Math Fundamentals PoW Packet

Wheels R Us

October 18, 2010 • http://mathforum.org/pows/

Welcome!

This packet contains applicable standards, a copy of the problem, the answer check, our solutions, teaching suggestions, and samples of student work that we received when the problem ran in October 2006 (Library PoW #4075). For a print-friendly version use the *Print this Problem* link on the problem page.

We invite you to visit the PoW discussion groups to explore this problem with colleagues. From the Teacher Office use the link to *PoW Members,* or use this URL to go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username and password.]

Are you making the most of your PoW Membership? If you have an Individual Teacher Membership consider registering for one of our (free) Orientation Sessions to learn more about the features of your membership. Teachers with Class or School or District Memberships also are encouraged to register for one of our online courses. View information, dates, and links to register here: http://mathforum.org/pd/

Standards

In ***Wheels R Us,*** solvers use basic conceptual knowledge of fractions and multiples to determine how many bicycles and tricycles Danielle has in her shop.

If your state has adopted the *Common Core State Standards*, this alignment may be helpful:

Grade 3: **Operations and Algebraic Thinking**

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Grade 3: **Number & Operations—Fractions**

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.

Grade 4: **Operations and Algebraic Thinking**

Use the four operations with whole numbers to solve problems.

3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

Additional alignment information can be found through the **Write Math with the Math Forum** service, where teachers can browse by NCTM and state standards, as well as popular textbook units, to find related problems.

The Problem

Wheels R Us

One fourth of the vehicles at Danielle's Cycle Shop are tricycles. The rest are bicycles. Danielle counted a total of 45 wheels in her shop.

1. How many bicycles does she have?
2. How many tricycles does she have?

Explain how you found your answer. Show how you know you are correct.

Extra: Danielle didn't count the pairs of training wheels on the back of 1/5 of the bikes. How many total wheels are there if training wheels are included?

Tell how you figured it out.

Answer Check

After students submit their solutions, we encourage them to check their answers by looking at the answer that we provide. Below is what they will see. You might use the accompanying questions as prompts to help students who are struggling, or to encourage those who have found a correct solution to improve their explanation.

Danielle has 15 bicycles in her shop. Now you can figure out how many tricycles she has!

If your answer does **not** match ours,
- did you think about even and odd numbers?
- do you know how many bicycles she has for each tricycle?
- what can you figure out about the total number of vehicles?
- did you check your arithmetic?

If you used guess-and-check, did you tell ...
- what numbers you tried?
- how you knew whether they worked or not?
- how you used that information to make a better next guess?

If your answer **does** match ours,
- is your explanation clear and complete?
- did you try the Extra?
- did you verify your answer with another method?

Our Solutions

Below are some ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. We often receive solutions from students who have used approaches we've not anticipated. I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1: Guess and check

If 1/4 of the vehicles are tricycles, there must be 3 times as many bicycles as tricycles. I decided to try 2 tricycles and 6 bicycles. I made a list of what I tried and how many wheels they would have. I multiplied the number of tricycles by 3, since each one has 3 wheels. I doubled the number of bicycles, since each has 2 wheels. My first combination only had 18 wheels.

I tried doubling the numbers of tricycles and using 3 times that many bicycles. That made 36 wheels altogether. I was getting closer.

I added one more tricycle and 3 more bicycles. That totaled 45 wheels. Danielle has 5 tricycles (5 * 3 = 15 wheels) and 15 bicycles (15 * 2 = 30 wheels). 15 + 30 = 45 wheels total.

Tricycles	T wheels	Bicycles	B wheels	Total wheels
2	6	6	12	18
4	12	12	24	36
5	15	15	30	45

Strategy 2: Proportional reasoning, with Extra

One fourth of Danielle's vehicles are tricycles, so I knew that, for each tricycle with 3 wheels, Danielle had 3 bicycles with 2 wheels each. One group of 1 tricycle and 3 bicycles has 9 wheels.

(1 * 3) + (3 * 2) = 3 + 6 = 9 wheels

In order to have 45 wheels, Danielle must have 5 of those groups of 9, or 5 tricycles and 15 bicycles.

To verify my answer, 5 tricycles + 15 bicycles = 20 vehicles total and 5 is 1/4 of 20.
5 tricycles have 15 wheels (5 * 3 = 15).
15 bicycles have 30 wheels (15 * 2 = 30).
15 tricycle wheels + 30 bicycle wheels = 45 wheels total.

Extra: 1/5 of 15 bicycles = 3 bicycles that have 2 training wheels each, or 6 additional wheels. 45 + 6 = 51 wheels total.

Strategy 3: Multiples and parity (odd/even properties)

I knew that the total number of vehicles had to be a multiple of 4, since 1/4 of them were tricycles. I also noticed that the total number of wheels, 45, is an odd number. That means that the number of tricycles has to be odd, because any number of bicycles has an even total of wheels, and an even number of tricycles has an even number of wheels.

I tested 3 tricycles and 9 bicycles. 3 * 3 = 9 wheels and 9 * 2 = 18 wheels.
9 + 18 = 27 wheels. Not enough.

I tried the next odd number of tricycles: 5 tricycles have 15 wheels (5 * 3 = 15). Three times as many bicycles would be 15, which have 30 wheels (15 * 2 = 30)
15 wheels + 30 wheels = 45 wheels.

Danielle has 5 tricycles and 15 bicycles.

Strategy 4: Work from extremes

I checked the extreme cases that would produce 45 wheels. If all the vehicles were tricycles, there would be 15 of them. 45 wheels / 3 wheels each = 15 vehicles.

If I used the maximum number of bicycles, there would have to be one tricycle (because 45 is an odd number) and 21 bicycles. 42 wheels / 2 wheels each bike = 21 bikes. This is a total of 22 vehicles.

I know the total number of vehicles has to be a multiple of 4, and that 1/4 of them have to be tricycles. There are only two multiples of 4 between 15 and 22, and they are 16 and 20. I tried 16 first.

1/4 of 16 = 4 tricycles
4 * 3 = 12 tricycle wheels
12 * 2 = 24 bicycle wheels
12 + 24 = 36 wheels total -- Not enough.

Then I tried 20 total vehicles.
1/4 of 20 = 5 tricycles
5 * 3 = 15 tricycle wheels
15 * 2 = 30 bicycle wheels
15 + 30 = 45 wheels total -- Correct!

Danielle has 15 bicycles and 5 tricycles.

Strategy 5: Algebraic

I used algebra to solve the problem. I let t stand for the number of tricycles. $3t$ represents the number of bicycles, since 1/4 of the vehicles are tricycles.

$(t * 3) + (3t * 2) = 45$ [tricycle wheels + bicycle wheels = total wheels]

$3t + 6t = 9t = 45$ [Combine like terms.]

$t = 5$ [Divide both sides by 9.]

Danielle has 5 tricycles and 3 * 5 = 15 bicycles.

To show that is correct, 5 tricycles + 15 bicycles = 20 vehicles total and 5 is 1/4 of 20.
5 tricycles have 15 wheels (5 * 3 = 15).
15 bicycles have 30 wheels (15 * 2 = 30).
15 tricycle wheels + 30 bicycle wheels = 45 wheels total.

Teaching Suggestions

In Wheels R Us solvers use basic conceptual knowledge of fractions and multiples to determine how many bicycles and tricycles Danielle has in her shop. It can be solved with a variety of approaches. The Extra extends the main problem and depends on a correct solution to it.

Consider introducing this problem by presenting students with the Scenario Only version (linked in the teacher's blue box on the problem page). By having a rich discussion about what students notice before

seeing the question, they have the opportunity to explore the story, practice math language, and develop a deeper understanding of the problem, which "primes" them for addressing the question.

Students who struggle with understanding the problem might benefit by drawing diagrams. Modeling the problem with linking cubes, grouped in 2s for bicycles and 3s for tricycles, might help children understand the distinction between vehicles and wheels as well as the ratio of bicycles to tricycles.

High-achieving students can be challenged to develop their own versions of the problem using different numbers and fractions. The process of finding a combination of numbers that works will require them to apply their skills with proportions and multiples in different ways.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources. If you would like one page to find all of the 2010-2011 Current Problems as we add them throughout the season, consider bookmarking this page: http://mathforum.org/pow/support/

Sample Student Solutions

Focus on Interpretation

In the solutions below, I've focused on Interpretation. This refers to whether students understand the question and the key mathematical ideas in the problem. I have printed below the criteria for the different levels of Interpretation from our problem-specific Scoring Rubric.

I have commented on what I noticed about each solution and suggested questions and prompts that might help the student move forward in their thinking.

Novice	Apprentice	Practitioner	Expert
Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that the number of tricycles is 1/4 of the total vehicles. Understands that each bicycle has 2 wheels, and each tricycle has 3 wheels. Understands that there are 45 wheels altogether Attempts to find how many bicycles and tricycles are in the shop.	Achieves at least Practitioner in Strategy. Understands the Extra. Attempts to find how many totals wheels there are.

Sara
Age 10

Interpretation
Novice

My answer for how many tricycles is 11

how I got that was I divided 4 from 45 because it said 1/4th of the bikes were tricycles so i divided 45 by 4 and I got 11. I subtracted 11 from 45 and I got 34 regular bicycles.

It appears that Sara confused wheels with vehicles, in finding 1/4 of the wheels. There is no evidence that she recognized that bicycles have 2 wheels and tricycles have 3. I would ask her how many wheels would be on 34 bicycles as a way to check whether her answer is reasonable. I'd make sure she understands the key vocabulary, and then ask her, if there were only one tricycle, how many bikes would there be?

Elizabeth and Alisha
Age 12

Interpretation
Apprentice

There are 4 trycycles and 18 bicycles.

We used a pie graph and split it into 4 pieces. Then, we put 10 wheels in each piece to start of and then we kept on adding until it sumed up to 45 wheels.

The strategy of using a pie graph divided into 4 sections is evidence that Elizabeth and Alisha understand that the number of tricycles is 1/4 of the total vehicles. Putting 10 wheels in each section indicates that they've ignored the number of wheels on each kind or vehicle. After confirming their reason for the 4 sections, I'd ask them to draw the correct number of wheels in each section to represent one vehicle per section. Making a table to keep track of total wheels as they added additional vehicles might help them to see the relationships between the numbers and possibly develop a more general strategy.

Anthony
Age 13

Interpretation
Apprentice

9 bikes 9 trycicles

danielle had 45 wheels so i put the wheels in groups of 2 bikes and groups of 3tricicles. once you have 27 talles for the ticicles and 18 talles for the bikes. so if you add 18+27 the answer would be 18+27=45. and that is how i got my answer.

Anthony understood the relative number of wheels on the two kinds of vehicles and found an equal number of bicycles and tricycles that used 45 wheels. He seemed to ignore the stated proportions. I'd ask him to read the first sentence of the problem aloud and explain what it means. Then I'd ask how me might adjust his strategy to satisfy that condition.

Hyunjin
Age 10

Interpretation
Apprentice

There are fifteen bicycles and five tricycles.

There are forty-five wheels total. And one fourth of them are tricycles. And the rest are bicycles. I found out the answer by dividing forty-five by three. Then I got fifteen. That is how I found out the amount of bicycles there were because only ONE FOURTH of the amount of vehicles were the tricycles. So fifteen had to be the amount of bicycles. Because then there would only be five bicycles, and that would be less than the tricycles. But there are supposed to be more bicycles than tricycles.

So the rest of the vehicles, five vehicles, should be the tricycles. Because the amount of bicycles was already found.

So it is like working backwards, instead of finding the rest amount of bicycles, I found the rest amount of tricycles.

So there are a total of twenty vehicles. Five tricycles and fifteen bicycles.

Hyunjin understood that there were a total of 45 wheels and the relative proportions of bicycles and tricycles. He has arrived at the correct answer, but his strategy doesn't make sense. He makes no reference to the number of wheels on bicycles and tricycles. I would check to make sure he knows and encourage him to find how many bicycles and total wheels there would be, if there were only one tricycle. How many total wheels? What if there were two tricycles?

Chad
Age 11

Interpretation
Practitioner

There were 15 bicycles and 5 tricycles.

I solved this problem by making a chart with one side with bicycles and one with tricycles and inserted a number. Then for bicycles I multiplied the number of bicycles I put in the chart by two since bicycles have two wheels. Then I did the same thing with the tricycles except I multiplied it by three since tricycles have three wheels and I maked sure that the number of bicycles was three times the number of tricycles since the tricycles were one fourth and the bicycles were three fourths. Then I added up the number of wheels together and they added up to fourty-five wheels and that is how I got my answer!

Chad understood all the key ideas and relationships of the problem. His strategy is sound. I'd like to know what number he chose to try first, what he learned from it, and how he used what he learned to make other tests. I'd ask him to provide the calculations that show his answer meets all the conditions of the problem.

Josie and Zach
Age 8 and 9

Interpretation
Practitioner

There are 5 tricycles and 15 bicycles in the shop. . .

We used cubes for the wheel. We put them into groups of 2's and 3's. Then wrote a chart with a space for tricycles and 3 spaces for bicycles. We put one group of 3 in the tricycles and 1 group of 2 in each space for bicycles. There were 5 tricycles and bicycles in each spot when the 45 cubes were used up.

2*15=30 wheels for bikes
3*5=15 wheels for trikes
15+30=45 wheels in all

Josie and Zach's strategy of using cubes and a chart with spaces is evidence that they understood the mathematical ideas of the problem. I like the fact that they showed the math that verifies their answer. I'd like them to construct a table showing the stages of their process and the total wheels at each stage.

Math Plus 7
Age 12

Interpretation
Expert

There are 5 tricycles and 15 bicycles. EXTRA. There are 51 wheels in total.

To start with, I made a table and added the same number of wheels (2 for bicycles and 3 for tricycles). However, I kept the ratio of 1 tricycle: 3 bicycles in the table to insure that it was correct. The table looks like this:

_____
1 tricycle (3 wheels)	3 bicycle (6 wheels)	Wheels: 9
2 tricycle (6 wheels)	6 bicycle (12wheels)	Wheels: 18
3 tricycle (9 wheels)	9 bicycle (18wheels)	Wheels: 27
~~~~~~

By now, I have found a pattern. The total number of wheels is the number of tricycles multiplied by 9.
In an equation, it would be: W/9=T. If W were 45 wheels, then the equation would become: 45/9=5.
So, the number of tricycles is 5.

This team made an organized list, which helped them discover a pattern. I'd like to know whether they could understand the reason for it. I'd also ask them to carry out the math for 5 tricycles and 15 bicycles to verify that the pattern holds.

For the Extra, they remove the bicycles with training wheels from the original count and add their total number of wheels back in.

The bicycles are easier to find out. If you remember, the ratio of tricycles to bicycles is 1:3. Therefore, if there are 5 tricycles, there are 15 bicycles.

~~~~~~~~~~~~~~~~~EXTRA~~~~~~~~~~~~~~~~~~~~~~

There are 1/5 of the bikes that have training wheels, and these bikes have 4 wheels. From the last part, I know that there are 15 bikes in the shop. 1/5 of 15 is 3, so 3 bikes have training wheels.

I subtract 3*2, or 6 wheels from the entire count of wheels, making the number 39(45-6). After that, I added the training wheels and the wheels that where just subtracted back there, making the grand total 51 wheels (39+[3*4]).

Kirina
Age 12

Interpretation
Expert

There are 15 bicycles and 5 tricycles from the total of 20 vehicles at Danielle's cycle shop. EXTRA: The total number of wheels would be 51.

I solved this problem by using the 'variable' method. I used t for tricycle, b for bicycle and v for the total amount of vehicles.

Since there are two wheels on a bicycle and three wheels on a tricycle, I came up with this: 2b+3t=45. I knew that since 1/4 of the vehicles were tricycles and that means that 3/4 of it is bicycles. To make the first sentence more detailed I added 1/4 and 3/4 comming up with this: 2 3/4V + 3 1/4V = 45. Then I simplified.

6/4V + 3/4V = 45 = 9/4V =45. I divided 45 by 9/4, and my answer was 20...20 vehicles.
20 divided by 4 equals 5...5 tricycles.
20 minus 5 equals 15...15 bicycles.

2b + 3t = 45
2 3/4V + 3 1/4V = 45
9/4V=45
45 divided by 9/4 = 20
20 divided by 4 = 5
20-5=15
15 bicycles, 5 ticycles, total of 20 vehicles.

Kirina's algebraic strategy is very insightful and based on sound reasoning. I would like to have seen verification that the answer to the main problem worked, as was done for the Extra.

EXTRA: I divided the amount of bicycles to 1/5. My answer was 3, that means that three of the bicycles had training wheels on the back. Then I multiplied three times two becuase there are two training wheels. My answer was six, six plus 45 equals 51.

15 x 1/5 =3
3x2=6
6+45=51

Scoring Rubric

The problem-specific scoring rubric we use to assess student solutions is a separate stand-alone document available from a link on the problem page. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Claire

claire@mathforum.org