



Algebra PoW Packet

Math Club Mystery

February 14, 2011 • <http://mathforum.org/pows/>

Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, and some samples of the student work we received in February 2007. It is Library Problem #4036. The text of the problem is included below. A print-friendly version is available using the “Print” link on the current AlgPoW problem page in the blue shaded box at the top of the page.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. To access the discussions [log in using your PoW username/password], choose one of these methods:

- from your **My PoW Work as a Teacher** area use the link to “PoW Member Discussions.”
- go to *algpow-teachers* directly: <http://mathforum.org/kb/forum.jspa?forumID=528>

Are you making the most of your PoW Membership? If you have an Individual Teacher Membership consider registering for one of our (free) Orientation Sessions to learn more about the features of your membership. Teachers with Class or School or District Memberships are welcome to take the free Orientation Session but also are encouraged to register for one of our online courses. View information, dates, and links to register here: <http://mathforum.org/pd/>.

Standards

In **Math Club Mystery** students are asked to find out how many students, teachers and parents went on a trip to the movies. The **key concept** is to set up equations that represent the given information and use those equations to determine, from the given information the possible combinations of students, teachers and parents that were on the trip.

If your state has adopted the [Common Core State Standards](#), this alignment may be helpful:

Algebra: Reasoning with Equations and Inequalities: Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Algebra: Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

Additional alignment information can be found through the [Write Math with the Math Forum](#) service, where teachers can browse by NCTM and individual state standards, as well as popular textbook chapters, to find related problems.

The Problem

Math Club Mystery

Members of the Math Club at Morganson Middle School sent us this problem:

The math club took a field trip to see the movie *An Inconvenient Truth*. A total of 28 people went on the trip, including students, teachers, and parent chaperones. There were more parents than teachers.



Movie tickets cost \$7, but the students got a discount and only had to pay \$3 each. The group paid a total of \$108 to get everyone in. Determine how many students, how many teachers, and how many parents went on the trip.

How much can you figure out and be certain of about how many students, teachers, and parents went on the trip?

Extra: State a mathematical relationship between two of the types of people (students, teachers, parents) that would allow you to be certain of all three answers if it had been part of the original problem.

Answer Check

After students submit their solution, they can choose to “check” their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn’t agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation:

You can determine with certainty the number of students who went on the trip and also the total number of teachers and parents who went. You cannot be certain about the individual numbers for teachers and parents.

If your answer **doesn’t match ours**:

- did you try using variables to represent the number of each type of person and writing equations based on the given information?
- did you realize that both the teachers and the parents paid \$7 to get into the movie?
- did you try solving your system of equations by elimination or combination?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you’re still stuck, leave a comment that tells us where you think you need help or where you’re having trouble.

If your answer **does match ours**,

- did you use algebraic techniques to find your answer?
- did you show and explain the thinking and work you did?
- is your explanation clear and complete? Would another student understand your solution?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

This problem is intended to have students use the given information to determine what they can about the number of teachers, students and parents who went to the movies. For any method, the key is that students take the given information, assign variables, and set up algebraic relationships that allow them to figure out what they can about the number of teachers, parents and students on the trip.

Method 1: Elimination/Combination

There are three types of people, so I’ll use a variable for each one:

Let s = the number of students who went on the trip
 t = the number of teachers who went on the trip
 p = the number of parents who went on the trip

I know that 28 people went in all, so $s + t + p = 28$.

Each student paid \$3, so the total paid by all students was $3s$ dollars.

All others paid \$7, so the total paid by the teachers and parents are $7t$ and $7p$ dollars respectively. The group paid \$108 total, so $3s + 7t + 7p = 108$.

I have two equations, so I’ll use the combination/elimination method to try and solve the system, multiplying the first equation by -7 :

$$\begin{aligned} -7(s + t + p) &= -7(28) \\ -7s - 7t - 7p &= -196 \end{aligned}$$

and then adding it to the second equation:

$$\begin{aligned} -7s - 7t - 7p &= -196 \\ 3s + 7t + 7p &= 108 \\ -4s &= -88 \\ s &= 22 \end{aligned}$$

There were 22 students on the trip. Substituting $s = 22$ into the first equation and simplifying:

$$\begin{aligned} s + t + p &= 28 \\ 22 + t + p &= 28 \\ t + p &= 6 \end{aligned}$$

I can be certain that 22 students went on the trip. I also know that the total number of teachers and parents was 6, but I can't be certain how many of each there were.

Method 2: Substitution

I will group the teachers and parents into a group called adults. Now I have students and adults to work with:

Let s = the number of students who went on the trip
 a = the number of adults who went on the trip

There were 28 people total, so $s + a = 28$. Each student paid \$3, so the total paid by the students was $3s$ dollars. Each adult paid \$7 so the total paid by the adults was $7a$. A total of \$108 was paid, so $3s + 7a = 108$.

I have a system of two equations and two variables, so I'll solve it by solving the first equation for a and substituting that into the second equation:

$$\begin{aligned} s + a &= 28 \\ a &= 28 - s \end{aligned}$$

$$\begin{aligned} 3s + 7a &= 108 \\ 3s + 7(28 - s) &= 108 \\ 3s + 196 - 7s &= 108 \\ -4s &= -88 \\ s &= 22 \end{aligned}$$

There were 22 students on the trip. Substituting $s = 22$ I can solve for a :

$$\begin{aligned} a &= 28 - s \\ a &= 28 - 22 \\ a &= 6 \end{aligned}$$

I can be certain that 22 students went on the trip. I also know that the total number of teachers and parents was 6, but I can't be certain how many of each there were.

I can't be certain about how many teachers and how many parents went on the trip beyond the fact that the total number was 6. However, I can do some further speculation based on interpreting the problem.

Start with a list of all possible arrangements of teachers and parents that total 6. Writing them as ordered pairs in the form (teachers, parents) I would have:

$$(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), \text{ and } (6,0).$$

The problem said there were more parents than teachers. That eliminates some of the possibilities above, leaving only:

$$(0,6), (1,5), \text{ and } (2,4).$$

Since the problem states that students, teachers, and parents went on the trip, I can discard the (0,6) option since that included no teachers.

So given the information in the problem, I can decide with certainty that there were 22 students and a total of 6 teachers and parents. I can further be confident that there were either 1 teacher and 5 parents or 2 teachers and 4 parents.

Extra:

As noted above, there are only two possible solutions to the problem with regard to the number of teachers and parents - 1 teacher and 5 parents or 2 teachers and 4 parents.

Possibilities include:

There were twice as many parents as teachers on the trip. (2,4).

There were four more parents than teachers on the trip. (1,5).

Method 3: Using One Variable

I decided to start by assigning a variable to the number of students.

Let x = the number of students who went on the trip
So, $28 - x$ = the number of adults

Since student tickets cost \$3, adult tickets cost \$7, and the total cost of the tickets for students and adults was \$108, I can set up the following equation:

$$3x + 7(28 - x) = 108$$

And then I can solve it:

$$3x + 7(28 - x) = 108$$

$$3x + 196 - 7x = 108$$

$$196 - 4x = 108$$

$$-4x = -88$$

$$x = 22$$

So the number of students is 22.

And the number of adults, equals $28 - x = 6$. So there are 6 adults.

Since there are more parents than teachers and I am assuming there is at least 1 teacher, which means that there must be either 4 parents and 2 teachers or 5 parents and 1 teacher.

I know for sure that there are 22 students and either 4 parents and 2 teachers or 5 parents and 1 teacher.

Teaching Suggestions

This is a great problem for thinking about what the limits are of what you can determine given a limited amount of information. Students can practice setting up relationships, conjecturing about potential solutions and solving simultaneous equations with 2 or 3 variables.

Since one of the main goals of this problem is to have students think about what they can and cannot determine with certainty about the number of students, parents and teachers, it may be useful to start by exploring the Scenario:

The math club took a field trip to see the movie *An Inconvenient Truth*. A total of 28 people went on the trip, including students, teachers, and parent chaperones. There were more parents than teachers.

Movie tickets cost \$7, but the students got a discount and only had to pay \$3 each.

By looking at the scenario, students can brainstorm about the relationships and equations that may be useful, create questions they'd like to answer and engage with the material.

One key decision in solving the problem is whether to treat each of the three person types (students, teachers, parents) individually or whether to put the teachers and parents into one group. Students may be able to set up two equations, but are likely to become concerned when they see they have three unknowns. If they have set up three variables and become stuck, it may be helpful to review with them the strategies for solving systems of equations and to think about which one might work best for the way they've set up the problem.

The problem is a good chance to practice the "Make a Mathematical Model" strategy and activities as outlined in our *Activity Series* area. Figuring out how to model a given situation in a mathematical way is a great skill that's practiced often in Algebra. A link to the *Activity Series* page is always available in the left menu when you're logged in, and links to details and examples of that strategy can be found in the blue box on the problem page. I've also modeled what that approach might look like in the *Our Solutions* section above.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the resources for the Current Problems as we add them throughout the 2010-2011 season, consider bookmarking this page:

<http://mathforum.org/pow/support/>

Student Solutions
Focus on Interpretation

In the solutions below, I've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement. The table below is an excerpt from the rubric for this problem showing the guidelines for scoring in Interpretation:

Novice	Apprentice	Practitioner	Expert
Shows understanding of few of the criteria listed in the Practitioner column.	Shows understanding of most but not all of the criteria listed in the Practitioner column.	Understands that: <ul style="list-style-type: none"> the 28 people are students, teachers or parents the cost of tickets is \$3 for students and \$7 for adults the total cost of the tickets for students, parents and teachers is \$108. the goal is to represent the situation in a way that will allow the determination of how many students, adults and teachers went on the trip 	Solves the main problem and the Extra correctly, and is at least a Practitioner in Strategy, or discusses possible interpretations of the results, leading to the conclusion that there were either 2 teachers and 4 parents or 1 teacher and 5 parents.

For each solution, I've included a comment about why I would score it as shown, as well as what I'd ask the student to work on when they revised their solution to help them move forward with solving the problem or improving their write-up of their work.

Layla
age 13
Interpretation
Novice

I think that there are 12 students on the trip. There are 8 parents and 8 teachers on the trip. What i did to get my answer was divide 108 divided by 3 and my answer was 36 then i divided 36 divided by 3 and i got 12.

I figured this out by dividing the price of the tickets.

Layla's solution represents an understanding that the total number of people on the trip is 28, but her explanation of how she came to 12 students does not indicate that she understands how to use the price of the tickets to make sense of the problem. I would ask her why she divided 108 by 3.

Paul
age 15
Interpretation
Novice

There was 70 students, 20 parents, and 18 teachers and all together in total is 108.

Well the movie tickets were \$7 for adults and \$3 for students so i came to the conclusion of

70-students
20-parents
18-teachers.

Paul seems to have confused the total cost of the tickets with the total number of people on the trip. I would point out that the number of people is 28 and that \$108 represents the cost of all the tickets together.

Lakiya
age 14
Interpretation
Novice

7 parents 5 teachers 8 students went to watch an inconvenient truth.

I did $7 \times 7 = 49$. Then i subtracted 108 from 49 and got 59. Next i did 5×7 and got 35. I added 35 and 49 and got 84. After that i did $8 \times 3 = 24$. Then i added $84 + 24$ and got 108. So ther will be 7 parents 5 Teachers and 8 Students going to the movies to see An Inconvenient Truth.

Lakiya has found a solution that accounts for the total cost of the tickets, but it seems she forgot to attend to the total number of people on the trip. I would ask her to explain more about how she got 7 parents, 5 teachers, and 8 students since I am having some trouble following her reasoning.

Treshell
age 14
Interpretation
Apprentice

It would be 22 kids, 2teachers, and 4 adults.

It will be 22kids, 2teachers, and 4 adults. I started by trying to figure out how many of each it would be. Then I put it down on a sheet of paper. I tried to count in my head but it was to many numbers. So I put it in the calculator and i got my answer.

Treshell has come up with an answer that works for the given constraints. However, she has not explained clearly what she did with the calculator and how she knew which things she tried worked and which did not. I would ask her if 2 teachers and 4 adults (I'm assuming she means parents) was the only combination that worked for the non-students tickets. Lastly, I'd remind her this is an algebra problem and even if she can solve it with her calculator, we do need to see the relationships algebraically.

Evan
age 14
Interpretation
Pracitioner

There were 22 students, 4 parents, and 2 teachers.

I used the guess and check method. These are the problems I tried.

19 students, 5 parents, and 4 teachers? No.
18 students, 6 parents, and 4 teachers? No.
15 students, 7 parents, and 6 teachers? No.
12 students, 9 parents, and 7 teachers? No.
10 students, 10 parents, and 8 teachers? No.
9 students, 10 parents, and 9 teachers? No.
21 students, 4 parents, and 3 teachers? No.
20 students, 5 parents, and 3 teachers? No.
22 students, 4 parents, and 2 teachers? Yes!

Evan has effectively interpreted the problem and used the information presented to solve the problem. He is therefore a Practitioner in Interpretation. For strategy, however, I would rate him as an Apprentice, since the Guess & Check strategy does not rely on an algebraic representation of the problem. I would encourage him to see if he could assign variables and come up with equations to represent this problem.

Elton
age 16

Interpretaton
Practitioner

I am certain about the number of students (22) who went on the trip and the total number of adults (parents and teachers) who went.

I started by creating two different equations using the info given to me. They were:

The amount of people who went; $s + p + t = 28$ people
The total cost; $\$3s + \$7a = 108$ dollars

I then formed a system of equations using the cost and an altered form of the number of people. I altered it by combining the teachers and parents.

$$\begin{aligned} s + a &= 28 \text{ people} \\ \$3s + \$7a &= 108 \text{ dollars} \end{aligned}$$

Now I solved for the number of adults by multiplying the top equation by -3.

$$\begin{array}{r} -3s - 3a = -84 \\ \$3s + \$7a = 108 \\ \hline 4a = 24/4 \\ \hline a = 6 \end{array} \qquad \begin{array}{r} s + 6 = 28 \text{ people} \\ - 6 = -6 \\ \hline s = 22 \end{array}$$

Through this sequence I now know that there are 22 students and 6 adults. It is impossible to solve for the number of parents and teachers without any more information. To check my work I plugged my answers into my cost equation.

$$\begin{aligned} \$3(22) + \$7(6) &= 108 \text{ dollars} \\ \$66 + \$42 &= 108 \text{ dollars} \\ \$108 &= 108 \text{ dollars} \end{aligned}$$

Elton's explanation of his work shows that he interpreted the given information in the problem at the Practitioner level, attending to the number of people, the cost of the tickets and the relationship between the two. His work would also earn him the Practitioner level at Completeness, Clarity and Strategy.

William
age 14

Interpretation
Practitioner

There are 22 children and 6 adult chaperones or teachers that attended the movie.

For the first part i did an equation for X
 x =The number of students
 $28-x$ =The number of parent chaperones or teachers

$$\begin{aligned} 108-3x &= 7(28-x) \\ 108-3x &= 196-7x \\ &+7x \quad +7x \\ 108+4x &= 196 \\ -108 \quad -108 & \\ 4x &= 88 \\ /4 \quad /4 & \\ x &= 22 \end{aligned}$$

$$\begin{aligned} x &= 22 \\ 28-x &= 6 \end{aligned}$$

William solved this problem nicely using only one variable. For Interpretation I would score him as a Practitioner and encourage him to explore the extra. For Completeness, I would score him as Apprentice and ask him to expand his explanation of the equation he set up adding more detail around how he used the ticket price and the original equation he created.

Kadeja
age 14

Interpretation
Expert

I'm certain there were 22 students and 6 adults on the trip. I can't be sure how many teachers or parents there were.

I decided to put teachers and parents together and call them adults. Then I picked two variables:

s = the number of students on the trip
 a = the number of adults on the trip

Since 28 people went, I wrote $s + a = 28$. Each student paid 3 dollars, so s students would pay $3s$ dollars. Each adult paid 7 dollars so a adults would pay $7a$ dollars. The total was 108 dollars, so $3s + 7a = 108$.

Now I had two equations. I decided to use elimination to get rid of one of the variables. I multiplied the first equation by -3 and then added it to the second equation.

$$\begin{array}{r} -3(s + a = 28) \rightarrow -3s - 3a = -84 \\ 3s + 7a = 108 \\ \hline 4a = 24 \\ a = 6 \end{array}$$

That means 6 adults went on the trip. I substituted 6 for a in the first equation to figure out s .

$$\begin{aligned} s + a &= 28 \\ s + 6 &= 28 \\ s &= 22 \end{aligned}$$

That means 22 students went on the trip. So I'm sure there were 22 students and 6 adults.

With the 6 adults, the problem said that parents and teachers went on the trip so there must be some of each. It also said there were more parents than teachers. That means there could be 4 parents and 2 teachers, or 5 parents and 1 teacher. But I can't be sure which one it was.

Extra: If the problem said "There were 2 more parents than teachers" then I could be sure that it had to be 4 parents and 2 teachers since 5 and 1 would not be true for that statement.

Kadeja did a terrific job with this problem. She assigned her variables clearly, explained her equations well and offered a great idea for the extra.

Scoring Rubric

A **problem-specific rubric**, to help in assessing student solutions, is available in the Teacher Support Materials on the Problem page when you are logged in as a teacher. As shown above, we consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good solutions.

We hope these packets are useful in helping you make the most of the Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Valerie (valerie@mathforum.org)