad to approximate the four solutions to $x^4 + 3x^2 + 1 = 0$. Yes, there are four solutions and not two. What happens, in general, to even functions? (Tougher: Find the four roots by factoring.)

22. Track down the solutions to $x^3 - 7x^2 + 15x - 9 = 0$. What does a "double root" look like in the complex plane? How many solutions does this equation have?

If the last three digits of your raffle ticket precede 5309, you win! Claim yo prize.

Poor Karen Carpenter. . . Ask Cal to give the punchline.

Review Your Stuff.

Historically, the final day is considered review. Because this is a self-reflective process of discovery, we think that an end product of this discovery might be some summarizing questions of what you might find valuable in this course. We would like these to get at what you think are important mathematical themes in the course, and also themes that might apply to what you teach. We hope this will be a valuable journey, but mostly we just want you to write two problems on any topics that have cropped up in the course. We may or may not use your review questions, depending on how much other material we write and on the color of the paper on which you submit your questions.

Is there really such a thing as a "self-reflective process of discovery"? Yes, there really is! Don't believe us? Ask Google. And don't forget to put quotes around that.

Tough Stuff.

23. Construct a regular pentagon using the quadratic factoring of $x^5 - 1$, then look for quadratic factorings of other polynomials in the form $x^n - 1$.

24. Find some connections between what we've been doing with tangent and Taylor series.

25. Use the Taylor series for $\sin x$, $\cos x$, and e^x to show that $e^{i\pi} + 1 = 0$. Woo!

26. What is one possible value of i^i? (Use a TI-Nspire in radian mode.) How on earth would one arrive at such a thing? Why did we say "one possible value"?

27. Solve this problem then claim your prize:
`http://www.claymath.org/millennium/Riemann_Hypothesis/`
Don't forget to give us 50% of your prize.

We can't bust heads like we used to, but we have our ways. One trick is to tell 'em stories that don't go anywhere, like the time I caught the ferry over to Shelbyville. I needed a new heel for my shoe, so I decided to go to Morganville, which is what they called Shelbyville in those days. So I tied an onion to my belt, which was the style at the time. Now, to take the ferry cost a nickel, and in those days, nickels had pictures of bumblebees on 'em. "Give me five bees for a quarter", you'd say. Now where were we? Oh yeah: the important thing was I had an onion on my belt, which was the style at the time. They didn't have white onions because of the war. The only thing you could get was those big yellow ones.