



The Math Forum: Problems of the Week

Problem Solving and Communication

Activity Series

Round 1: Understanding the Problem

Understanding the problem is the first stage of most descriptions of problem solving (for example, mathematician George Polya's (1945) *How to Solve It* or math educators Stephen Krulik and Jesse Rudnick's (1987) read and explore stage described in their book, *Problem Solving: A Handbook for Teachers*). But what does it mean to understand a problem? How do problem solvers get better at understanding the problem? How do we get past the "I don't get it" stage?

We have found that taking time to notice and wonder and focus on understanding before focusing on solving also gives students a place to begin when they are stuck or are not confident in their problem-solving ability. Encouraging the habits of noticing, wondering, and understanding early in the year can help create a classroom culture that focuses on problem solving as a creative process rather than as being just about getting to the right answer quickly. In addition, it gives every student a way to contribute mathematically: even if they are not sure how they would solve the problem, they can share interesting noticings and wonderings. Again, the focus is on thinking and creativity, not correct answers.

Included in this document are several activities that support students to develop strategies for understanding challenging math problems, along with examples of possible student work to help teachers plan facilitation strategies.

Goals for the Understanding the Problem strategy

Different techniques for understanding the problem can lead us to ideas for solving a problem we have never used before. Good problem-solvers use this problem-solving strategy and may come back to it often as they're working on the problem to refine their strategy, see if they can find better solutions, or find other, even more interesting questions. Specifically, good problem-solvers:

- Use various methods to make sense of a problem from different perspectives.
- Pull out the relevant information.
- Figure out what counts as an answer.
- Connect to prior knowledge and experience.
- Identify questions and relationships that make the problem interesting.

Writing Goals

Writing is an integral part of understanding the problem and builds momentum in thought. It helps the problem solver organize their information and articulate the questions they will address on their journey towards the solution. Specifically, when trying to understand the problem, good problem-solvers might:

- Create an organized list of what they noticed and what they wondered about the problem.
- Paraphrase the problem in their own words.
- Question or hypothesize, focusing thinking on specific parts of the problem (e.g., I'll know I'm right when ...; I could solve the problem if ...; the part that makes it hard is ...; I'll need to use the fact that ...).

Activities

I. Noticings/Wonderings

Format: pairs/small groups, big-group brainstorming or go-round activity, or think-pair-share.

Make a list of all of the mathematical information and relationships you notice about the problem, and everything you wonder about the problem. Your noticings may include:

- The quantities that can be counted or measured.
- Relationships between quantities.
- Information that is not given in the problem.
- Key words from the problem.

Your wonderings may include:

- I wonder what will happen if ...
- I wonder what this word means ...
- I wonder if this pattern will continue ...

Sample Activity: Forget the Question

Present the problem to students without the question. If you would like a copy of the scenario without the question to project for the class or to print for individuals or groups, we'll be providing one for most of the problems this year, including this one. Just look for "Scenario Only" listed under the Teacher Support Materials for this problem.

As a whole class, or in small groups, have students go around quickly and offer one mathematical relationship or bit of mathematical information that they notice, or a mathematical question they are wondering about. Record each noticing and wondering for later use. Continue around the group until no one has anything more to offer.

Note: It is useful to refer back to and update your class list of noticings/wonderings throughout the process of understanding the problem and solving the problem. Students should be encouraged to keep their list, and/or the class list, available and update it as they work.

Key Outcomes

- Student ownership and understanding of the question to be solved.
- Momentum toward a solution path stimulated by all of the mathematical quantities and relationships noticed.
- Slowing down the thinking process and surfacing all of the information and questions that are too easily passed over or dismissed.
- Articulation of specific sub-problems or questions students need to answer or learn more about in order to solve the problem.
- Identifying other questions and features of the problem that may be even more interesting and challenging for students.

II. Extracting the Information and Question

Format: whole group brainstorming, individual brainstorming, or think-pair-share.

List as mathematically and succinctly as possible the key information that may be useful in solving the problem and state what will count as an answer.

- Identify and list important information given in the problem:
 - What quantities are given?
 - What terms are important?
 - What constraints are given?
- Predict as much as you can about the final answer:
 - What will the units of the solution be (what will be counted or measured)?
 - What justification is needed/what am I trying to prove?
 - Can I figure out upper and lower bounds?
 - Could the answer be negative? Could it be a non-integer?

Sample Activity: PoW IQ

"PoW IQ" stands for extracting the Information in the problem, and understanding the Question.

Have each student make a table with very compact mathematical information from the problem in the left column, calculations or mathematical relationships they see in the middle column, and questions they should explore for the final answer in the right column.

Key Outcomes

- Student ownership and understanding of the constraints a full solution requires.

- Articulating mathematical information in a simple, compact format that makes patterns and relationships visible and moves students toward possible solution paths.

III. Paraphrasing

Format: think-pair-share or students working individually.

The goal of paraphrasing a problem is to have students analyze the language of a problem and make clear the mathematics behind the problem situation. Some prompts you might use with students are:

- Identify the key words/phrases in the problem. How would you define them?
- How would you rewrite the problem?
- Same math idea/Different math story: How could you put the problem in a different scenario, while preserving the math behind the problem?

Sample Activity: In Your Own Words

Read the problem as a group. Discuss what the words mean, make sense of the situation and of the question that must be answered. Put away the problem and any notes and individually paraphrase the problem, using one of the prompts above.

Key Outcomes

- Learn to make sure the problem makes sense to you and that you know what has to be figured out.
- Put your thoughts in writing so you can compare your thinking to the original problem statement and see what you may be missing or changing without realizing it.

IV. Acting It Out

Format: students working in small groups.

This approach often requires the most teacher/expert support to ensure the tools or manipulatives that support the investigation are available, and that students are making sure their modeling of the problem fits the necessary constraints. Some problems do not easily lend themselves to this approach. Acting the problem out would probably be used in tandem with one of the activities above, since it doesn't end in a written representation of students' understanding of the problem. Some considerations include:

- If students are physically acting out the problem by using actual materials from the problem situation or using virtual manipulatives, they might want to act out a simpler version first (for example, if the problem is about 100 lockers, they might want to act it out with 20 first).
- If physically acting out the problem is going to be hard, students might draw a rough sketch instead (in a rough sketch, you draw the situation imperfectly and don't worry about representing it accurately because you are just trying to get a sense of relationships).
- Drawing a rough sketch is also a good way to plan how you are going to act out the problem.
- You might choose to purposely do the problem "wrong," by which we mean picking a number for an unknown quantity without worrying about being correct or being sure you have found every possible answer. In either case, the focus is on understanding the problem scenario and key relationships, rather than trying to get a full solution to the problem.

Sample Activity: Try It!

Working in small groups, students make a plan and then act out the problem. Before they begin, students should choose who wants to be the *narrator/checker* and who wants to be the *noticer/recorder*.

Step 1: The *narrator* reads the problem and the group brainstorms ways to act it out, The *noticer* is responsible for making sure that any new quantities, constraints, mathematical relationships used in the brainstorming are pointed out and recorded.

Step 2: The *narrator* makes sure that the group checks the acting it out ideas to see that they fit all of the constraints in the problem. The group picks one of the acting-it-out approaches.

Step 3: As the group acts out the problem, the *noticer* records any relationships or quantities that come up that aren't already on the list of noticings and wonderings. The *narrator/checker* watches carefully to compare the acting out to what the problem says.

Step 4: The group decides whether they have enough understanding to try solving the problem. If not, they can try another way to act out the problem, or improve the design of the approach they used.

Step 5 (optional): The different groups perform their versions of the problem for each other.

Key Outcomes:

- Use visual and physical intelligence to develop a sense of what is going on in the problem.
- Identify relationships in the problem.
- Check understanding of key steps.
- Figure out an answer or a good estimate and use this to start thinking of explanations about why it works that way.

References

Krulik, S., & Rudnick, J. A. (1987). *Problem solving: A handbook for teachers* (2nd ed.). Boston: Allyn and Bacon.

Polya, G. (1945). *How to solve it*. Princeton, New Jersey: Princeton University Press.



Noticings/Wonderings (Forget the Question)

Make a list of all of the mathematical information and relationships you notice and everything you are wondering.

I notice...	I wonder...

If you get stuck, you might try to notice

- The quantities (known or unknown counts or measurements).
- Relationships between quantities.
- Information that is not given in the problem but that might be related or that the problem reminds you of.
- Key words from the problem.

Your wonderings may include:

- I wonder what will happen if ...
- I wonder what this word means ...
- I wonder if this pattern will continue ...
- What does this mean?
- What do they want?
- Does it have to be that way?
- Do I need to figure that out?
- How does this situation work?
- Is there another way to think of it?
- How will I know if this is true?
- What is a good way to express that?
- When is this true?



Extracting the Information and Question (PoW IQ)

“PoW IQ” stands for extracting the **I**nformation in the problem, and understanding the **Q**uestion.

Information from the Problem (as mathematically as you can)	Calculations or Mathematical Relationships	The Question and Predictions/Considerations

If you get stuck, for the **Information** column, you might:

- Identify and list important information given in the problem:
 - What quantities are given?
 - What terms are important?
 - What constraints are given?

For the **Question** column, you might:

- Predict as much as you can about the final answer:
 - What will the units of the solution be (what will be counted or measured)?
 - What justification is needed/what am I trying to prove?
 - Can I figure out upper and lower bounds?
 - Could the answer be negative? Could it be a non-integer?



Paraphrasing (In Your Own Words)

Step 1: Read the problem as a group. Discuss what the words mean, make sense of the situation and of the question that must be answered.

Step 2: Put away the problem and any notes and individually paraphrase the problem, using one of the three prompts below.

Prompt 1: Identify the key words/phrases in the problem. How would you define them?

Prompt 2: How would you rewrite the problem?

Prompt 3: Same math idea/Different math story: How could you put the problem in a different scenario, while preserving the math behind the problem?



Acting It Out (Try It!)

Before you begin, choose who wants to be the *narrator/checker* and who wants to be the *noticer/recorder*.

Step 1: The *narrator* reads the problem and the group brainstorms ways to act it out. The *noticer* is responsible for making sure that any new quantities, constraints, or mathematical relationships used in the brainstorming are pointed out and recorded.

Step 2: The *narrator* makes sure that the group checks the acting-it-out ideas to see that they fit all of the constraints in the problem. The group picks one of the acting-it-out approaches.

Step 3: As the group acts out the problem, the *noticer* records any relationships or quantities that come up that aren't already on the list of noticings and wonderings. The *narrator/checker* watches carefully to compare the acting out to what the problem says.

Step 4: The group decides whether they have enough understanding to try solving the problem. If not, they can try another way to act out the problem, or improve the design of the approach they used.

Step 5 (optional): The different groups perform their versions of the problem for each other.

Noticings: (quantities, constraints, mathematical relationships)

Constraints we checked:

We are ready to move on because:

OR

We need to try again and think about: