

Teacher Professional Development in the Teaching and Learning of Functions

Teacher professional development occurs both during the initial preparation of prospective teachers and for experienced teachers. The focus in both should be on content and pedagogy. This brief considers ways to improve and deepen teachers' mathematical content knowledge and their instructional strategies for teaching functions, using an example to illustrate some of the considerations. Other issues about functions in relation to the preparation of teachers are also raised, including the role of technology.

Function is a core concept in the development of mathematics, but it is a concept that can be approached at very different levels. As a result, many challenges have to be addressed by teachers, for example, helping students make connections between different representations of functions. In the 2009 Park City Mathematics Institute (PCMI) International Seminar discussions about practices in the different countries represented, many problems and issues related to the teaching and learning of functions were discovered. To address these, it is crucial that those involved with the teaching of functions at all levels revisit and revise the approach, the language, the practice and the methodology surrounding the teaching of functions.

Background

For pre-service teachers, preparation for the teaching of functions involves both the mathematical content and the methodological and technological approaches to be used in their future teaching. Of necessity the preparation involves teachers of teachers of mathematics and professors of mathematics and of pedagogy. For currently practicing teachers, preparation for the teaching of functions entails professional development so they continue learning and improving their knowledge of and the didactics for the teaching and learning of functions. And both those who prepare teachers and those who deliver professional development for practicing teachers should pay attention to emerging issues involved in the teaching of functions.

All should recognize that the teaching of functions must take technology into account. Technology presents both new opportunities and new challenges for traditional ideas of functions in terms of their representations and uses (See technology brief, 2009).

Two Major Components in Teaching and Learning of Functions

In this brief we identify two major components necessary for the teaching and learning of functions: mathematical content knowledge and didactic knowledge.

1. Mathematical Content Knowledge

Most mathematics professors know the mathematics of functions. However, they need to consider how prospective teachers learn functions and how their learning may be improved with the use of new methodological and technological approaches. Efforts should be made to help professors responsible for teaching teachers or prospective

teachers consider ways and approaches to think about what a function is and how it may be taught in schools.

Just as professors know about functions, teachers also have a content knowledge of functions. However, teachers in schools usually teach the way they were taught at the university. As a result, it is imperative that mathematics professors consider different methods and sequences for the teaching and learning of functions. For example, the language of functions is problematic. When is informal language acceptable and when is formal language needed?

Ways to improve and deepen teachers' mathematical content knowledge of functions include the following:

- Courses/workshops given by experts for developing teachers' content knowledge about functions. Such courses might include modeling of situations with explicit uses of functions to solve problems in sciences, mathematics, social sciences and other disciplines.
- Competitions, demonstrations/exhibitions, written materials/articles about the content, methodology and use of technology in teaching functions. These activities can be used as learning experiences for teachers and may involve students' school work to exhibit explicit uses of functions.
- Forums/electronic communication dedicated to study of functions. For example the work of the PCMI International Seminar is exhibited at www.mathforum.org/pcmi. Such forums and electronic communications provide a method to efficiently share knowledge and techniques locally and across the world.

2. Didactics for the Learning and Teaching of Functions

School teachers have to know and understand how students conceptualize the notion of function and realize the obstacles and misunderstandings of students. Teachers need to identify problem situations involving mathematics, model the situations by mathematizing them, and then use mathematized models to find acceptable solutions to the situations or problems. Modeling problem situations helps to broaden the concept image of function (Hazzan and Goldenberg, 1997).

The didactics aspect includes modeling from one domain of mathematics to another one; deciding about the dependencies of variables in functions, and even the choice of variable. Function can be treated as unifying concept among different mathematical domains; teachers need to understand this perspective and help students make the connections. The following problem might be used with teachers to illustrate how function can be a connection between geometry and algebra while using technology.

Given a trapezoid $OABC$ where $OA = 4$ cm, $AB = 2$ cm, and $OC = 6$ cm with right angle AOC ; M is a point moving on the segment AO . Triangle MBC and rectangle $MNPO$ are such that N is on segment BC and P is on

segment OC . Students are asked to examine the area S_1 of the triangle MBC and the area S_2 of the rectangle $MNPO$.

Figure 1 shows the trapezoid $OABC$ with triangle MBC (and its altitude MH) and rectangle $MNPO$. Additionally, the right side of the screen shows beginning calculations of the areas for one location of M .

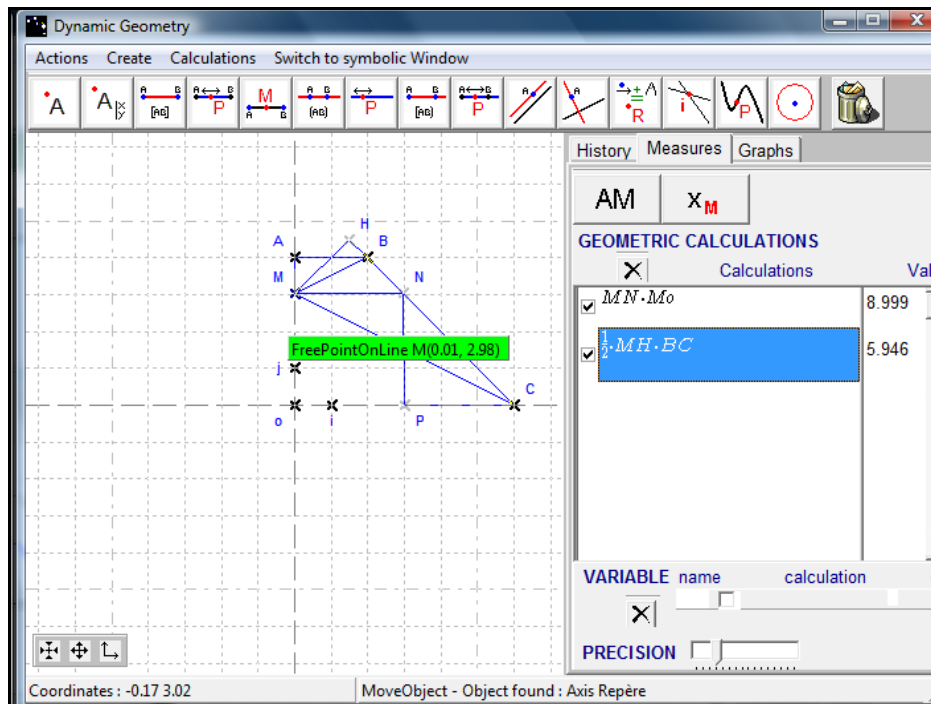


Figure 1: Trapezoid $OABC$ created using Dynamic Geometry

One aspect of the professional development might be to investigate the value of the following questions to promote student understanding of function:

1. Calculate S_1 and S_2 when $OM = 1$ cm, $OM = 2.5$ cm, $OM = 3$ cm.
2. Determine the functions expressing the areas S_1 and S_2 .
3. Find M such that $S_1 = 4.5$.
4. Find M such that $S_1 = S_2$.
5. Find M such that $S_1 > S_2$ or $S_1 < S_2$.
6. Find M such that the area of $OMNP$ is maximum.

Question 2, requires a decision on one variable that could be used as the independent variable to express the areas as different functions. Figure 2 shows one choice of variable, x , the length of AM , some simplifications of the functions, and the resulting graphs of the functions.

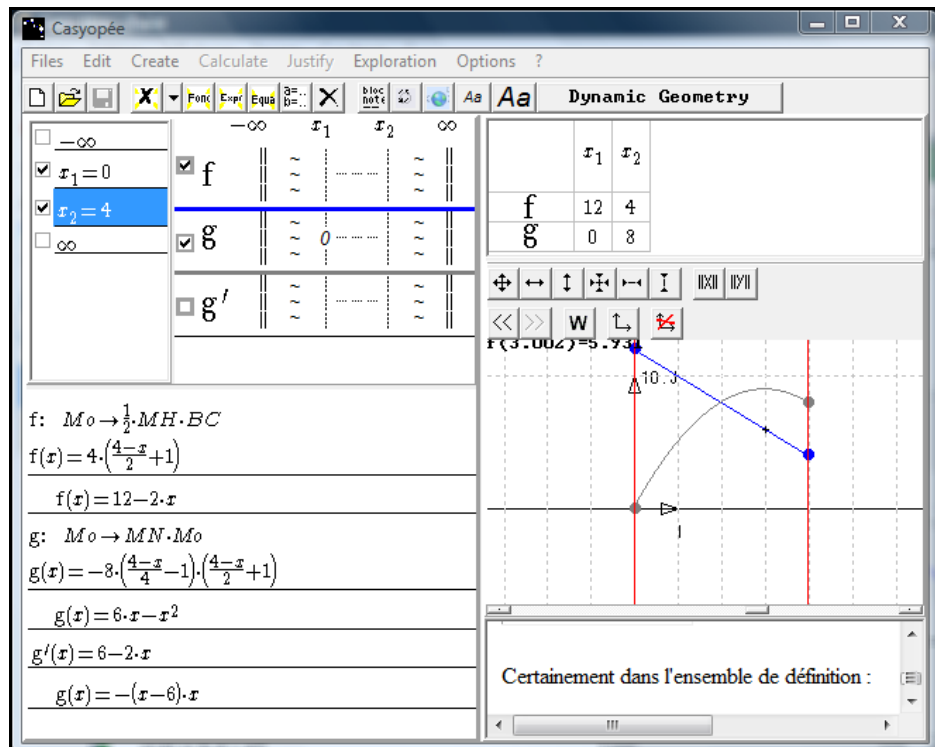


Figure 2. Calculations from Figure 1

The Casyopée software (see http://jb.lagrange.free.fr/site/index.php?option=com_content&task=view&id=23&Itemid=37.) used in Figures 1 and 2 simultaneously illustrates the variability in graphical, symbolic and geometric representations and makes visible the connections among these representations. Note that in this problem, the students construct the geometrical figure on dynamic geometrical window first, choose a variable to determine the functions, and then explore these functions in an algebraic window. There are many choices of variable among different dependencies; one was used in the figures. Teachers may need to help students choose a variable to aid in the understanding of function.

Giving teachers such an example can be used as an opportunity for them to discuss different representations of functions and how to think about the dependency of quantities (Hazzan and Goldenberg, 1997; Lagrange and Artigue, 2009). Questions that teachers might consider are the numerical aspects of the functions involved and the connection among geometrical and algebraic representations (Thompson, 1994 and Duval, 2006).

A useful but time-intensive effort to help teachers with didactics is to have experts or peers visit classes in schools or work in groups in lesson-study type situations to analyze how to improve teaching and learning of function. This method may require support of administrators, but it allows teachers to develop their understandings of students' learning processes about functions and any arising misconceptions.

Other Issues Related to Professional Development and the Teaching of Function

While the use of technology for teaching and learning about functions has much promise, it brings with it significant issues of professional development. Issues include:

1. Functions can be used to model many real life situations. However, teachers need to learn how to discover the hidden functional relations behind these situations. While such modeling can improve motivation and attitude of both teachers and students, providing an environment where this can happen may be a challenge. For example, extracting functions out of real life situations and making them appropriate mathematically for students may be difficult for many teachers.
2. Most teachers are committed to improving learning - including the learning of functions. However, overcoming the traditional modes of teaching functions may entail changing attitude and engaging support from ministries and departments of education. Teachers at all levels of mathematics education need to work together to promote this change.
3. While the use of technology for teaching and learning about functions has much promise, it brings with it significant issues of professional development. These issues include:
 - a. Technology can enrich the cultures of diverse people and improve the teaching of functions. As a result, teachers need to know how to use the technology and be comfortable with its use in their classrooms. Students may help teachers learn technology, but the teachers will have to help the students use the technology to learn mathematics, in particular functions. Technology gives students representations of functions. Hence, the role of the teachers changes. Their preparation should enable them to ask the questions that help students understand for example, the properties, definitions, and representations of functions and to solve problems involving functions.
 - b. For many teachers and students, the use of technology to study functions may be or seem easier than without the technology. In reality for some, the understanding of the technology itself, with its own complexity, may cause issues in the learning of functions. Professional development should ensure that these issues are recognized and addressed for teachers.
 - c. Use of technology can contribute to cultural development. However, it can be extremely difficult. A major challenge in developing countries is how to introduce and share the benefit of adequate use of technology with native people whose languages and cultures are traditional. Intercultural and bilingual education may create situations where the lack of language and culture about functions pose issues. Teachers have to be prepared to work on these issues.

Epilogue

Changing teaching and learning about functions is not trivial. Many of the concepts are

subtle and to make them visible for teachers demand a review of functions at all levels from that at the professoriate to the classroom. It also demands that professional development consider the use of technology and may very well demand a reconsideration of how function fits into the school curriculum.

References

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