



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Round 15: Planning and Reflecting

We have presented the following problem-solving strategies:

- *Understanding the Problem*
- *Guess and Check*
- *Solve a Simpler Problem*
- *Making a Table*
- *Cases*
- *Logical Reasoning*
- *Change the Representation*
- *Make a Mathematical Model*
- *Working Backwards*

As students get comfortable with more strategies, they begin to recognize and use multiple approaches in their problem-solving process. Following Polya (1945), it is common to break the problem-solving process down into four phases: Understanding the Problem, Making a Plan, Carrying Out the Plan, and Checking/Reflecting. Two of the most common issues in problem solving are (1) forgetting part of the problem or the ideas you have discovered that might be useful and (2) getting stuck, trying the same thing over and over without making new progress. Planning and reflecting can help you both solve problems and learn from your experience. What does it mean to get good at making a plan? How do I know when to start carrying out the plan? How do I know if I'm on a dead-end path? How do I effectively check my work?

The activities we have presented so far this year could all be organized into the four stages of problem solving.

1. **Understanding the Problem** (e.g. *Noticing and Wondering* from round 1)
2. **Making a Plan:** (e.g. *Building up or Breaking Down* from round 13)
3. **Carrying out the Plan:** (e.g. *Representing* from round 12)
4. **Checking/Reflecting:** (e.g. *Testing 1, 2, 3, 4* from round 7)

In addition, the structure of most of the activity series documents has been “(understand) plan, do, reflect and revise.” The questions featured in the first activity of each activity series can be helpful in planning and selecting strategies (for example, for *Solve a Simpler Problem*, students asked themselves “what makes this problem hard?”). The middle activities often helped students to organize their work as they carried out the given strategy. Finally, the reflection and revision questions in the final activity offer good ways for groups of students to revise together.

The activities below focus in particular on the planning and reflecting stages, particularly on helping students organize and record for their own and others' use the thinking that they do. These activities may be most useful to students who have learned some of the individual strategies covered earlier in the year.

The activities are written so that you can use them with problems of your choosing.

Problem-Solving Goals

Planning helps problem solvers to:

- Use what they notice and wonder effectively.
- Select an efficient strategy.
- Step back from their problem solving and evaluate what they have figured out so far and whether they are on a useful path.

Communication Goals

Problem solvers use writing to help them plan by:

- Writing down what they notice and wonder so that they can make use of their ideas and come back to them if they get stuck.
- Organizing and recording their planning so that they can:
 - Refer back to it easily to get other ideas and remember other parts of the problem solving process
 - Share it with others.

Activities

I. Drafting a Plan

Format: students working in groups of two to four.

This activity is designed to get students talking and planning together before they dive into a particular strategy. When we reflect on what we notice and what we hope to get from our strategy, we are better able to evaluate whether a strategy is working for us, and, if the strategy is not working, we have a list of other noticings and other possible strategies to make use of.

Sample Activity:

- 1) As you generate your lists of noticings and wonderings, you might want to write each noticing or wondering on a separate index card (or sticky note, spreadsheet cell, mind-map bubble, etc.) so that you can move them around later to reorganize your plan and easily add in new items.
- 2) Organize your noticings and wonderings into categories, for example:
 - a. **Info to use** – List the information that you want to use and the info you want to figure out in your plan:
 - i. Quantities (things you could count or measure) or objects (lines, points, shapes, etc.).
 - ii. Relationships between quantities (equals, less or more than, added or subtracted, multiplied by, parallel to, perpendicular to, etc.).
 - b. **Strategies:** Identify possible approaches to the problem (e.g. draw a picture of _____, make a table, make guesses for this quantity and check them by _____, simplify the problem by _____, play the movie backwards, etc.).
 - c. **Parking Lot:** Other things you noticed or wondered about, but aren't sure will be useful right away. Ideas about the solution or extensions of the problem (e.g. estimates, constraints, ways to check your answer, wonderings about what would happen next, new problems you've thought of, etc.).
- 3) Arrange the **Info** and **Strategy** index cards that fit together into a plan that you want to try. Make sure your plan includes all of the questions you have to answer for the problem and a way to check your answers. Does your plan show what you will know or will have figured out after each step?

Key Outcomes

- Work together to articulate and develop their noticing and wondering.
- Organize noticings and wonderings that will help with various strategies.
- Figure out what you should accomplish at each step of your plan.
- Form a “big picture” where you can see what you are trying and what else you might use if that doesn't work.

II. Where Am I?

Format: students working individually or in pairs.

This activity helps students remember to periodically stop and reflect back on their plan, to help them get “unstuck” or remember what else they need to do. If they have the whole plan in front of them--the strategies they have not selected and the noticings and wonderings they have not yet used--this can help them be more flexible and aware as they execute their strategy. It also helps them keep track of the interesting ideas that they discover as they work on the problem. In the best circumstances, problem solving is not just about getting a final answer, but about having mathematical thoughts worth pursuing.

Sample Activity:

- 1) Carry out your plan. As you notice new facts or wonder about new questions, add these to your original list.
- 2) Work on your selected strategy until:
 - a. You get stuck.
 - b. You figure something new or useful out.
 - c. You have done enough initial exploration of your strategy (often five to ten minutes is enough time to get started and be ready to think about how it is going).
- 3) **Step Back** - Look back at your plan and your original lists. Think about one or more of these questions:
 - a. Are there strategies or noticings and wonderings that I haven't used that might help me?
 - b. What did I hope to get out of my chosen strategy? Is it working out? What have I figured out so far?
 - c. Where am I in my plan? Is there anything I need to remember to do or to use? Is it time to try something else?
- 4) Continue in this way, working for a while on a strategy in steps #1 and #2, and then stepping back to ask the questions in step #3 and think about where you are and what else you want to keep in mind or make use of.

Key Outcomes

- Develop flexibility in carrying out problem solving.
- Develop tactics for getting “unstuck.”
- Increase awareness of multiple strategies and solution paths.

III. Reflection: Telling the Story

Format: students working in pairs (or each partnership pairing up with another partnership).

Finding a solution is really only half of what problem solving is about. Reflection gives you a chance to appreciate what you have accomplished and to make sure your new ideas and advice will be there when you want them next time. We use reflection to both get better at problem solving and to understand math concepts more deeply by noticing connections between different strategies and ideas.

After you have found a solution, your plan can now be used as an outline from which to tell the story of what you learned. In order to get better at noticing, wondering, and planning, you can reflect on what went well and what you wish you had done differently or what habits you might want to develop. Telling the story that connects noticings or insights to the strategies that you used and how you eventually solved the problem lets you reflect on how you can get unstuck and find more efficient or more interesting solutions.

Sample Activity:

- 1) Look back over your plan and what you did as you worked on the problem. Talk about the insights and breakthroughs you had. What did you notice that led you to those breakthroughs? How would you do that better next time?
- 2) Discuss the different strategies that were used. What can you learn about the math of this problem by showing how the different approaches fit together?
- 3) What math questions and ideas did you not get a chance yet to explore? What are the interesting noticings and wonderings that you might want to come back to and think about some more?

Key Outcomes:

- Identify ways to improve problem solving.
- Deepen the understanding of the math in the problem.
- Appreciate and hold onto the interesting ideas and questions you came up with.

References

- Polya, G. (1945). *How to solve it*. Princeton, New Jersey: Princeton University Press.