The Math Forum: Problems of the Week

Problem Solving and Communication Activity Series

Round 16: Getting Unstuck

If you never get stuck, then you are not solving interesting enough problems. Getting stuck (and, we hope, getting "unstuck"), is at the heart of problem solving. Challenging problems require that they be represented in many ways, approached with a variety of strategies, and checked again and again and again. The famous story of the solution of Fermat's last theorem illustrates getting stuck (and unstuck) very well.

Fermat's last theorem is so easy to state that students with a basic understanding of exponents can understand what the problem is asking: "If an integer n is greater than 2, then the equation $a^n + b^n = c^n$ has no solutions in non-zero integers a, b, and c."

Its proof took over 300 years, and came from multiple branches of mathematics that were not even imagined at the time Fermat jotted down the theorem. Andrew Wiles, who proved the theorem in 1994 after working on it for 8 years, at first submitted an incorrect proof, but when colleagues spotted the error, he was able to revise his work and ultimately prove this most famous of theorems.

Whether you are stuck on the current Problem of the Week, a famous unsolved conjecture, or (like we are at the Math Forum as I write this) a math problem that arose from a situation in your everyday life, you can learn to get better at getting unstuck. Getting unstuck might involve

- Identifying where you are stuck and why.
- Finding new ways of representing the problem or new strategies to help you simplify or better understand the problem.
- Reflecting on your process so far and what you hope to do next.
- Checking your strategies and solutions to make sure they make sense, fit the problem, and are accurate.
- Going beyond the specific problem to learn more about problem solving and get better at getting unstuck (or not getting stuck at all) the next time you solve a similar problem.

The activities below help students to evaluate where they are stuck, and provide reflective questions and tasks that they can learn to use to get unstuck. We refer to several activities published in previous activity series that have developed approaches useful when stuck such as deepening one's understanding of the problem, trying a variety of strategies, and using methods of testing and reflecting on one's work. We also offer some new insights and activities specific to being stuck, and go a little deeper with some previous strategies.

The activities are written so that you can use them with problems of your choosing. We include a separate section afterward to show what it might look like when students apply these activities to the current Geometry Problem of the Week.

Problem-Solving Goals

Good problem solvers get unstuck by:

- Being specific about where they are stuck or what is challenging about the problem they are solving.
- Representing problems in multiple ways.
- Applying multiple strategies to deepen understanding of problems, such as solving a simpler version, using guess and check to get a feel for the problem, or looking at specific cases.
- Checking their work and evaluating their strategies through collaboration, reflection, and other evaluation strategies (See *Round 7: Guess and Check Revisted*, "Testing 1, 2, 3, 4").
- Thinking about how the problem situation could be simulated or could have been developed.

Communication Goals

Problem solvers who are stuck use writing and communication to help them:

- Identify and communicate where they are stuck.
- Flag ideas they may need to come back to by recording:

- What they noticed and wondered.
- Approaches or strategies they may have abandoned too soon.
- Information from the problem they have not used yet.
- Prior knowledge of similar problems or situations.

- Represent the problem in multiple ways; paraphrase or rewrite the problem.
- Record their thinking and possible solution paths so they can check their own work, figure out possible assumptions that could be questioned, and easily share their work with others for checking and collaboration.

Activities

I. Kinds of Stuck

Format: students working individually.

"Kinds of Stuck" serves as a pre-activity to help students figure out which of the following activities are most relevant to where they are in their problem solving.

Sample Activity

It's easy to tell when you are stuck: you just don't know what to do. But it's important to think about where you are stuck, because different kinds of stuck lead to different ways of getting unstuck. Pick one of the following statements that best describes the kind of stuck you are facing.

1) I don't know what to do first. I have no idea how to approach this problem.
2) I have thought of something to try, and even started trying a strategy, but it doesn't seem to be working, or it's really ugly and I think there's got to be a better way.
3) I have a solution or a strategy that I think will work, but I'm not sure if it's right, or even how to tell if it makes sense.

Key Outcomes

- Identify more specifically where you are stuck.
- Prepare to choose a line of reflection or task to help you get unstuck.

II. I Don't Know What to Do

Format: Students working with a partner or small group.

When students don't have ideas of where to begin the problem, it is usually because they have not made or allowed connections between the problem and their own knowledge and experience. In this stage it is important to focus on making those connections and to not worry about solving the problem. All we really care about at this point is getting our brain in gear. We do this because it works, but we also do it because we know that one of the most enjoyable aspects of problem solving is the math that one discovers along the way. This is a phase where we as teachers can do more to help students celebrate their ability to generate ideas and make connections.

Sample Activity

When we don't see how to solve a problem, we try to forget about solving the problem for now. All we want is to get our brain in gear. We ask ourselves these three questions:

1. **What is going on in this problem?**
2. **What can I try?**
3. **What does this remind me of?**

How we answer these questions:

1. **What is going on in this problem?** – Our favorite activity for understanding the problem is *Notice and Wonder*, where we make lists of every quantity, mathematical relationship and question we can find in the problem. We also like to *Make a Picture* or *Act It Out*, using pencil and paper or manipulatives. Our goal is to have as many ideas as we can.

2. **What can I try?** – *Guess, Calculate, and Check* is one of our favorite ways to understand a problem. We don't even care at first if we make a good guess. We just want to have some things to try and this will help us think about some parts of the problem and not worry about solving the whole thing.

3. **What does this remind me of?** – We like to use *Simpler Problem* to make a version of the problem that we recognize or think we can solve. What makes this problem hard? What would make our problem easier? This gives us patterns and ideas to try with our original problem. We also try to *Change the Representation*, and turn this problem into a situation that we know better.

Key Outcomes

- Identify key quantities and relationships in the problem.
- Do some calculations or drawings that give us more ideas.
- Connect what I know and have done before to this problem.

III. This isn't Working Out

Format: Students working individually, then moving to pair or small-group work.

It is common with interesting and challenging problems that at some point we keep trying and trying an approach that we think will work, but it doesn't feel as if we're making any real progress. Or we tried a lot of approaches and none seemed helpful. The key at this point is to step back and get some perspective, take a break, approach the problem with some fresh eyes.

Sample Activity:

Why isn't it working? – Just thinking about this question can help you get unstuck. As a class, brainstorm a list of ideas for why you might be stuck when you are trying an approach that isn't working out.

Use the list to identify something you can try now to gain new perspective on your work.

Here is the list we run through in our heads of common reasons our approach isn't working:

- **More ideas are needed** – Interesting problems usually require us to combine different ideas in order to solve them. Make sure you **try different strategies** and bring the information they give you back to the approach that you think is most promising.
- **Missed something. Revisit your work.** – Often when we have an approach that we think should work, there are some facts and questions that we don't pay much attention to along the way. We might skip something because it seems like too much effort or not so useful. Try to keep a "parking lot" page of ideas and other problems that you can come back to when you are stuck. Make a note of places where you might be making an assumption that does not have to be true.
- **Don't know how. Ask for help.** – Sometimes we see an approach that should work, but we just don't know enough math to actually do it. This is a good situation for asking for help. We don't need someone to show us how to solve the problem, we just need some help doing one part of it.
- **Made a mistake. Talk it through with someone else.** – One of the most frustrating ways to get stuck is to make a mistake and not know it. It might be a simple mistake, but it can confuse all the rest of your work. Find a partner or group to whom you can explain your work. Often you will catch your own mistakes as you try to explain and show your calculations.
- **Get a fresh start** – After we have worked on a problem for a long time, we might have a paper that is full of notes and calculations. This can make it hard to see what you learned and what your new thinking is revealing. Often when we decide to *Change the Representation* we also get a clean sheet or workspace. It can also help to take a break, if we have been working a long time. We might keep thinking about the problem, but we're relaxing and doing other things too.

Key Outcomes:

- Keep good notes, even of ideas and questions that do not seem so helpful now.
- Figure out why you might be stuck.
- Reframe the problem to help you find a fresh perspective.

IV. I Don't Know if I'm Right

Format: Students working in small groups.

One place to have students begin if they think they have a solution but are not sure how to check if they are correct, is to do the *Testing 1, 2, 3, 4* activity from *Round 7: Guess and Check (Revisited)*. In the activity below we are more focused on understanding how an approach works than on correcting mistakes. It is possible to get the correct answer but still use methods that don't quite make sense. We also know that we might solve a problem, but not really understand all of the ideas.

Sample Activity

Getting an answer is only part of the fun of problem solving for us. We like to be able to use what we did to think up new problems and to have more powerful approaches the next time. In order to do this we find lots of ways to ask about the parts we're still not sure about.

Identify one of questions below that you can use to tell your group something you're still not quite sure about.

- **What's in the parking lot?** – Sometimes you notice or wonder something while doing the problem that you don't use right away. You might assume something you're not sure of, or think of an alternative strategy you're not going to try yet. Once you've found a solution can be a good time to come back to the parking lot and see if there are interesting mathematical ideas, other cases to try, etc.
- **Would this always work?** – Sometimes an explanation makes sense but you think there are situations where it may not work. This is often a good time to think about *Cases* and to try the solutions your group found with a special case.
- **Can you say what he said?** – In order to make sure you understand an idea from your group, you might try to explain it in your own words, or give an example of how it works, or ask someone else in the group to explain it in different words.
- **What can I do with this?** – Try to describe or make up another problem where you could use what you learned in solving this problem.

Key Outcomes:

- Continue thinking about key ideas or concepts in a problem even after generating a possible solution.
- Explain multiple solution paths in your own words.
- Articulate problem-solving insights generated while solving challenging problems.