Current Algebra PoW

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The Math Forum
@ Drexel University

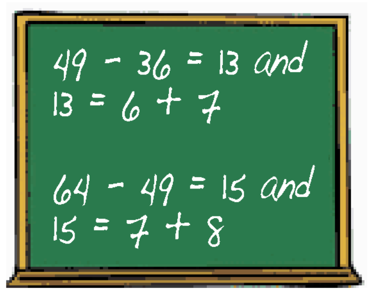
The Math Forum's PoWs provide non-routine constructed response problems. The Algebra problems target concepts typically learned in an Algebra I class. Memberships and mentoring options are available at the individual, class, school, and district levels.

Alvin's Theorem – posted March 24, 2007

Ms. Powers asked her class to look at a list of square numbers and see if they could find any interesting patterns. After a few minutes, Alvin raised his hand and said, "Ms. Powers, I think I've found something cool. If I take two consecutive squares and subtract them, the difference is always the sum of two consecutive integers."

"Show the class what you mean by that, Alvin," said the teacher.

Alvin wrote the following on the board:



Turning to the class, he said, "I call this 'Alvin's Theorem.'"

Ms. Powers smiled and said, "Very good, but if you want to call it a theorem, you must be able to prove that it's true for every possible pair of consecutive square numbers. Can you do that using algebra?"

Alvin worked for a while and then said, "Yes, I can do that, too. Here's how."

What might Alvin have written on the board next?

Extra: Later, Alvin found a similar pattern for the difference of every other square, such as 49 - 25 or 64 - 36. What pattern involving a sum do you think he discovered this time?

Learn more about the PoWs in Booth 2425 or at ***http://mathforum.org/problems_puzzles_landing.html***

Algebra Problem of the Week Scoring Rubric for *Alvin's Theorem*

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	shows little understanding of the concepts involved - see the Practitioner column	shows understanding of most but not all of the concepts in the Practitioner column (for example, understands the vocabulary in the problem and tries to prove Alvin's Theorem, but does so by writing more specific examples)	understands what "consecutive squares" and "consecutive integers" mean understands that proof requires a general case with variable(s), not just more specific examples attempts to show a proof	solves the main problem and the Extra correctly, and is at least a Practitioner in Strategy
Strategy	has few ideas that will lead them toward a successful solution	picks an incorrect strategy, or relies on luck to get the right answer isn't sure how to express consecutive integers using variables hasn't written an expression for the difference of consecutive squares hasn't shown how their expression proves the theorem (i.e. might not simplify)	picks a sound strategy—success achieved through skill, not luck uses algebraic techniques to solve the problem – this might include: • expressing consecutive integers as x and x+1 or x–1 and x • writing an expression for the difference of consecutive squares and simplifying	uses two separate strategies or an unusual or sophisticated strategy (for example, after $(x + 1)^2 - (x)^2$ they might expand and simplify, and also factor as a difference of squares to simplify)
Accuracy	work contains many errors	work is mostly accurate, with a few errors	work is accurate and contains no arithmetic mistakes uses appropriate vocabulary	generally not possible - can't be more accurate than Practitioner
Communication				
Completeness	has written very little that tells or shows how they found their answer	does not define variable(s) explains the steps used to find the answer but shows very few of the calculations and work OR shows the work but does not explain the thinking behind it	defines variable(s) explains and shows all of the steps taken to solve the problem	adds in useful extensions and further explanation of some of the ideas involved the additions are helpful, not just "I'll say more to get more credit"
Clarity	explanation is very difficult to read and follow	explanation isn't entirely unclear, but would be hard for another student to follow explanation is long and is written entirely in one paragraph explanation contains many spelling and typing errors	explains all of the steps in such a way that another student would understand them makes an effort to check their formatting, spelling, and typing (a few errors are fine as long as they don't make it hard to read)	formats things exceptionally clearly answer is very readable and appealing
Reflection	The items in the columns to the right are considered reflective. They could be in the solution or the comment left after viewing the Math Forum's answer.	checked answer in some way (in addition to viewing the answer provided by the Math Forum) reflected on the reasonableness of their answer	connected the problem to prior problems or experiences explained where they are stuck summarized the process they used	commented on and explained the ease or difficulty of the problem revised and improved their work
	did nothing reflective	did one reflective thing	did two reflective things	did three or more reflective things *or* did an exceptional job with two of them