



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Solve a Simpler Problem

Sometimes we see a way to solve a math problem, but it looks like it would take too long or be too complicated. Maybe the numbers are too big or there are too many lines or there is an extra step and we are not sure how to handle it.

Solve a Simpler Problem is a technique that can be used in several ways to solve challenging problems. In some situations you can see how to work the problem with easier numbers. This may show you an approach that you can try with the more difficult numbers. Second, you can choose to break the original problem into smaller steps, finding answers for parts of the problem, and then putting those together for the whole solution. Finally, students may see a way to change this hard problem into one that they have solved before.

The activities below provide a way to learn how to make problems simpler and to figure out the connection of those results to the original problem. The activities are written so that you can use them with problems of your choosing. We include a separate section afterward to show what it might look like when students apply these activities to the current Algebra Problem of the Week.

Problem-Solving Goals

Setting up simpler problems can help problem-solvers:

- More fully understand the problem.
- Find patterns.
- Discover and generalize important relationships.
- Focus on the specific mathematics that they need to figure out or learn.

Communication Goals

Writing can be a way to become clear about what you know and what you need to figure out. The *Simpler Problem* strategy can be used to drill down to the smallest, most specific question or issue that would allow you to make progress on a problem. The goal of all of the strategies in this series is to develop skills that build confidence in your own thinking. We use the writing to get us to the point where we are ready to learn, where there is an idea or piece of information that we are prepared to use in order to make progress. We are trying to overcome the habit of asking broad questions too early that lead to being shown what to do without being ready to learn or remember it. Students that attempt to *Solve a Simpler Problem* can use the writing process to:

- Organize what they understand and what is hard or confusing.
- Develop good questions that are specific and that help you focus your investigation or enable a helper to give useful answers that support learning.
- Focus on promising ideas that you might skip over as you think about the harder problem.

Note: There is an implicit assumption throughout this *Activity Series* that users know that they can also be making use of strategies that have been introduced in prior weeks. In particular, *Understanding the Problem* was developed as a set of strategies that is always useful and to some extent assumed to be in use, even when focusing on a new strategy, such as *Simpler Problem*.

Activities

I. Making it Simpler

Format: students working individually or in pairs, and then sharing in groups of 4-6 students.

One first step in making a problem simpler is to identify what's hard about the problem. This may seem obvious, but taking the time to write these aspects down can enable effective brainstorming about good ways to change the problem.

Sample Activity

Work individually or in pairs to begin filling in the blanks of the following prompts for just a few minutes. Then share ideas with the larger group of 4-6 students. At this point, you should be focusing on brainstorming and generating as many ideas for making the problem simpler as you can. Don't solve the problem yet.

Fill in as many of the blanks as you can in the following *Simpler Problem* sentences:

- 1) One thing that makes this problem hard is _____.
- 2) I can make a simpler problem to solve by _____.

It is a good idea when brainstorming in the group to write down the ideas stimulated by hearing from each other. After brainstorming, discuss which simplification idea each of you thinks is most promising or interesting.

Key Outcomes

- Recognize elements that make math problems harder and to not get stuck because of them.
- Explore some different ways of making problems simpler.
- Know how to break problems down into smaller problems and find a place to start working it out.
- Learn to use brainstorming to open up ideas before you try to solve a problem.

II. Working on the Simpler Problem

Format: Individually or in pairs (see above).

The focus in this activity is on using writing to organize the problem solving activity, to notice patterns or ideas that make the solution possible, and to ask specific questions that need to be answered in order to make progress,

Sample Activity

1. Pick a promising idea from the brainstorming session and solve one or more simpler problems. Use the following to guide you as you work on the problems:
 - **Organize your problem solving** using an outline or a table so that you can see the different steps and keep them in order.
 - **Highlight** the steps where you noticed a pattern or something helpful in solving the problem.
 - When you come to a place where you are stuck, **write a question** that explains where you are in your approach and what you need to know.
2. Rejoin your group of 4-6 students. Go around and each person to **share one thing they noticed** when doing the simpler problems that might be useful in solving the problem. Then go around and give each person a chance to **ask one specific question** for which they need an idea or some information.

Key Outcomes

- Ways to organize problem solving to keep track of sub-problems and iterations.
- Learning to narrow down questions to get specific, meaningful help and prepare to learn something.
- Using writing to separate what you know and can do from what you still need to figure out.

III. Learning from the Simpler Problem

Format: Students working individually or pairs and then sharing with the whole group.

Sample Activity:

1. Use the best ideas from your group and finish solving the problem as best as you can.
2. Write a very clear explanation of the insights that enabled you to solve the problem. What were the key ideas that you used in solving the problem?
3. Submit your write-up to the PoW online. (Optional)
4. Use a jigsaw or gallery walk format to share your explanation with classmates and to appreciate their insights. (Optional)

Key Outcomes:

- Consolidate understandings of ways to make problems simpler.
- Learn clear organization and presentation of ideas.
- Understand how solutions to simpler problems can be used to solve more complex problems, i.e. generate useful patterns, provide pieces you can put together for a solution, show another way of thinking about the problem, etc.