



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Round 13: Make a Mathematical Model

A mathematical model is a way to describe a situation, usually real-world, using numeric and mathematical relationships. Mathematical models usually have inputs, operations on those inputs, certain parameters or constants that make the operations fit the particular situation, and outputs that result from performing the operations on the inputs.

While students in Algebra and beyond use mathematical models with functions and variables, students of all ages describe situations mathematically. An Algebra student might mathematize the idea “apples cost fifty cents each,” as $p(a) = 0.5a$, but an elementary student would be expressing the same idea, as mathematically as possible, when they say, “divide the number of apples by 2, because it is 2 for a \$1, and \$.50 if one is left over.”

Sometimes in problem solving, coming up with the mathematical model to use is at the heart of the problem. Problem-solvers are engaged in noticing quantities and relationships, selecting operations to describe the relationships, and fitting those operations to the specific scenario by setting parameters.

Other times, the operations and relationships are given in the problem, and the problem solver’s job is to organize the information and apply it to determine a final answer.

In either case, identifying quantities and relationships, and recording information as mathematically as possible are key components of making mathematical models.

The activities below help students to organize and write succinctly the mathematical quantities and relationships they observe.

The activities are written so that you can use them with problems of your choosing.

Problem-Solving Goals

Making a mathematical model can help problem-solvers:

- Organize information given in a problem to streamline the problem-solving process.
- Represent the essential mathematical aspects of a system.
- Develop rules and algorithms that can be generalized to multiple situations.
- Generalize and formalize insights.

Communication Goals

Problem-solvers who are making mathematical models use the writing process to:

- Help them organize the quantities and relationships they observe.
- Label and define quantities clearly.
- Make sure their math is represented as succinctly and symbolically as is age-appropriate.

Activities

I. Organizing Quantities and Relationships

Format: students working individually or in pairs, then sharing with groups of 4-6.

If students have not recorded their “Noticing and Wondering” from the *Understanding the Problem* activities, they will need to do that before doing step one of the sample activity below.

Sample Activity

Mathematical models pay more attention to the **quantities** and the **relationships** between them than they do to the specific **values**. Mathematical models can be built by identifying the key **quantities** in a situation and putting them together in a **relationship** using operations such as addition, subtraction, multiplication, division, etc.

Example: A simple model for an apple farmer's profit:

$$(\text{Apple's price} \times \text{number sold}) - (\text{Apple's cost to grow} \times \text{number of apples}) = \text{farmer's profit}$$

Once you have written the model, it is easy to plug in different values for your quantities to see what happens.

Work individually or in pairs on the following activities for just a few minutes. Then share ideas with the larger group of 4-6 students.

- 1) Organize your list of noticings and wonderings into these two categories: "Quantities (things I can count or measure)" and "Relationships Between Quantities" (addition, subtraction, multiplication, division, equality, etc.)
- 2) If you haven't already, write the values that you know next to the names or descriptions of the quantities, for example in parentheses. Be sure to include units. Example: cost of an apple (50 cents)
- 3) Write mathematical sentences or equations for the relationships that you understand already, using the names or descriptions of quantities (don't plug in the values yet).
- 4) Share your lists and sentences with your group. Identify the quantities whose values you still need to figure out and the relationships that you don't know how to write as a mathematical sentence yet.

Key Outcomes

- Organize and clearly label known information.
- Begin grouping known quantities into relationships.
- Identify which relationships and quantities are known, and which are unknown.

II. Choose Your Approach: Building Up or Breaking Down

Format: Individually and then in pairs or teams.

When making a mathematical model, some people focus on breaking down the quantities or relationships they don't know, by wondering, "Where does this come from? What is it based on?" Other people focus on building up relationships from known quantities and given information. They wonder, "How are these known things related? How can I express this relationship?" Sometimes you start with one kind of wondering, and switch to another if you get stuck or have a new insight into the problem.

Sample Activity: Choose your Approach

Look over your responses from activity 1, *Organizing Quantities and Relationships*. 1) Do you see quantities that you can combine into relationships to form a mathematical model? 2) Can you see ways to break the unknown quantities and relationships into parts that you can figure out?

Pick one of the following approaches to build your model:

- 1) Building Up:** If there are quantities that you have figured out, and you want to make a model with them, you might use the following questions:
 - What is the relationship between these quantities?
 - What operations would I use to combine these quantities? How would I label the result?
- 2) Breaking Down:** If you have a general model for what you need to figure out in this problem, but don't know the value of some of the quantities, then you might try to break down the unknown quantities and relationships with the following questions:
 - Is this quantity that I don't know the value of related to some other quantity whose value I do know?
 - Can I break this relationship down into smaller parts until I get to something I know the value of?

When you get to a stopping point (a model that you think works) or to a place where you're not sure how to keep going, share your work with a partner and see if you can learn anything from each other's approach.

Key Outcomes

- Organize the noticed quantities into increasingly comprehensive mathematical models.

- Work step-by-step to continuously refine or expand models.
- Explore multiple methods of approaching a modeling problem.

III. Checking the Model

Format: Students working in pairs.

Multiple strategies are presented below for checking models. Students should complete as many as possible.

Sample Activity:

Compare: With your partner, compare your models. Are they exactly the same? Do they give the same results when you plug in the same values? Is it possible to rewrite the operations so that the models become exactly the same?

If the models are different or give different results, check your work. Is the arithmetic correct? Are the assumptions the same? Do you agree about the relationships?

Try Values: Identify the “constraints” of the problem – the rules or information given in the problem that the result should match. Try plugging some values into your model. Do the outcomes fit the constraints?

Reflect: If the problem had started with different values, could you use your model to work with those different values or would you have to change it? Does your model work for all the different situations of this type of problem?

Key Outcomes:

- Use different strategies to check the models.
- Make sure that models fit all given constraints.
- Evaluate models on the basis of the ease of generalizing them.