The Math Forum: Problems of the Week

Problem Solving and Communication Activity Series

Round 12: Change the Representation

All math problems, whether they are word problems, arithmetic problems, equations to solve, etc., come to us in a particular representation. Word problems are represented in story form, using words and often referencing a particular context. Arithmetic problems are represented numerically. Equations are represented using mathematical symbols. Each representation has benefits to the problem solver. For example, word problems allow students to apply their knowledge of the given context, which can allow them to check that their approaches are reasonable. Numeric and symbolic representations can make it easy for students to manipulate objects in the problem, and to quickly see patterns. Visual and physical representations, such as manipulatives, diagrams, and graphs, can often help students gain new insights into the problem and provide them with additional tools for solving it. Changing the representation can mean use of a different form of representation (e.g. using a line drawing for a word problem) or it can mean trying different ways of presenting the information in the same form (e.g. rewriting all of the numbers as fractions with a numerator of 1). Considering multiple representations and choosing representations that fit the problem well are important problem-solving skills.

The activities below help students to brainstorm and work with multiple representations, and compare what they learned about the problem using their different representations.

The activities are written so that you can use them with problems of your choosing.

Problem-Solving Goals

Changing the representation of a problem can help problem-solvers:

- Strengthen their understanding of the problem.
- Gain new insights into the problem or solution.
- Provide additional tools for solving the problem.
- Find multiple solution paths, leading to deeper mathematical connections.

Communication Goals

Changing the representation of a problem requires that students change how they communicate about the problem and find different ways to express the same idea or information. They might:

- Paraphrase the problem in terms of a different representation.
- Re-tell the story of the problem with a different context.
- Organize the numerical and calculation strategies using a table or other organizational method.
- Use mathematical symbols to restate the problem succinctly.
- Use diagrams to communicate the math in the problem.
- Represent the problem graphically.

Activities

I. Brainstorm Representations

Format: students working individually or in pairs, then sharing with groups of 4-6.

There are many ways to represent math problems and mathematical ideas. Math problems are often represented in words. Math can be represented visually, through graphs, diagrams, and sketches. Tables and expressions can be used to represent math ideas numerically. Mathematical ideas are often represented symbolically, with operations, numbers, variables, and functions. Each representation can help you understand and solve the problem in different ways. Problems represented in words help you to make sense of the problem and use your knowledge of real-world situations. Graphical representations can lead to new insights or problem-solving methods. Numerical

representations can help you find patterns and generate strategies. Symbolic representations represent math ideas clearly and succinctly, and help you to manipulate the mathematical objects.

Sample Activity

Work individually or in pairs to begin filling in the blanks of the following prompts for just a few minutes. Then share ideas with the larger group of 4-6 students. The first question asks you to think about the math ideas in the problem, which might get you thinking about other representations you know. The second asks you to think as creatively as you can.

1) The main mathematical ideas and relationships in this problem are _____.

2) I could represent this problem by: (drawing a picture, using some blocks, making a graph, writing some equations, telling a different story, writing the numbers or expressions in a different way).

3) The idea I want to try is _____.

Share your thoughts with your group. If the ideas you hear spark other ideas, record those too. Try to brainstorm as many possible representations as you can.

Suggestions

These suggestions can be used as a place to start with a student who is struggling.

- Think of or make up another story or situation that could be used to present a math problem just like this.
- Think of manipulatives like blocks or chips or algeblocks or fraction bars or the number line or a drawing that you could use to model and explore this problem.
- Think about substitutions you could make for some of the quantities in the problem (rewriting whole numbers as fractions; express all of the quantities in terms of the smallest item; use a different expression that has the same value but might make the calculations easier)

Key Outcomes

- Identify key mathematical ideas in the problem that can be represented.
- Think of ways that mathematical ideas are sometimes represented.
- Generate multiple possible representations for the problem to be solved.

II. Representing

Format: Individually and then in pairs or teams.

The focus in this activity is on using writing to organize the problem solving activity, to notice patterns or ideas that make the solution possible, and to ask specific questions that need to be answered in order to make progress,

Sample Activity

Step 1: Pick the representation that seems most useful or stimulating and play it out in detail. Use your new representation to explore the problem and solve it if you can.

Step 2: Explain to your partner how your new representation works:

- What did you notice about the problem when you changed the representation? What new information, relationships, patterns, and approaches occurred to you?
- What have you been able to figure out for a solution so far?
- If you are stuck, which representation shows best where and why you are stuck? Ask your partner(s) if they have a way of representing the problem that helps you get unstuck.

Step 3: Check your solution with the original problem to make sure it fits with all of the information and constraints of that situation.

Key Outcomes

- Play out a particular representation as fully as possible.
- Consider multiple representations when you get stuck.
- Work with others to see multiple perspectives and fresh ideas.

III. Comparing

Format: Students working individually or pairs and then sharing with the whole group.

Sample Activity:

Step 1: Within your group of 4-6, compare the different solution paths you generated. Did different representations lead to different insights? In what ways are the different solutions similar?

Step 2: Select the representations that provided the most insight into the problem or key steps in the solution. Prepare a presentation for the class on how each representation helped you get to a key insight into the problem.

Step 3 (optional): Submit your write-up to the PoW online.

Step 4 (optional): Use a jigsaw or gallery walk format to share your explanation with classmates and to appreciate their insights.

Key Outcomes:

- Compare insights generated by multiple representations. Identify the similarities and the differences in the contributions from each representation.
- Evaluate how well different representations fit a given problem. Figure out how to recognize when a particular way of changing the presentation of the problem will be useful.