Understanding the Problem

Starting with Round 6, we are cycling back through the four key problem-solving strategies we began the year with: (1) Understanding the Problem (first two rounds), (2) Guess and Check, (3) Solve a Simpler Problem, and (4) Tables and Patterns. As we return to each strategy, we have updated the activities to help students deepen their focus and improve their problem-solving skills. In this document, you will notice:

• We added a “parking lot” to the Noticing/Wondering activity.
• We focus on “implications” in the Extracting the Information and Question activity.
• We focus on “key words” in the Paraphrasing activity.
• We focus on mathematically-dense drawings in the Act It Out activity.

The activities are written so that you can use them with problems of your choosing. We include examples afterward to show what this strategy might look like when students apply these activities to the current Math Fundamentals Problem of the Week.

Problem Solving Goals

Different techniques for Understanding the Problem can lead us to ideas for solving a problem we have never used before. Good problem-solvers use this problem-solving strategy and may come back to it often as they’re working on the problem, to refine their strategy, see if they can find better solutions, or find other, even more interesting questions. Specifically, good problem-solvers:

• Use various methods to make sense of a problem from different perspectives.
• Pull out the relevant information.
• Connect to prior knowledge and experience.
• Represent problems in multiple ways.
• Focus their representations on mathematically-relevant information.

Communication Goals

Writing is an integral part of Understanding the Problem and builds momentum in thought. It helps the problem solver organize their information and articulate the questions they will address on their journey towards the solution. Specifically, when trying to understand the problem, good problem-solvers might write:

• Create an organized list of what they noticed and what they wondered about the problem.
• Record implications or background knowledge that they apply to a problem.
• Paraphrase the problem in their own words.
• Make clear drawings and well-labeled diagrams.

Activities

I. Noticing/Wondering

Format: big-group brainstorming or go-round activity.

Step 1: Make a list of all of the mathematical information and relationships you notice about the problem, and everything you wonder about the problem. Your noticing may include:

• The quantities that can be counted or measured.
• Relationships between quantities.
• Information that is not given in the problem.
• Key words from the problem.
Your wondering may include:
• I wonder what will happen if …
• I wonder what this word means …
• I wonder if this pattern will continue …
• I wonder whether …

Sample Activity: Noticing/Wondering in a Parking Lot

Step 1: As a whole class, or in a few groups (5 or more students), have students go around quickly and offer one mathematical relationship or bit of mathematical information that they notice, or a mathematical question they are wondering about. Record each noticing and wondering. Continue around the group until no one has anything more to offer.

Tell students that as the group is reporting their noticing/wondering, they may be having reactions like:
• Wow, I never noticed that!
• I wonder if ____ is important to the problem.
• I noticed ____, which “answers” the wondering ____.
• I wonder if you really need to figure out ____ to solve the problem.

Rather than share those ideas during the brainstorming, jot them down or remember them for Step 2.

Step 2: Discuss as a class or in their group, the following questions:
• How did other people think about the problem differently from you?
• What assumptions did they make?
• What’s different from how you thought about it and what you noticed?

Step 3: Make a “parking lot.” As you discussed different ways of understanding the problem, which issues needed to be resolved. Are there ones you can put aside? Are there issues you can “agree to disagree” about? Write those issues on a separate list, called the “Parking Lot.” Once you have solved the problem, refer back to the parking lot to see if you resolved the issues, or if you need to explore them further.

Key Outcomes
• Student ownership and understanding of the question to be solved.
• Momentum toward a solution path stimulated by all of the mathematical quantities and relationships noticed.
• Slow down the thinking process and surface all of the information and questions that are too easily passed over or dismissed.
• Articulation of specific sub-problems or questions students need to answer or learn more about in order to solve the problem.
• Identify students’ own and others’ assumptions.
• Identify assumptions that can appropriately simplify the problem.
• Identify other questions and features of the problem that may be even more interesting and challenging for students.
• Distinguish between more and less relevant information.

II. Extracting the Information and Question

Format: whole group brainstorming, individual brainstorming, or think-pair-share.

List the key mathematical information that may be useful in solving the problem as succinctly as possible and state what will count as an answer.

• Identify and list important information given in the problem:
  o What quantities are given?
  o What terms are important?
  o What constraints are given?
• Write down implications of the given information:
  o What do you already know about the key terms or ideas?
  o What calculations could you do?
  o What relationships can you describe?
• Predict as much as you can about the final answer:
  o What will the units of the solution be (what will be counted or measured)?
  o What justification is needed/what am I trying to prove?
  o Can I figure out upper and lower bounds?
  o Could the answer be negative? Could it be a non-integer?
Sample Activity: PoW IQ

“PoW IQ” stands for extracting the Information in the problem, and understanding the Question.

Have each student make a table with the mathematical information from the problem in the left column, calculations or mathematical relationships they see in the middle column, and questions they should explore for the final answer in the right column.

Key Outcomes

- Student ownership and understanding of the constraints a full solution requires.
- Articulate mathematical information in a simple, compact format that makes patterns and relationships visible and moves students toward possible solution paths.
- Make visible the background knowledge that students bring to a problem.

III. Paraphrasing

Format: think-pair-share or students working individually.

The goal of paraphrasing a problem is to have students analyze the language of a problem and make clear the mathematics behind the problem situation. Some prompts you might use with students are:

- Identify the key words/phrases in the problem. How would you define them?
- How would you rewrite the problem?
- Same math idea/Different math story: How could you put the problem in a different scenario, while preserving the math behind the problem?

Sample Activity: Key Words

Highlight words or phrases you think are going to be really important to understanding and solving the problem. Write each word on a sheet of paper, with several lines between each. Below each word, sketch or write what you think it means. Then write as many things you need to find out about that word as you can, or as many things as you can think of that the word tells you about the problem.

Key Outcomes

- Learn to make sure the problem makes sense to you and that you know what has to be figured out.
- Put your thoughts in writing so you can compare your thinking to the original problem statement and see what you may be missing or changing without realizing it.

IV. Acting it Out

Format: small groups.

This approach often requires the most teacher/expert support to ensure the tools or manipulatives that support the investigation are available, and that students are making sure their modeling of the problem fits the necessary constraints. Some examples include:

- Physically acting out the problem by using actual materials from the problem situation or using virtual manipulatives (You might act out a simpler version of the problem, for example, using smaller numbers).
- Drawing a rough sketch (This is different from drawing a picture as a strategy to solve the problem, since you know you might be drawing it imperfectly and less representative, but you are just trying to get a sense of relationships).
- Doing the problem “wrong”: similar to a quick version of guess and check, doing the problem wrong can refer to working through the problem by guessing a number for an unknown quantity, or trying to find an answer that works without being sure you have found every possible answer. In either case, the focus is on understanding the problem scenario and key relationships, rather than trying to get a full solution to the problem.

In the sample activity below, the focus is on the concept of a rough sketch. What visual information is important to include? How can that information be simplified or abstracted? What are effective ways to use labels? How accurate does the sketch have to be?
Sample Activity: Rough Sketches

Step 1: Imagine a movie or photograph in your head of the problem. Imagine the story of the problem or the perfect diagram for it. Think of what in the movie or photograph you could count or measure. Try to imagine removing from the movie or photograph all of the “extra” non-mathematical stuff. What’s left?

Step 2: Draw a rough sketch of what was left. Represent what you can count or measure, and leave out as much of the “extra” information as you can. Label your sketch with numbers, letters, or words if you can.

Step 3: Share your drawing with your partner or group. Compare your drawing with the other drawings. What are the similarities? What are the differences? What countable things do you see? What measurable things do you see? What would you label? What might you do differently?

Step 4: As a group, try to make the simplest, clearest sketch possible. Use it as you try to solve the problem. How is it helpful?

Key Outcomes:
- Use visual and physical intelligence to develop a sense of what is going on in the problem.
- Figure out an answer or a good estimate and use this to start thinking of explanations about why it works that way.
- Develop techniques for representing information abstractly.
- Become more comfortable with drawings that are not to scale or not representative.

Examples: Wooden Legs (FunPoW)

Wooden Legs

Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables.

She has a supply of 31 legs and wants to use them all to make stools and tables.

Find all the possible ways she can use all 31 legs.

Explain how you solved the problem and how you know you have found all solutions.

Extra: Wendy sells her furniture to the local toy store. She gets $2 for each stool and $3 for each table. Of all the ways you found, which would earn her the most money?

Be sure to explain how you know.

The goal of these lessons is for the students to experience generating their own understanding of the problem. While it’s tempting to steer them towards certain key ideas, we want students to experience the gain in confidence that comes from being able to rely on their own resources in order to get going. As a result, we tend to hold back on suggestions and focus on supporting the student’s own thinking. If students are stuck, or we feel their ideas need further probing and clarifying, we might help with facilitating questions that reinforce the problem-solving strategies. Check out the “funpow-teachers” discussion group (http://mathforum.org/kb/forum.jspa?forumID=526) for conversations about this problem in which teachers can share questions, student solutions, and implementation ideas.

If we do facilitate by asking some strategy questions, then at the end of the session we often ask students to notice the questions and suggestions we asked so that they can begin to do that for themselves: Which were helpful? Could you see how you could use these with other problems? Which questions would you put on a class list of “Ways to get Unstuck in Understanding the Problem”?

Below you’ll find examples of student thinking, both to provide illustrations of the activities we described above, and to anticipate student thinking that may come up in your class that you might want to probe or question further.
I. Noticing/Wondering

Noticing/Wondering students may generate:

- I notice Wendy makes stools with 3 legs and tables with 4 legs.
- I notice Wendy has 31 legs to build with.
- I notice that the same kind of leg is used for the stool and for the table.
- I notice Wendy wants to use all 31 legs in making stools and tables.
- I notice the question asks to find all the different ways Wendy can use the legs.
- I wonder how many different ways there are for Wendy to make stools and tables.
- I wonder how I can know that I found all of the ways.
- I wonder if she will make more stools or tables.
- I wonder how I should organize my work.
- I wonder how I could use some manipulatives to help me solve the problem.

Reactions students may have:

- I was not sure if we were supposed to find more than one answer until someone said, “I wonder how many different ways there are for Wendy to make stools and tables.”
- When someone asked about using manipulatives, it made me wonder what I should use so the manipulatives look like wooden legs.
- I was not sure how to organize my work and was glad to hear someone else say they were not sure either because 31 is a lot of legs to keep track of.

II. Extracting the Information and Question

PoW IQ notes students may generate:

<table>
<thead>
<tr>
<th>Information</th>
<th>Implications</th>
<th>Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wendy has 31 total wooden legs. Wendy will use all 31 wooden legs to make tables and stools. Wendy makes stools with 3 legs. Wendy makes tables with 4 legs. The question asks how many different ways can Wendy use the 31 total legs to make stools and tables.</td>
<td>The number of legs must total 31, no matter how many stools and tables Wendy makes. There are no combinations of just tables or just stools that will use all the legs because 31 is not a multiple of 3 or 4. I can list multiples of 3 and multiples of 4 smaller than 31 to see what it is possible to use.</td>
<td>How many combinations of 3 legs and 4 legs can I make that use all 31 legs? How many tables and stools can be made with 31 legs? Can I make more than one combination of tables and stools that uses all 31 wooden legs?</td>
</tr>
</tbody>
</table>

III. Paraphrasing

Key Words and Phrases

- “Same kind of legs” – This tells us that 31 legs are used for making stools and tables.
- “Wants to use them all” – We should try to use all the legs so that none are left over.
- “Find all the possible ways” – There are different combinations of stools and tables that will use all 31 legs and we need to find each of these combinations.
- 3-legged stools – students need to understand this means a stool made from exactly 3 of the wooden legs.
- 4-legged tables – students need to understand this means a table made from exactly 4 of the wooden legs.
- Supply of 31 legs – students need to understand that Wendy has 31 individual legs to use for making her tables and stools.
IV. Acting it Out

Possible student drawings:

- Students draw tables and stools to keep track of them.

I started drawing stools and tables and counting legs. I determined that 5 stools and 4 tables use 31 wooden legs.

- Students use tallies or toothpicks to represent the 31 wooden legs, grouping them together for a table or stool.

First I drew the most stools I could have, which is 10 stools made from three legs each:

```
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
```

Then, I knew that I had to have 31 total legs, and I only had used 30, so I added one leg to the last group and made it a table, with 4 legs.

```
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l
  l l l l
```

Therefore, 9 stools and 1 table use 31 wooden legs.

- Students use triangle and square pattern blocks to represent stools and tables, respectively.

I used a triangle to represent 3-legged stools and a square to represent 4-legged tables. I knew I just had to keep track of the legs in my head and multiply the squares by 4 and the triangle by 3. I could also just count the corners on each.

```
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
```

5 tables and 3 stools use 29 wooden legs

This was not enough, so I tried adding a square and taking out a triangle.

```
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
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  ▲ ▲ ▲
```

6 tables and 2 stools use 30 wooden legs

This was not enough, so I tried added another square and took out another triangle because when I did that the last time, the total number of legs went up by one.

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  ▲ ▲ ▲
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  ▲ ▲ ▲
  ▲ ▲ ▲
  ▲ ▲ ▲
```

7 tables and 1 stool use 31 wooden legs
Noticings/Wonderings (Forget the Question)

Make a list of all of the mathematical information and relationships you notice about the situation and everything you wonder about the situation.

<table>
<thead>
<tr>
<th>I notice...</th>
<th>I wonder...</th>
</tr>
</thead>
</table>

Reactions to the Noticings and Wonderings I hear:

You might jot down things like:
- Wow, I never noticed that!
- I wonder if ___ is important to the problem.
- I noticed ____, which "answers" the wondering ____.
- I wonder if you really need to figure out ____ to solve the problem.

My Parking Lot

(noticings and wonderings I would like to come back to, but can put aside for now).
Extracting the Information and Question (PoW IQ)

“PoW IQ” stands for extracting the Information in the problem, and understanding the Question.

<table>
<thead>
<tr>
<th>Information from the Problem (as mathematically as you can)</th>
<th>Calculations or Mathematical Relationships</th>
<th>The Question and Predictions/Considerations</th>
</tr>
</thead>
</table>

If you get stuck, for the Information column, you might:

- Identify and list important information given in the problem:
  - What quantities are given?
  - What terms are important?
  - What constraints are given?

For the Calculations/Relationships column, you might:

- Write down implications of the given information:
  - What do you already know about the key terms or ideas?
  - What calculations could you do?
  - What relationships can you describe?

For the Question column, you might:

- Predict as much as you can about the final answer:
  - What will the units of the solution be (what will be counted or measured)?
  - What justification is needed/what am I trying to prove?
  - Can I figure out upper and lower bounds?
  - Could the answer be negative? Could it be a non-integer?
Paraphrasing (In Your Own Words)

Step 1: Highlight words or phrases you think are going to be really important to understanding and solving the problem.

Step 2: Write each word on a sheet of paper, with several lines between each. Below each word, sketch or write what you think it means.

Step 3: Write what you need to find out about that word, or as many things as you can think of that the word tells you about the problem.

Word 1: _____________________________
Meaning: _______________________________________________________________________________________________
________________________________________________________________________________________________________
Questions / What the word tells me: ______________________________________________________________________
________________________________________________________________________________________________________

Word 2: _____________________________
Meaning: _______________________________________________________________________________________________
________________________________________________________________________________________________________
Questions / What the word tells me: ______________________________________________________________________
________________________________________________________________________________________________________

Word 3: _____________________________
Meaning: _______________________________________________________________________________________________
________________________________________________________________________________________________________
Questions / What the word tells me: ______________________________________________________________________
________________________________________________________________________________________________________

Word 4: _____________________________
Meaning: _______________________________________________________________________________________________
________________________________________________________________________________________________________
Questions / What the word tells me: ______________________________________________________________________
________________________________________________________________________________________________________
Acting It Out (Rough Sketches)

**Step 1:** Imagine a movie or photograph in your head of the problem. Imagine the story of the problem or an illustration for it. Think of what in the movie or photograph you could count or measure. Re-imagine the movie, focusing on the quantities you could count or measure. Try removing all of the “extra” non-mathematical stuff that you can’t count or measure. What’s left?

**Step 2:** Draw a sketch of the mathematical movie or picture. Show the stuff you can count or measure and leave out as much of the “extra” information as you can. You might use a sequential storyboard, shapes, icons, a graph, a table, fraction bars, etc. Label your sketch with numbers, letters, or words, and/or use the key below.

**Sketch Key:** Create a key for your sketch that includes:

- The names and values of the quantities represented.

- If possible, a mathematical representation of the relationships represented.
Acting It Out (Rough Sketches), continued

Step 3: Share your drawing with your partner or group.

Compare your drawing with the other drawings noting the similarity or differences in:

• The names and values of the quantities and how they were represented.

• The relationships between the quantities and how they were represented.

Reflect on any improvements you want to make to your sketch:

• What would you label?

• What might you do differently so that the quantities and relationships are clearer or more accurate?

Step 4: As a group, try to make the simplest, clearest sketch possible. Use it as you try to solve the problem. How is it helpful?
Wooden Legs

Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables.

She has a supply of 31 legs.
Wooden Legs

Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables.

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Find all the possible ways she can use all 31 legs.

Explain how you solved the problem and how you know you have found all solutions.

**Extra:** Wendy sells her furniture to the local toy store. She gets $2 for each stool and $3 for each table. Of all the ways you found, which would earn her the most money?

Be sure to explain how you know.