



## The Math Forum: Problems of the Week

# *Problem Solving and Communication*

## *Activity Series*

### Solve a Simpler Problem

Sometimes we see a way to solve a math problem, but it looks like it would take too long or be too complicated. Maybe the numbers are too big or there are too many lines or there is an extra step and we are not sure how to handle it. Other times the problem seems too hard to tackle all at once, or we aren't sure we really understand it.

*Solve a Simpler Problem* is a technique that can be used in several ways to solve challenging problems. In some situations you can see how to work the problem with easier numbers. This may show you an approach that you can try with the more difficult numbers. For other problems you might choose to break the original problem into smaller steps, finding answers for parts of the problem, and then putting those together for the whole solution. A third simpler problem method often used by mathematicians is to change a hard problem into one that you have solved before.

The activities below help students learn ways to make a simpler problem, to solve the simpler problems, to identify the key questions or struggles they encounter, and to apply the insights they gain to the original problem. The activities are written so that you can use them with problems of your choosing.

#### Problem-Solving Goals

Setting up simpler problems can help problem-solvers:

- More fully understand the problem.
- Find patterns.
- Discover and generalize important relationships.
- Focus on the specific mathematics that they need to figure out or learn.

#### Communication Goals

Writing can be a way to become clear about what you know and what you need to figure out. The *Simpler Problem* strategy can be used to drill down to the smallest, most specific question or issue that would allow you to make progress on a problem. The goal of all of the strategies in this series is to develop skills that build confidence in your own thinking. We use the writing to get us to the point where we are ready to learn, where there is an idea or piece of information that we are prepared to use in order to make progress. We are trying to overcome the habit of asking broad questions too early that lead to being shown what to do without being ready to learn or remember it. Students that attempt to *Solve a Simpler Problem* can use the writing process to:

- Organize what they understand and what is hard or confusing.
- Develop good questions that are specific and that help you focus your investigation or enable a helper to give useful answers that support learning.
- Focus on promising ideas that you might skip over as you think about the harder problem.

**Note:** There is an implicit assumption throughout this *Activity Series* that users know that they can also be making use of strategies that have been introduced in prior weeks. In particular, *Understanding the Problem* was developed as a set of strategies that is always useful and to some extent assumed to be in use, even when focusing on a new strategy, such as *Simpler Problem*.

# Activities

## I. Making it Simpler

**Format:** students working individually or in pairs, and then sharing in groups of 4-6 students.

One first step in making a problem simpler is to identify what's hard about the problem. This may seem obvious, but taking the time to write these aspects down can enable effective brainstorming about good ways to change the problem.

### Sample Activity

Work individually or in pairs to begin filling in the blanks of the following prompts for just a few minutes. Then share ideas with the larger group of 4-6 students. At this point, you should be focusing on brainstorming and generating as many ideas for making the problem simpler as you can. Don't solve the problem yet.

Fill in as many of the blanks as you can in the following *Simpler Problem* sentences:

- 1) One thing that makes this problem hard is \_\_\_\_\_.
- 2) I can make a simpler problem to solve by \_\_\_\_\_.

It is a good idea when brainstorming in the group to write down the ideas stimulated by hearing from each other. After brainstorming, discuss which simplification idea each of you thinks is most promising or interesting.

### Key Outcomes

- Recognize elements that make math problems harder and find ways to not get stuck because of them.
- Explore some different ways of making problems simpler.
- Know how to break a problem down into smaller problems and find a place to start working it out.
- Learn to use brainstorming to open up ideas before you try to solve a problem.

## II. Working on the Simpler Problem

**Format:** Students working in pairs.

The focus in this activity is on using writing to organize the problem solving activity, to notice patterns or ideas that make the solution possible, and to ask specific questions that need to be answered in order to make progress. Student partnerships will help each other by asking clarifying questions and focusing each others' thinking on key aspects of working on and learning from simpler problems.

### Sample Activity

- 1) Students work on their own carrying out their simpler problem plan.
- 2) Whenever one of the partners gets to a place where they are stuck, or figures something out, they should stop and share their work and ask their partner for help. The partner asks the student who is at a stopping point any useful questions from the following list, and any other questions they have:
  - a. Was what you tried simpler? Why?
  - b. What did you learn from that simpler problem?
  - c. Do you see anything it would be good to record in a table or organized fashion?
  - d. Do you have any hypotheses you could test out on another simpler version (or the main problem)?
  - e. What can you try next?
- 3) Students continue to work in this fashion until they solve the problem.

### Key Outcomes

- Learn ways to organize problem solving to keep track of sub-problems and iterations.
- Learn a set of reflective questions to help you make use of simpler problem strategies.
- Use writing to separate what you know and can do from what you still need to figure out.

## III. Learning from the Simpler Problem

**Format:** Students sharing with groups of 4-6, perhaps selected so that groups have a diversity of strategies.

**Sample Activity:**

- 1) Share how you used the simpler problem strategy with your group. Make sure you can tell:
  - a. What different simpler problems you tried.
  - b. What you learned from each simpler problem.
  - c. How you decided what to try next.[If you're not ready to share this concisely, practice it with your partner first]
- 2) Listen to what others share. Think about what people did that was similar, and what people did that was different.
- 3) Talk about the similarities and differences you noticed. Did you or your group mates use:
  - a. Solving the problem with simpler numbers?
  - b. Solving sub-problems?
  - c. Changing the problem to make it more like a problem you've solved before?
  - d. Another simplifying strategy?
- 4) Submit your write-up to the PoW online. (optional)
- 5) Use a jigsaw or gallery walk format to share your explanation with the whole class and to appreciate their insights. (optional)

**Key Outcomes:**

- Consolidate understandings of ways to make problems simpler.
- Learn clear organization and presentation of ideas.
- Understand how solutions to simpler problems can be used to solve more complex problems, i.e. generate useful patterns, provide pieces you can put together for a solution, show another way of thinking about the problem, etc.



## ***Making it Simpler***

On your own or with a partner, spend just a few minutes filling in as many of the blanks of the following prompts as you can. Focus on brainstorming and generating lots of ideas for making the problem simpler. Don't solve the problem yet.

One thing that makes this problem hard is \_\_\_\_\_.

Another thing that makes this problem hard is \_\_\_\_\_.

Another thing that makes this problem hard is \_\_\_\_\_.

Another thing that makes this problem hard is \_\_\_\_\_.

Another thing that makes this problem hard is \_\_\_\_\_.

Another thing that makes this problem hard is \_\_\_\_\_.

Another thing that makes this problem hard is \_\_\_\_\_.

I can make a simpler problem to solve by:

Share your brainstorms with your larger group. Add any new ideas to your own list above.

After brainstorming, discuss which simplification idea each of you thinks is most promising or interesting.

If you are stuck, here are three ways mathematicians sometimes make problems simpler.

- Try simpler numbers or smaller numbers of items at first.
- Break the problem down into sub-problems that they can tackle.
- Re-phrase the problem or the scenario so it reminds them of a problem they've solved before.



## ***Working with the Simpler Problem***

- 1) Fill in the blanks and record your work.

My simpler version is to try \_\_\_\_\_  
Here is my work:

- 2) When you get to a stuck place or a stopping point, tell your partner what you've done, and have them ask you some questions. When your partner asks you for help, you can use these questions to help you get started:
- Was what you tried simpler? Why?
  - What did you learn from that simpler problem?
  - Do you see anything you need to record in a table or organized fashion?
  - Do you have any hypotheses you could test out on another simpler version (or the main problem)?
  - What can you try next?
  - \_\_\_\_\_
  - \_\_\_\_\_
- 3) Work on recording, testing your hypotheses, and trying next steps. When you get to a stopping or stuck place, repeat step 2 with your partner.

