



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Round 20: Understanding the Problem

(focus: Wondering)

In our third time through *Understanding the Problem* we focus on wondering, on those questions that problem solvers ask as they try to understand the problem and find a solution path. It is usually the wondering that leads us to a solution. First we notice. Then we wonder about meaning and implications and possibilities. Then we pursue those questions and work our way to a solution.

As we've worked on noticing and wondering with students, we've noticed that their wondering is often interesting but could be more mathematical. Persistence is an important part of the process, such as investigating aspects we wonder about, even if they don't seem promising. It helps to have a repertoire of reflective questions (wonderings) that help us get unstuck, connect to prior knowledge, and use our noticings to solve the problem. In this round we have shared a *lot* of questions that have been useful to us.

The first activity contains a long list of great problem-solving questions. Students can use them to self-reflect (perhaps they can be on a handout or poster) and to ask their partners good questions. As teachers, we can also use them to prompt struggling students. After asking a student a good question, we take a moment to reflect with them on what we asked them and how it helped, so they can begin to ask it themselves.

The second activity focuses on how good wondering questions can help students at particular phases of problem solving:

- When they're missing something.
- When they're trying to connect their noticing and wondering to prior knowledge.
- When they have generated noticings but need to apply them to the solution.

We've provided some specific questions and questioning structures students can use when they or we have identified how they're stuck.

Finally, we provide a structure for students to reflect on the wondering questions that helped them, and give them a task to organize different types of wondering questions. This helps them see how different questions can be helpful at different times, and to remember when different kinds of questions are helpful.

The activities are written so that you can use them with problems of your choosing. We include a separate section afterward to show what it might look like when students apply these activities to the current Algebra Problem of the Week.

Problem-Solving Goals

Focusing on wondering can help problem solvers:

- More fully understand the problem.
- Develop a repertoire of meta-cognitive questioning techniques.
- Have an entry point into selecting and carrying out a strategy.

Communication Goals

Students focusing on wondering can use the writing process to:

- Keep track of ideas that come up that they might choose not to address right away (parking lot).
- Organize answers to questions they ask themselves, for later use in carrying out the problem.

Activities

I. Wonderama

Format: Students working individually or in pairs.

Sample Activity

Write down as many wonderings as you can about the mathematical quantities and relationships in this problem.

To help, here are some wonderings we find ourselves using a lot:

- What does this mean?
- What do they want?
- Does it have to be that way?
- Do I need to figure that out?
- How does this situation work?
- Is there another way to think of it?
- How will I know if this is true?
- What is a good way to express that?
- When is this true?

From the strategies:

- What guess can I make?
- How would I check if my answer was wrong?
- How could I make this simpler?
- What must be true? What might be true? What can't be true?
- What are different ways I can represent this situation?
- What cases can I think of?
- What model can I make for this situation?

Spend some time wondering, and write down new noticings that come to you as you explore the questions you think of. It can be helpful to work with a partner, who can ask you questions and help you record your ideas.

Key Outcomes

- Generate questions that give momentum to problem solving.
- Use wonderings to drive focused noticings.

II. Wondering out of the Wilderness

Format: Students working in pairs.

After we have been working on a problem for a while, we often need to do some more wondering in order to get some fresh ideas or to see how we can use the thinking we have done so far. There are three questions that we use, often together. What am I missing? How does this fit what I have learned before? Again, what am I trying to figure out?

Sample Activities for “What am I missing?”

“What am I missing?” can be a hard question to answer, since you don't know what you are missing. But there are tricks we use to help us discover what we're missing. Here are typical ways we wind up missing things and activities that can help us make progress:

- A. It might be that you **already noticed** the thing you're missing, but you didn't really see it yet or you just need to think about it some more.

What to do:

- Read your list of noticings and wonderings to your partner while they read over your shoulder. Explain anything you think is unclear. Add anything you left out.
 - As you read them the list, you will probably say things that you didn't write down. Your partner will notice those things and tell you what they were. Write them down and play those out more.
 - If you had an idea but it seems too hard or complicated to write it down and play it out, ask your partner to do it with you. Often a hard approach will give us some clues, even if we can't finish it.
- B. It might be that you **made assumptions**: early decisions about things that seem obvious or right.
- Try to ask these assumptions questions about everything you are trying:

"Why did I think/do/mean ____? Could it be different?"
 - Ask your partner to notice and ask about assumptions when you think you aren't finding enough of them. Your partner will try to spot any time you *assumed* something had to be true or couldn't be true, and ask you, "How do you know? Could it be different?"
- C. It might be that you **don't need** the information you are missing.
- Imagine for a moment that you don't need it. Try to tell your partner another way to use the information that you do have.

Sample Activity for "How does this fit what I learned before?"

Play it out: When we think about what this situation reminds us of, we often go very quickly, in order to search for good ideas. Sometimes we need to come back and think about the connections more carefully. See what happens if you use the following questions to really think through the connection of this problem to strategies or approaches that you have used before or that this reminds you of:

- What are the ways I know to get this kind of info or to do this? Have I really tried them all? Really?
- How might this be different than the kind of situation I've seen before? How would I deal with that or know if it matters?

Sample Activity for: "Again, what am I trying to figure out?"

Use the following wonderings to say again what you are trying to figure out in order to remember, to connect ideas, and to see how your understanding of the problem is changing as you make progress.

- How does the stuff that I notice and wonder connect to what I have to figure out?
- Is there another way I could say what I'm trying to figure out, based on my noticings and wonderings?
- What do I need to figure out before I can solve the problem?
- What counts as an answer? Do I have an answer that is sort of good enough? What would it take to make it better?

Key Outcomes

- Get unstuck by uncovering missing quantities and relationships.
- Activate prior knowledge and use it to look for clues and generate strategies.
- Connect noticings and wonderings back to the original problem.
- Generate sub-problems or reframe the problem in terms of given information.

III. Wonder What We Did

Format: Students working in pairs after they have solved the problem.

Look over your work. Take turns interviewing each other using the following questions.

- What questions did you ask yourself that led to new ideas and things to do that helped you find a solution?
- What kinds of questions are those? Can you give them a name or organize them so that you remember to use them next time you need them?

Key Outcomes:

- Reflect on the usefulness of different types of wonderings.
- Think about the relationship between different ways of understanding or representing the problem, and the different strategies they lead to.