



Pre-Algebra PoW Packet

The Oracle's Oration

August 24, 2009 • <http://mathforum.org/pows/>

Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, a problem-specific scoring rubric, and some samples of the student work we received in November 2002, when *The Oracle's Oration* first appeared. It is Library Problem #2768.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From your My PoW area use the link to “PoW Members” or use this URL to go to *prealgpow-teachers* directly: <http://mathforum.org/kb/forum.jspa?forumID=527> [Log in using your PoW username/password.]

The Problem

In *The Oracle's Oration*, students are asked to explain how payment should be distributed based on the number of rolls two people shared with the group of three.

The text of the problem is included below. A print-friendly version is available from the “Print this Problem” link on the current PreAlgPoW problem page.

The Oracle's Oration

Two travelers stop by the side of the road to eat. The first, Amani, has seven bread rolls, while the second, Biagio, has five. A third person, Caleb, stops and asks if Amani and Biagio would be willing to share their rolls. They are, and each of the three eats four rolls.

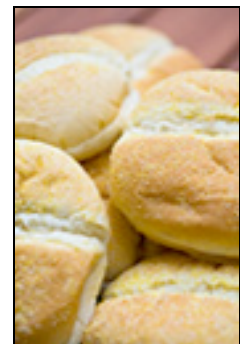
Upon departing, Caleb leaves behind twelve silver pieces as payment for the rolls. Amani thinks that she should have seven of the pieces since she began with seven rolls, and that Biagio should get five, since he started with five rolls. Biagio, however, thinks that they should each get six pieces of silver, since they shared the rolls equally.

Since they can't agree, they go to the Oracle. Once they have explained the situation, the Oracle speaks:

“Amani, you should get nine silver pieces. Biagio, you should get three. I've determined this because”

How did the Oracle explain this?

Extra: Is there another equitable decision the Oracle could have made? What would it have been and how would the Oracle have explained it?



Answer Check

The Oracle's explanation was based on the number of rolls Amani gave to Caleb and the number of rolls Biagio gave to Caleb. (The details you include will depend on what kind of math you used to figure out your answer.)

If your answer **doesn't** match ours,

- did you try acting it out?
- did you try drawing a picture or making a table?
- did you consider the number of rolls Amani gave up compared to the number of rolls Biagio gave up?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you try the Extra?
- have you clearly shown and explained the work you did?
- did you explain your work as well as you could?
- did you make any mistakes along the way? If so, how did you find and fix them?
- what hints would you give another student?

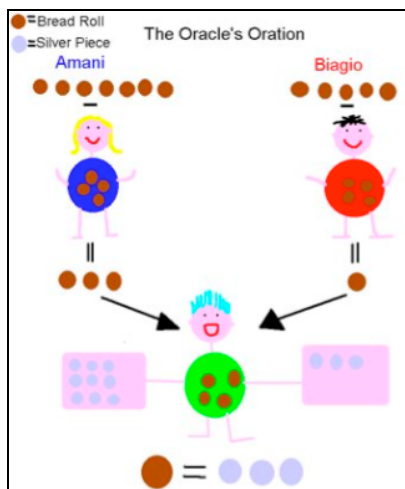
Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

The key concepts are ratios and proportional reasoning.

Method 1: Acting It Out / Drawing a Picture

I could draw a picture or three students could act it out like the picture illustrates:



After eating her share of 4 rolls, Amani had 3 rolls left. After eating his share of rolls, Biagio had 1 roll left. Amani gave 3 rolls to Caleb and Biagio gave 1. The Oracle divided the 12 pieces of silver that Caleb gave to him into 4 sets of 3.

Amani, giving 3 rolls, should get 3 sets of the silver pieces; $\frac{3}{4}$ of 12 is 9 silver pieces. Biagio, giving 1 roll, should get 1 set of the silver pieces; $\frac{1}{4}$ of 12 is 3 silver pieces.

Method 2: Using Ratios

There is a total of twelve rolls to evenly distribute between the three travelers. Amani had seven rolls to begin with, ate four, and sacrificed three. Biagio had five to begin with, ate four, and sacrificed one. This means that for every three rolls that Amani sacrificed, Biagio only had to sacrifice one. That gives a 3:1 ratio.

The money left by Caleb should be shared using the same ratio in which the rolls were shared, 3:1. Twelve silver pieces divided using this ratio equates to the ratio 9:3. This gives Amani nine silver pieces and Biagio three.

Method 3: Using Percentages

Amani had 7 rolls and Biagio had 5 rolls. Caleb had none. To divide the rolls equally between the three people, Amani would give Caleb 3 rolls and Biagio would give Caleb 1 roll and they would each have 4 rolls.

Since Amani gave Caleb 3 of the 4 shared rolls, that was 75% of the rolls. Biagio gave 1 roll, which was 25%. So, to divide the silver coins fairly, Amani would receive 75% of the silver coins and Biagio would receive 25% of the silver coins. 75% of 12 equals 9 and 25% of 12 equals 3, so Amani would get 9 silver coins and Biagio would get 3 silver coins.

Method 4: Using Fractions

Amani started with 7 rolls. She ate 4 herself and shared 3 rolls. Biagio started with 5 rolls, ate 4, too, and shared 1. Amani's 3 leftover rolls plus Biagio's 1 roll made up the four rolls Caleb ate.

Once I knew how many of Amani's and Biagio's rolls Caleb ate, I figured out how much each deserved in payment from Caleb. Remember, Caleb is only paying for the 4 rolls he ate, not all 12 rolls. Since 3

of Caleb's 4 rolls came from Amani, we can say that Amani gave $\frac{3}{4}$ of what Caleb ate. And 1 of Caleb's rolls came from Biagio, so Biagio gave $\frac{1}{4}$ of what Caleb ate:

$$\frac{3 \text{ from Amani}}{4 \text{ Caleb ate}} + \frac{1 \text{ from Biagio}}{4 \text{ Caleb ate}} = \frac{4 \text{ from Amani \& Biagio}}{4 \text{ Caleb ate}}$$

Caleb left 12 silver pieces as payment. Amani gave Caleb $\frac{3}{4}$ of what he ate, so Amani should be paid $\frac{3}{4}$ of the 12 silver pieces Caleb left. To calculate what $\frac{3}{4}$ of 12 is, I multiplied the total payment of 12 silver pieces by $\frac{3}{4}$ and found Amani should be paid 9 silver pieces:

$$\frac{3}{4} \times 12 = \frac{3}{4} \times \frac{12}{1} = \frac{36}{4} = 9$$

Biagio gave Caleb $\frac{1}{4}$ of what he ate, so he deserves $\frac{1}{4}$ of the twelve silver pieces. To calculate Biagio's payment I multiplied $\frac{1}{4}$ times the twelve silver pieces:

$$\frac{1}{4} \times 12 = \frac{1}{4} \times \frac{12}{1} = \frac{12}{4} = 3$$

Therefore, Amani gets paid 9 silver pieces, and Biagio gets paid 3.

Method 5: Using a Table

I made a chart that helped me picture the problem better. It looked like this:

	Rolls	Rolls To Caleb	Rolls Left
Amani	7	3	4
Biago	5	1	4
Caleb	0	0	4

Since the profit is 12 silver pieces, I divided 12 by 4 to see how much each roll of bread was worth. $12/4 = 3$, meaning each roll was worth 3 silver pieces.

Amani provided 3 of the 4 rolls given to Caleb meaning she should get $\frac{3}{4}$ of the profit. $\frac{3}{4}$ of 12 is 9. Biagio provided 1 of the 4 rolls given to Caleb meaning he should get $\frac{1}{4}$ of the profit. $\frac{1}{4}$ of 12 is 3.

Method 6: Using Cost Per Roll

Caleb left behind 12 silver pieces as payment for 4 rolls he ate. If we calculate the price of each roll, then Caleb paid 3 silver pieces for each roll because

$$12 \text{ silver pieces} \div 4 \text{ rolls} = 3 \text{ silver pieces per roll}$$

Amani had 7 rolls and she ate 4 rolls. So she gave $7 - 4 = 3$ rolls to Caleb, therefore, Amani should get

$$3 \times 3 = 9 \text{ silver pieces}$$

Biagio had 5 rolls and he ate 4 rolls. So he gave $5 - 4 = 1$ roll to Caleb, therefore, Biagio should get

$$3 \times 1 = 3 \text{ silver pieces}$$

Teaching Suggestions

We chose this problem to be the first Pre-Algebra PoW of the 2009-2010 season because we are hoping that it presents some challenges that lead to interesting conversations but, at the same time, remains reachable for many students.

One of our favorite classroom activities is to just "notice" and "wonder" about a problem with the final question removed. In case you would like a copy of the problem to either project for the class or to print for individuals or groups, we'll be providing one for most of the problems this year, including this one. Just look for "Scenario Only" listed under the Teacher Support Materials for this problem.

Many students solved this problem successfully when we first used it in 2002 using the variety of strategies listed above. The 2002 version of this problem did not include the Extra. When we recently talked about the problem in the office, however, we wondered if students might have ideas about other ways to distribute the 12 silver pieces that would also make sense. We'd be very interested to hear from you or your students as you discuss this additional feature of the problem.

At one point as I was reading one of the submissions from 2002, I remember that I thought one student

had a new way to explain how this worked that I hadn't yet thought about. The method in question involves adding 2 to Biagio's amount and subtracting 2 from Caleb's amount. As I read through the solution archive to select student solutions to include below, there were several students who used this idea to think about the problem. Interestingly, this method works with 12 silver pieces but if you try a similar problem with 24 silver pieces it doesn't work! This thinking, along with the idea that the Oracle is somewhat magical, helped me realize that acting out the problem or drawing a picture could help students realize that there is actually no magic involved at all.

The Problem Solving and Communication Activity Series document for this problem contains ideas and activities to help students experience generating their own understanding of the problem.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the resources for the Current Problems as we add them throughout the 2009-2010 season, consider bookmarking this page:

<http://mathforum.org/pow/support/>

Sample Student Solutions

Focus on Interpretation

In the solutions below, we've focused on students' "interpretation" of the problem, meaning that they interpret the problem correctly and attempt to solve all of the parts. Because this is the first Pre-Algebra PoW of the season, we've included more Novice and Apprentice solutions than we would normally. Our hope is that they help provide insight into conversations you might have with your students as they start their 2009-2010 year of problem solving and communication.

With our new PoW environment and our continued offering of both these Packets and the Activity Series documents, we invite you to consider registering to participate in one of our online professional development courses. View information here: <http://mathforum.org/pd/>

Also join us in conversations with the PoW community using our discussion: *prealgpow-teachers*: <http://mathforum.org/kb/forum.jspa?forumID=527> [Log in using your PoW username/password.]

Kaylee and Melanie

age 13
Interpretation
Novice

To get that answer, Oracle subtracted 2 pices from Biagio's 5 rolls, and added 2 to Amani's 7.

The Oracle gave Amani 9 silver pieces and Biagio only 3 because Amani had 2 more bread rolls to start than Biagio and he thought it was fair that Amani got more silver pieces because she had more rolls to start.

This is an example of what I mentioned above in the Teaching Suggestions. Kaylee and Melanie thought of an arithmetic solution to the mystery. Does it really work, though?

I would ask Kaylee and Melanie if their method would work if Caleb left 24 silver pieces as payment. If the Oracle then said that Amani should have 18 pieces of silver and Biagio should have 6, could the subtraction method be used to explain it?

Brittney
age 13
Interpretation
Novice

Amani should get 9 pieces of silver because he shared more. and the other person shared less so he shall get less money. i used guess and check and got it right.

Brittany has noticed that Amani shared more with Caleb than Biagio. She might almost be at the Apprentice level instead of where I've included her at the Novice level.

After reinforcing her idea that Amani shared more, I would ask her to act out or draw a picture or a table of the scenario.

Matt
age 11
Interpretation
Novice

Orcale determined this by knowing that Anami had 7 and Biagio had 5 rolls of bread.

I think that the bread must have cost a lot because Anami got silver 9 pieces and Biagio only got 3 silver pieces, even though Anami brought 7 rolls and Biagio had 5 rolls. I got this answer by knowing that it wouldn't be fair if Anami brought 7 rolls and Biagio brought 5 rolls and Anami got 9 silvers and Biagio got three silvers. That is my explanation for solving the Orcale's Oration.

Matt like Brittany has one bit of information that could almost move him to being considered an Apprentice. He mentions the cost of the rolls of bread. That is a great starting point!

I might ask Matt what Caleb paid for with his 12 silver pieces. Perhaps he would realize that Caleb paid for 4 rolls and that would help him think about who actually gave Caleb those rolls.

Huskies
average age 10
Interpretation
Novice

Amani gave two pieces of bread to Caleb, so the Oracle gave her two additional pieces of silver. Amani also gave Biagio one piece of bread for them all to be even. So the Oracle gave her another piece of silver. Biagio received two pieces of silver for giving Caleb two pieces of bread. There was an extra piece of silver left over so the Oracle gave that piece for Biagio.

I notice that the Huskies thought that Amani gave three rolls away but rather than giving all three to Caleb they decided that only two went to Caleb and one went to Biagio.

I wonder why they only thought there were three silver pieces. Might it help for them to act out the scenario?

Patrick
age 13

Interpretation
Novice

Oracle miscounted.
He thought that amani brought 9 of the rolls and biagio brought 3. Thats why i think that oracle gave amani 9 and biagio 3.

Patrick seems convinced the Oracle did not really have a reason for the answer. I wonder if Patrick thinks that we are trying to trick him! We're not. It can be assumed that the Oracle did not miscount but it will be interesting for Patrick (or students who think like Patrick) to convince themselves of this.

I would encourage him to draw a picture or act out the problem.

Math Brains
average age 10

Interpretation
Apprentice

we got the answer by finding out the first problem.that was $3*4=12$ so that was our start.
so with the roles some people who brought the roles got to eat them.
so with the silver pices the people tha did not get any roles they get the silver pices.

*The Math Brains actually do have a start. It would be helpful if they explained what they were thinking when they wrote $3*4=12$. Are they thinking about the rolls or the silver pieces or both?*

IM Crushers
average age 13

Interpretation
Apprentice

The Oracle triped the amount that each of them gave Caleb.
Since Amani started out with 7 rolls and Biagio started out with 5 rolls, they decided to give Caleb some. In order for everyone to get an equal amount, Amani gives Caleb 3 and Biagio gives him 1. Now they all have 4 rolls. When Caleb leaves, he also leaves behind 12 silver pieces for Amani and Biagio to split. Amani and Biagio couldn't come to a agreement so they went to the Oracle. The oracle said that Amani gets 9 and Biagio gets 3. The Oracle said that because he tripled the amount each of them gave Caleb. I think the reason why the Oracle did that was because there are 3 people and since there were three the oracle just tripled it.

The IM Crushers noticed so many details of the problem.

I wonder if they could explain more about what they meant by "he tripled the amount each of them gave Caleb."

Mark
age 15

Interpretation
Practitioner

Amani gave up three rolls, and Biagio gave up one

If we add 3 and one we get 4. Amani gave up 3 out of the 4 rolls.... $3/4$ of the total amount, and Biagio gave up 1 roll.... $1/4$ of the total amount. Since there was 12 silver pieces given up. Amani should get $3/4$ of that amount, $3/4$ converted into $x/12$ gives us $x=9$, after we subtract that from 12, we get Biagio should get 3

Mark isn't as complete and clear as he could be but he seems to have a firm grasp of the fractions involved.

To improve his completeness score I might ask him to explain how " $3/4$ converted into $x/12$ gives us $x=9$."

Stacey
age 11

Interpretation
Practitioner

Armani should receive four silver pieces and Biagio should receive 3.

People	Original Rolls	Rolls Consumed	Rolls Given
Armani	7	4	3
Biagio	5	4	1
Caleb	0	4	-4

Since Caleb paid the boys 12 silver pieces, we use the property of multiplication to multiply all of the rolls given by three. This keeps the proportion the same. This gives you the result that is Armani gave 9, Biagio gave 3 and Caleb gave -12. That negative twelve is how many rolls he gave if multiplied by 3. This is also the amount of silver pieces that he gave, so Armani would receive 9 pieces, and Biagio would get 3.

Stacey's use of a table helped her organize the things she noticed. The table also led her to represent Caleb's payment as a negative number, which is pretty neat.

Her explanation, "Since Caleb paid the boys 12 silver pieces, we use the property of multiplication to multiply all of the rolls given by three" is a little unclear. I might ask her, "How did you know to multiply by three?"

Ben
age 13

Interpretation
Practitioner

The Oracle said, "Armani should get nine silver pieces and Biagio should get three. I have determined this because each roll that Armani gave Caleb is worth three silver pieces. Armani gave Caleb three rolls, each three silver pieces ($3 \times 3 = 9$). Therefore, Armani would pay nine silver pieces. Biagio gave one roll to Caleb which was worth three silver pieces ($3 \times 1 = 3$). Thus, Biagio got paid three silver pieces."

Amani has 7 rolls.
ate 4 rolls.
gave 3 rolls to Caleb.

Biagio has 5 rolls.
ate 4 rolls.
gave 1 roll to Caleb.

I know that:

Caleb ate four rolls
Caleb gave twelve silver pieces to Armani and Biagio.
According to how many silver pieces Caleb paid, each of his rolls is worth three silver pieces. ($12 \text{ divided by } 4 = 3$)

Using that information, I know that:

The 3 rolls that Armani gave Caleb cost 9 silver pieces. ($3 \times 3 = 9$)
The 1 roll that Biagio gave Caleb cost 3 silver pieces. ($3 \times 1 = 3$)

Ben did a nice job showing what he noticed and wondered and the conclusions he drew.

I might ask him to tackle the Extra, by asking him if he wondered or thought about different ways the Oracle could have answered. Does he think the Oracle's way is the fairest way? What makes something fair?

Scoring Rubric

On the last page is the **problem-specific rubric**, to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Pre-Algebra PoWs. Please let me know if you have ideas for making them more useful.

~ Suzanne <suzanne@mathforum.org>

Pre-Algebra Scoring Rubric for Oracle's Oration

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	does none or one of the things listed under Practitioner	does two of the things listed under Practitioner	understands the Oracle's explanation was based on the # of rolls Amani and Biagio each gave to Caleb understands that there are 12 rolls in all understands that there are 12 silver pieces in all	is at least a Practitioner in Strategy and has successfully answered the Extra
Strategy	does not have any ideas about how to solve the problem	has some ideas about how to solve the problem, but isn't quite there	has a strategy that relies on skill, not luck might use ratios, fractions, or percentages might use a table might act it out or draw a picture	uses two separate strategies
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	work is accurate and contains no arithmetic mistakes	[not normally available for this category]
Communication				
Completeness	has written nothing that tells you how they found their answer	shows work without an explanation or explains everything without showing the numbers doesn't include enough information for another student to follow	attempts to explain all of the steps taken to solve the problem, which might include <ul style="list-style-type: none"> • details of how they used ratios, fractions or percentages • visually showing in a picture the exchange • describing how the scene was acted out 	adds in useful extensions and further explanation of some ideas involved
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely long and written in one paragraph lots of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing
Reflection	<i>The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:</i> does nothing reflective	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer does one reflective thing	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used does two reflective things	comments on and explains the ease or difficulty of the problem revises their answer and improves anything does three or more reflective things or an great job with two