



# Pre-Algebra PoW Packet

## *Measuring Melons*

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### Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, a problem-specific scoring rubric, and some samples of the student work we received in April 2006, when *Measuring Melons* first appeared. It is LibraryPoW #3704.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to “PoW Members” or use this URL to go to *prealgpow-teachers* directly: <http://mathforum.org/kb/forum.jspa?forumID=527> [Log in using your PoW username/password.]

### The Problem

In *Measuring Melons*, students are shown three photos and are asked to determine how much each type of fruit weighs.

The text of the problem is included below. A print-friendly version is available from the “Print this Problem” link on the current PreAlgPoW problem page.

### *Measuring Melons*

Jerson is selling fruit at the Farmers’ Market. To attract customers to his booth, he has made up a contest! He has these photos of combinations of fruit on display. His contest is to find out how much each fruit weighs.



8.25 lbs.



9.5 lbs.



11.5 lbs.

*Note:* Assume that both of the melons weigh the same, both of the pineapples weigh the same, and both of the limes weigh the same.

Melanie looks at his photos and tells him, “I know how much one melon weighs.”

How might Melanie have figured out how much one melon weighs? How much does each type of fruit weigh?

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## Answer Check

One melon weighs 3.5 lbs. Be sure to figure out the weight of the lime and the pineapple, too.

If your answer does **not** match ours,

- might a table help you keep track of things?
- can you find how much one pineapple and one lime weigh together?
- can you learn anything by doubling the first scale?

If you used Guess and Check, did you tell . . .

- what numbers you tried?
- how you knew whether they worked or not?
- how you decided what to try next?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- are you confident that you could solve another problem like this successfully?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- what hints would you give another student trying to solve this problem?

*Revise* your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

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## Our Solutions

The key concept is algebraic reasoning.

### Method 1: Guess and Check

First, let's say the melon weighs 4 pounds. Then from the third picture, one pineapple weighs 3.75 pounds. Then from the first picture, a lime is 0.5 pounds. Let's check the second picture: 2 pineapples plus 2 limes is  $7.5 + 1$ , or 8.5, which isn't enough.

Probably the pineapple should weigh more. Let's say it weighs 4 pounds. Then from the third picture, two melons is 7.5, so one is 3.75. From picture 2, the lime is 0.75 pounds. Checking picture 1,  $3.75 + 4 + 0.75 = 8.5$ . That's too much.

Let's make the melon 3.5. Then from picture 3, the pineapple is  $11.5 - 7$ , or 4.5. Picture 2 says the lime is  $9.5 - 9$ , divided by 2. That's 0.25. Checking picture 1,  $3.5 + 4.5 + 0.25 = 8.25$ . Yeah!

### Method 2: Logical Reasoning

Looking at the second picture, we can take half of each side and still have something that's equal. So a pineapple and a lime weigh 4.75 pounds.

Since a pineapple and a lime weigh 4.75 pounds, then from picture 1, we can see that a melon weighs 8.25 pounds minus 4.75 pounds, which is 3.5 pounds.

In picture 3, if we take away the two melons, we will find the weight of a single pineapple.  $11.5 - 3.5 - 3.5 = 4.5$ .

Now go back to picture 2. Two pineapples weigh 9 pounds, so two limes will be  $9.5 - 9$  pounds, or 0.5 pounds. That means one lime is 0.25 pounds.

That's it! A melon is 3.5 pounds. A pineapple is 4.5 pounds. And a lime is 0.25 pounds.

**Note:** There will probably be a number of variations on this method, and that's fine. Just make sure all the steps are well-reasoned, not just guesses. One other version has students noticing that pictures 1 and 3 mean that the weight difference between a melon and a lime is 3.25 pounds, but then they often guess and check from there.

### Method 3: Algebra

Assign variables for the three fruits: m for melon, p for pineapple, and l for lime. Write three equations based on the information in the pictures.

$$m + p + l = 8.25 \quad [1]$$

$$p + p + l + l = 9.5 \quad [2]$$

$$p + m + m = 11.5 \quad [3]$$

We can divide both sides of [2] to find out how much p + l is.

$$\begin{aligned} p + p + l + l &= 9.5 \\ p + l &= 4.75 \text{ pounds} \end{aligned}$$

We can substitute this into [1] to find m.

$$\begin{aligned} m + p + l &= 8.25 \\ m + 4.75 &= 8.25 \\ m &= 3.5 \text{ pounds} \end{aligned}$$

From that, we can use [3] to figure out how much p is.

$$\begin{aligned} p + m + m &= 11.5 \\ p + 3.5 + 3.5 &= 11.5 \\ p + 7 &= 11.5 \\ p &= 4.5 \text{ pounds} \end{aligned}$$

We can substitute that back into [2] to find l.

$$\begin{aligned} p + p + l + l &= 9.5 \\ 4.5 + 4.5 + l + l &= 9.5 \\ 9 + 2l &= 9.5 \\ 2l &= 0.5 \\ l &= 0.25 \end{aligned}$$

So we know that a lime is 0.25 pounds, a pineapple is 4.5 pounds, and a melon is 3.5 pounds.

## Teaching Suggestions

Many students used either guess and check or logical reasoning to solve this problem. I wonder how clearly a student needs to describe how the “looking at the photos” method works so that it is clear that a sound strategy was used?

Can a student look at the lime compared to the other fruit and decide that it weighs a quarter pound? I wonder if a student refers to a real life experience (knowing that four limes weigh about a pound) if that would be considered reasonable?

I wonder if you were to only show the three photos without any of the text, how students might describe what they see? After they have explained what they notice and they move to taking some notes, I wonder how many students would tire of writing the word “pineapple” and instead use an abbreviation? Their next step might be to naturally move to using variables and then algebraic techniques to solve the problem. If some students select that strategy and show it to their classmates, explaining it clearly and completely, it might help them move forward in their algebraic thinking.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources:

<http://mathforum.org/prealgpow/puzzles/supportpage.ehtml?puzzle=404>

The Problem Solving and Communication Activity Series document for this problem contains ideas and activities to help students experience the idea of making a simpler problem:

[http://mathforum.org/pow/support/activityseries/prealgpow\\_psc.404.pdf](http://mathforum.org/pow/support/activityseries/prealgpow_psc.404.pdf)

## Sample Student Solutions

### Focus on Clarity

In the solutions below, we’ve focused on students’ “clarity” of the problem, meaning that they explain the steps in such a way that a fellow student would understand, and make an effort to check formatting, vocabulary, and spelling.

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**Jacob**  
age 14

Clarity  
**Novice**

one melon = 3.5 lbs.

I wrote 3 problems, substituted and used the distributive property to find my answer.

*Jacob has written a clear sentence for his explanation but would a classmate really know what he did?*

*I would notice that he's used some appropriate math vocabulary, including "substituted" and "the distributive property." I would acknowledge that it sounds like he may have been solving some equations.*

*Perhaps using the term "equations" would prompt him to use that word rather than "problems."*

*I would wonder what equations he wrote.*

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**Kyle**  
age 16

Clarity  
**Novice**

In the picture, the melons are on a scale, and maybe she could see the weight on the scale in the picture.

I looked at the picture and I saw a scale, and maybe she did too.

*While it seemed that Jacob might be thinking of an algebraic method, Kyle has started to describe what he sees in the picture.*

*I would notice that he has mentioned the scale. I would wonder what numbers the scale displays in each of the three pictures.*

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**KK**  
age 11

Clarity  
**Apprentice**

The pineapple equals 4.5 lbs., the melon equals 3.5 lbs. and the lime equals .25 lbs.

First I looked at the picture and I knew right away that the pineapple had to be heavier than the melon and lime. I tried putting 4 lbs. for the pineapple and 3 lbs. for the melon. I then put .5 lbs. for the lime. I added up the 4 lbs. for the pineapple and the 3lbs. for the melon. I got 7 lbs. I then added up the .5 lbs. for the lime and I got 7.5 lbs. That doesn't add up. Next I tried 4.5 lbs. for the pineapple and 3.5 lbs. for the melon. 4.5 plus 3.5 equals 8 lbs. Since the total of the 3 put together is 8.25 lbs., and the 2 I have equals 8 lbs. that means that the lime has to equal .25 lbs.

Now for the next picture. I added 4.5 and 4.5 lbs. together because there were 2 pineapples. I then added the two limes and I got 9.5 lbs. That means they have a 2/3 chance of working. For the last picture, I added the 2 melons, (3.5 lbs.) and I got 7 lbs. I then added the pineapple (4.5 lbs.) It all equaled 11.5 lbs. That means the weights for the pineapple, melon, and lime are right. So the answers are 4.5 lbs. for the pineapple (4 lbs. 8 ounces) 3.5 lbs. for the melon, (3 lbs. 8 ounces,) and the lime is .25 of a lb. (4 ounces.) Plus the pineapple is heavier than the melon.

*KK's description of this successful guess and check strategy is complete, but it's hard to follow.*

*I wonder if it would make KK's solution easier to follow if the calculations were written one calculation per line, with units.*

**Jesse**  
age 13  
  
Clarity  
**Apprentice**

the melons weigh 3.5, the pineapples weigh 4.5, and the limes weigh .25 to figure this problem out, i used the first two pictures. if one of each kind of fruit is 1.25 less than two pineapples and two limes, then one melon is 1.25 less than one pineapple plus one lime. using the next two pictures, i see that two melons is 2 bigger than one pineapple and two limes. so if 1 melon < pineapple + lime < melon + melon by 2, and first inequality is a 1.25 difference. then i see that the first and last pictures are 3.25 different. melon+lime is 3.25 less than two melons so one lime is 3.25 less than one melon. therefore, one pineapple must be bigger than one melon if one pineapple plus one lime is 1.25 bigger than melon and one lime is 3.25 less than one melon, the difference balance, divided by two, since we're comparing two different things, one pineapple must be one more than a melon. now, we can guess about the melon, and estimate that it's 4. that makes the lime .75, and the pineapple 5.  $4+5+.75$  is larger than 8.25, so that doesn't work. too high. now let's say that the melon is equal to 3 and one half, making the pineapple four and one half, and the lime .25.  $3.5+4.5+.25=8.25$ , which is correct. this works for the other two pictures, so it must be the answer

*Jesse's method of solving the problem by figuring out differences between different combinations of fruit is effective.*

*Similar to K K, Jesse has not separated his text to help the reader but, in addition, the solution is difficult to follow, especially this part: "if one pineapple plus one lime is 1.25 bigger than melon and one lime is 3.25 less than one melon, the difference balance, divided by two, since we're comparing two different things, one pineapple must be one more than a melon".*

*I would ask Jesse, "Could you clarify what you mean when you say, 'the difference balance'? Why did you divide it by two?"*

**Stephen**  
age 14  
  
Clarity  
**Apprentice**

Melon=3.5lbs

pineapple=4.5lbs

lime=0.25lbs

x=one melon's weight

y=one pineapple's weight

z=one lime's weight

$x+y+z=8.25$  lbs ----- a

$2y+2z=9.5$  lbs -----b

$y+2x=11.5$  lbs -----c

First, you have to make the equation with only one unknown quantity

So,  $(ax)-b \rightarrow 2x=(2 \times 8.25)-9.5$ , therefore,  $x=?$

If you prove this equation. The x will be 3.5lbs.

Then, to know the value of y you have to put the value of x into c

then, if you prove it, the y is 4.5lbs {the solve,  $y+(2 \times 3.5)=11.5$ }

Then to know the value of z you have to put the value of y into b

then,  $z=0.25$ . {the solve,  $(2 \times 4.5)+(2 \times z)=9.5$ }

So one melon is 3.5lbs and one pineapple is 4.5lbs and the last, lime is 0.25lbs.

Mr. Compton, I did it myself. My mother helped my grammar skills

*Stephen's algebraic solution is nearly complete, and it was helpful how he labeled his equations.*

*One thing that made Stephen's solution hard to read was his use of x as a variable and as the multiplication sign.*

*I would ask him what other ways he might show multiplication.*

*I would also suggest that he show some more of the simplifications and algebraic manipulations that he did; for example, to clarify "So,  $(ax)-b \rightarrow 2x=(2 \times 8.25)-9.5$ "*

**Channing**  
age 12

Clarity  
**Practitioner**

Melanie could have found out how much each melon weighs by finding the weight of one pineapple and one lime by looking at the second picture, then subtracting it from the first picture. A melon weighs 3.5 pounds, a pineapple weighs 4.5 pounds, and a lime weighs 0.25 pounds.

To find the weight of a melon, you first look at the second picture. If two pineapples and two lime weigh 9.5 pounds, so half that, one pineapple and one lime, must weigh half of 9.5 pounds, or 4.75 pounds. Next, look at the first picture. If a melon, a pineapple, and a lime weigh 8.25 pounds, then a melon, a pineapple, and a lime minus the pineapple and lime must be  $8.25 - 4.75$ , or 3.5, pounds. The melon is the only fruit left on the scale, so it must weigh 3.5 pounds.

To find the weight of a pineapple, look at the third picture. If a pineapple and two melons weigh 11.5 pounds, then a pineapple and two melons minus the two melons weighs  $11.5 - (3.5 \times 2)$ , or 4.5, pounds.

To find the weight of a lime, simply use what we already know. A pineapple and a lime together weighs 4.75 pounds, so subtract the pineapple, to find out that a lime weighs  $4.75 - 4.5$ , or 0.25, pounds.

Then I went back to Problem 1. Since the area of the whole pattern is 128 square units, the number of bricks required to cover it was  $128/2=64$ .

*Channing did a good job of explaining her logical reasoning strategy, based on looking at the information presented visually.*

*I would ask Channing to tell me more about how she got started on the problem. Did splitting the second scale in half jump out at her, or did she try some other things first?*

*If one of her classmates wasn't sure where to begin, is there a tiny hint she might give? Not just, "divide the second scale in half," but something that gets them thinking even more?*

**Ishita**  
age 13

Clarity  
**Practitioner**

The pineapple weighs 4 pounds. The lime weighs .75 pounds. The melon weighs 3.75 pounds.

I made  $p$ =pineapple,  $l$ =lime and  $m$ =melon.

Then I set up three equations:

$$p+l+m=11.5$$

$$2p+2l=9.5$$

$$p+2m=11.5$$

Then I solved for  $p$  using the last equation.

$$p+2m=11.5$$

$$p=-2m+11.5$$

Then I substituted  $p$  in for all the other  $p$ 's in the other two equations and I got:

$$-2m+11.5+l+m=8.25$$

and

$$2(-2m+11.5)+2l=9.5$$

I solved the first equation and got  $-m+l=-3$

Then I solved for  $l$ :  $l=m-3$

Then I solved for the other equation and I ended up with  $-4m+2l=9.5$

Then I substituted  $l$  in for that equation and I ended up with  $m=3.5$

After I found out what  $m$  equalled, I just substituted  $m$  in for all the other  $m$ 's in the other two equations. Then I ended up with  $l=.75$  and  $p=4$

*Ishita did a great job laying out a clear algebraic solution. I like how the variables are labeled, the manipulations are well laid out, and the text is interspersed to explain what's going on.*

*In addition to suggesting that another variable beside  $l$  might be easier to read (I looks a lot like 1), I would suggest that Ishita find a way to check the final answer of limes = .75 pounds and pineapples = 4 pounds.*

*I might even say that I also got 3.5 pounds for the melons, but that my answers for the other two are different, and wonder how that could be.*

**T K**  
age 13

Clarity  
**Expert**

One melon weighs 3.5 pounds, one pineapple weighs 4.5 pounds, and one lime weighs .25 pounds.

The first picture may be the most important. It tells you the combined weight of all three fruits.

$$\text{melon} + \text{pineapple} + \text{lime} = 8.25 \text{ lbs.}$$

If you find the weights of any two fruits combined, you can find the weight of the remaining fruit. Subtract the weight of the two fruits from the total weight, and you have the weight of the last fruit.

$$8.25 - (\text{pineapple} + \text{lime}) = \text{melon}$$

$$8.25 - (\text{pineapple} + \text{melon}) = \text{lime}$$

$$8.25 - (\text{melon} + \text{lime}) = \text{pineapple}$$

The only two fruits that you can find the net weight of are the pineapple and the lime. In the second picture, you are able to divide the weight in half even without knowing what each fruit weighs.

On the scale, there are 2 pineapples (P) and two limes (L), which is equivalent to ---  $2(P+L)$  ---

So if...

$$2(\text{pineapple} + \text{lime}) = 9.5 \text{ lbs.}$$

Then...

$$1(\text{pineapple} + \text{lime}) = 4.75 \text{ lbs.}$$

Now that you know what one pineapple and one lime weigh together, you can find out the melon's weight from the first picture

$$8.25 - 4.75 = \text{melon} \text{ -----> melon} = 3.5 \text{ pounds}$$

In the third picture,

$$11.5 \text{ pounds} - 2 \text{ melons} = \text{pineapple}$$

So...

$$11.5 \text{ pounds} - 7 \text{ pounds} = 4.5 \text{ pounds} \text{ -----> pineapple} = 4.5 \text{ pounds}$$

Subtract the weight of the pineapple from the weight of the lime and the pineapple and you have the weight of the lime.

$$(\text{lime} + \text{pineapple}) - \text{pineapple} = \text{lime}$$

$$4.75 \text{ pounds} - 4.5 \text{ pounds} = .25 \text{ pounds} \text{ -----> lime} = .25 \text{ pounds}$$

There you have it.

*T K did a great job explaining all of the steps in the problem, and justifying why each step made sense. I really like how, even though the strategy is similar to a logical reasoning strategy (rather than solving a system of equations purely algebraically), T K uses algebraic notation to make the explanation very clear.*

*I think that to stretch T K's thinking and build on the observation about the importance of the scale telling the combined weight of all three fruits, I would tell T K about another solution I saw in which the first step was to double the equation  $p + \text{lime} + m = 8.25$ . I would ask what that student might have done next.*

## Scoring Rubric

On the last page is the **problem-specific rubric**, to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on any problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Pre-Algebra PoWs. Please let me know if you have ideas for making them more useful.

~ Lillian and Suzanne

<lillian@mathforum.org> and <suzanne@mathforum.org>

# Pre-Algebra Scoring Rubric for *Measuring Melons*

For each category, choose the level that *best describes* the student's work

|                        | Novice  | Apprentice  | Practitioner   | Expert  |
|------------------------|---|---|--|---|
| <b>Problem Solving</b> |   |   |  |   |
| <b>Interpretation</b>  | doesn't find the weights of any fruit<br>doesn't seem to understand the situation   | only finds the weight of one or two kinds of fruit<br>finds an answer that only works for two of the combinations   | understands that the pictures give weights of different combinations of fruit<br>understands that all three combinations of fruit must work at the same time<br>attempts to find all three weights | there is no Extra, so there's no way to achieve Expert in this category   |
| <b>Strategy</b>        | does not have any ideas about how to solve the problem  | does guess and check without being careful – might have gotten lucky  | has a strategy that relies on skill, not luck<br>if they guess and check, they check well  | uses algebra <i>or</i> uses the other two methods shown in the Expected Solution  |
| <b>Accuracy</b>        | has made many errors  | makes a few errors that lead to an incorrect answer   | makes no arithmetic mistakes that really matter (small stuff and typos don't count)  | [not normally available for this category]  |
| <b>Communication</b>   |   |   |  |   |
| <b>Completeness</b>    | has written nothing that tells you how they found their answer  | shows work without an explanation <i>or</i> explains everything without showing the numbers<br>doesn't include enough information for another student to follow | attempts to explain all of the steps taken to solve the problem<br>if they use guess and check, explains what they guessed, how they checked, and how they made next "guess"                       | adds in useful extensions and further explanation of some of the ideas involved   |
| <b>Clarity</b>         | explanation is very difficult to read and follow  | another student wouldn't be able to follow their explanation entirely<br>long and written in one paragraph<br>lots of spelling errors/typos                     | explains all of the steps mentioned in such a way that another student would understand<br>makes an effort to check their formatting, spelling, and typing (a few errors are fine)                 | formats things exceptionally clearly<br>answer is very readable and appealing   |
| <b>Reflection</b>      | <i>The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:</i><br><br>does nothing reflective | checks their answer (not the same as viewing our "answer check")<br>reflects on the reasonableness of their answer<br><br>does one reflective thing             | connects the problem to prior knowledge or experience<br>explains where they're stuck<br>summarizes the process they used<br><br>does two reflective things  | comments on and explains the ease or difficulty of the problem<br><b>revising their answer and improving anything</b><br><br>does three or more reflective things or does an exceptional job with two |