Motivation: Issues in Math Education

- **Goal:** help students learn math with understanding

- **Issue:**
  - Students are told about math and asked to remember facts
  - Shapes students’ perception of math:
    - Math as an uncontestable body of truth
    - No need to think, just plug in the formula!
  - There seems to be a gulf of separation between what mathematicians practice/do and what we teach as math in the classroom

Learning Math with Understanding I

- Triangle area formula
- \[ \text{Area } \triangle ABC = \frac{b \times h}{2} \]
  - \( b \): base length
  - \( h \): height

adapted from Mathematician’s Lament (Lockhart, 2009)
Learning Math with Understanding II

Let’s start with a rectangle

Imagine a triangle inside the rectangle

How much of the box does the triangle take up?

There is as much space inside the triangle as outside; so it takes half the box!

How about if we consider a triangle like this? and so on…
## Learning Math with Understanding III

<table>
<thead>
<tr>
<th>Approach 1</th>
<th>Approach 2</th>
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<tbody>
<tr>
<td>○ Math objects like triangle, base, height and area are simply treated as given</td>
<td>○ Constructs a new object by using a familiar object</td>
</tr>
<tr>
<td>○ The formula of the area is stated as a fact, a question is asked and answered at the same time</td>
<td>○ The area is introduced through a question</td>
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<tr>
<td></td>
<td>○ The height is introduced as a significant problem solving step, not as a given</td>
</tr>
<tr>
<td>Memorizing, rule following</td>
<td>Reasoning, rule eliciting</td>
</tr>
</tbody>
</table>
Multimodal Interaction Spaces in CSCL

- Integration of two or more online communication technologies
  - e.g. text-chat and a shared workspace

- Many CSCL applications and commercial suites like Wimba, Eluminate offer multiple modalities

Belvedere (Suthers et al., 2003)

Epsilon (Soller & Lesgold, 2003)
Overview of Related CSCL Research

<table>
<thead>
<tr>
<th>System</th>
<th>Features</th>
<th>Group Size</th>
<th>Task</th>
<th>Focus</th>
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<tbody>
<tr>
<td>EPSILON</td>
<td>Chat with sentence openers, shared workspace</td>
<td>Triads</td>
<td>Object-oriented design</td>
<td>Automated assessment of effective knowledge sharing cases from sequence of codes</td>
</tr>
<tr>
<td>Representation v2</td>
<td>Chat and shared workspace</td>
<td>Dyads</td>
<td>Database design</td>
<td>A formal model that captures all connections among single actions across dual interaction spaces</td>
</tr>
<tr>
<td>C-Chene</td>
<td>Chat with sentence openers, shared workspace, strict turn taking control</td>
<td>Dyads</td>
<td>Physics problems on the transfer of energy</td>
<td>Facilitating joint attention via strict turn-taking control and sentence openers</td>
</tr>
<tr>
<td>Traffic Simulator</td>
<td>Chat, shared simulation, interaction meters</td>
<td>Dyads</td>
<td>Optimizing traffic flow</td>
<td>The relationship between task and interaction regulation on effective joint work</td>
</tr>
<tr>
<td>MOO</td>
<td>Shared whiteboard and chat integrated into a moo environment</td>
<td>Dyads</td>
<td>Resolving a murder mystery</td>
<td>The relationship between grounding and problem solving. The use of whiteboard as an extended shared memory for the group.</td>
</tr>
<tr>
<td>Belvedere</td>
<td>Chat and shared concept maps</td>
<td>Dyads</td>
<td>Developing argument maps to describe the spread of a disease</td>
<td>Deictic uses of representational proxies. (Guiding partner's attention to a node by direct manipulation and/or via verbal references in chat)</td>
</tr>
</tbody>
</table>
Analytical Approaches in CSCL to Study Multimodal Interaction

- **Content Analysis**
  - Identify a uniform unit of analysis (e.g. chat line)
  - Categorize/code units based on
    - their semantic contents
    - rules devised from a theory of communication (e.g. common ground theory, speech acts)

- **Bottom-up approach to characterize collaboration**
  - Compare groups based on code distributions
  - Correlate codes with outcomes (e.g. post-test scores) to characterize successful collaboration

- **Design features that make this kind of analysis possible**
  - Constrain user actions to partially automate coding
    - Sentence openers, strict turn taking control
  - Studies mainly focus on dyads
  - Tasks have well-defined ontologies (e.g. Entity-Relationship diagrams) that allow modeling of user actions and correct solutions
Typical VMT Use Scenario

- Groups larger than dyads (3-6)
  - Sequential relationships among actions becomes non-trivial due to multiple threads unfolding across modalities
  - Coding/counting ignores sequential organization

- Open-ended tasks
  - We aim to engage students with inquiry-based math learning
  - Groups are encouraged to raise questions and to come up with their own problems

- In short, we are interested in studying interactional achievements of small groups in complex computer mediations “in the wild” (Hutchins, 1995)
For the original problem:
sides: $N(N+3)$
squares: $\frac{n(n-1)}{2}$

We also found formulas for a diamond-like arrangement of the squares:
sides: $(n^2+(n-1)^2)2+n*3-2$  
squares: $n^2+(n-1)^2$

By "sides" we mean the three squares a side of the diamond is comprised of.

We decided that while an explicit formula to calculate the number of squares or sides is clearer for calculating, a recursive formula is easier when one is trying to determine how a particular series or pattern grows.

Next, we did a hexagon made of triangles. $n$ is the side length, again. 
The number of sides is: $9n(n+1)-6n$  
The number of triangles is: $6n^2$
The object of the Game of Pig is to be the first player to reach 100 points or more. Each turn of the game consists of one or more rolls of a die. In a turn, you repeatedly roll the die and record the sum of your rolls until either you decide to stop or a 1 is rolled. If you roll a 1, your score for that turn is zero. If you choose to stop before rolling a 1, your score is the sum of all the numbers you rolled in that turn. Your turn total is added to your overall score, and the next player’s turn begins.

Do you think it is better to choose to roll the die few times or many times? Play several games with your chat-room partners, using the random-integer or dice function of your graphing calculator to simulate the roll of a die. Record the outcomes. Do your game results support your answer? Explain.

With your chat-room partners, develop a strategy to determine when to stop and save your score. Justify why you use this strategy.
# Overview of Instructional Technology

<table>
<thead>
<tr>
<th>IT Type</th>
<th>Theory of Learning</th>
<th>Model of Instruction</th>
<th>Epistemological Stance w.r.t. Math Objects</th>
<th>Research Issue</th>
<th>Research Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAI (60s)</td>
<td>Behaviorism</td>
<td>Programmed instruction</td>
<td>Platonism, absolutism</td>
<td>Instructional Efficacy</td>
<td>Pre/post test design, with a Focus on individuals</td>
</tr>
<tr>
<td>ITS (70s)</td>
<td>Information processing theory of cognition</td>
<td>One-on-one tutoring</td>
<td>Logic-based modeling, Formalism</td>
<td>Instructional Competence</td>
<td>Pre/post test design with more fine grained measures, with a focus on individuals</td>
</tr>
<tr>
<td>Logo (70s)</td>
<td>Cognitive Constructivism</td>
<td>Discovery based learning</td>
<td>Intuitionism, Subjective constructions of individuals.</td>
<td>Instructional Transfer</td>
<td>Protocol analysis (think aloud sessions), design evaluations, pre/post test design, with a focus on the development of an individual across different modeling activities</td>
</tr>
<tr>
<td>CSCL (90s)</td>
<td>Knowledge Building, Situated Learning, Social constructivism</td>
<td>Collaborative Learning</td>
<td>Social co-construction in situ</td>
<td>Instruction as Enacted practice</td>
<td>Design-based research Discourse Analysis Conversation Analysis Focus on social interaction and practices within collectivities</td>
</tr>
</tbody>
</table>
## Evolution of the VMT System

<table>
<thead>
<tr>
<th>Software</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
<th>Stage 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>New Features</strong></td>
<td>Basic text chat</td>
<td>○ Text Chat</td>
<td>○ Text Chat</td>
<td>○ Stage 3 features +</td>
<td>○ Stage 4 features +</td>
</tr>
<tr>
<td></td>
<td></td>
<td>○ Shared drawings</td>
<td>○ Shared drawings</td>
<td>○ Awareness</td>
<td>○ Tabs</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>○ Referencing</td>
<td>● wiki pages</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>● summary tab</td>
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<td></td>
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<td>● shared browser</td>
</tr>
</tbody>
</table>

Tasks: Math problems designed to stimulate thinking about underlying math concepts rather than rote application of formulas (Combinatorics, Probability, Geometry, Algebra, Calculus)

Users: Middle/High School and College students from US, Singapore, Brazil

Group size: 3-6

Duration: ~1 hour long online sessions facilitated by a VMT member
The goal of this line of analytic work is to discover the commonsense understandings and procedures group members use to organize their conduct in particular interactional settings.

Commonsense understandings and procedures are subjected to analytical scrutiny because they “enable actors to recognize and act on their real world circumstances, grasp the intentions and motivations of others, and achieve mutual understandings” (Goodwin and Heritage, 1990 p. 285).

Members’ shared competencies in organizing their conduct not only allow them to produce their own actions, but also to interpret the actions of others (Garfinkel & Sacks, 1970).

Since members enact these understandings/procedures in their situated actions, researchers can discover them through detailed analysis of members’ sequentially organized conduct (Schegloff & Sacks, 1973).
Task Design Rationale

- Allows various solution approaches
  - ranging from simple counting to more advanced methods (e.g. recursion, combinatorics)

- Has both algebraic and geometric aspects
  - various features of the environment could potentially be put into use

- Has an open-ended nature
  - requires teams to discuss and agree upon what would be a mathematically interesting pattern

- Encourages reflection on group work
  - via wiki summaries