Algebra PoW Packet
Apple Orchard

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Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, and teaching suggestions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *algpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=528 [Log in using your PoW username/password.].

The Problem

In ***Apple Orchard***, students are given the total number of apples picked and relationships between the quantities of various types of apples, and are asked to determine how many of each kind was picked. Solving the problem requires some facility with algebraic expressions, ratios, and solving a first-degree equation. More details are available in the "Our Solutions" section below.

The text of the problem is included below. A print-friendly version is available using the "Print this Problem" link on the problem page.

Apple Orchard

On a recent trip to a local orchard, the Nomial family picked four different kinds of apples - Braeburn, Cortland, Fuji, and Rome. When they were done, they discovered that they had picked

* a total of 360 apples,
* twice as many Braeburn as Fuji,
* twice as many Cortland as Rome,
* 50% more Fuji than Rome.

How many of each kind of apple did they pick?

Extra: The Factore family went to the same orchard and picked the same four kinds of apples. When they finished, they found

* they had picked four times as many Fuji as Rome,
* they had 25% more Braeburn than Fuji,
* the number of Cortland exactly equaled the difference between the numbers of Fuji and Rome.

What fraction of their total apples are Fuji?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation:

The Nomials picked 72 Fuji apples. Be sure that you've also determined how many of each of the other types they picked.

If your answer **doesn't match** our answer,

* did you realize that 50% more means one and a half times as much and not twice as much? For example, 50% more than 10 would be 15.
* did you try making a table and guessing numbers to find the patterns in the problem?
* did you try writing expressions for the number of each type of apple using the same variable for all of them?
* if you used a different variable for each apple type, did you try making substitutions based on the given relationships?
* did you use the fact that the total number of apples is 360 to write an equation?

* did you check your arithmetic?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does match** ours,

* is your explanation clear and complete? Did you show all the work that you did so that it's easy to follow?
* did you make any mistakes along the way? If so, how did you find and fix them?
* are there any hints that you would give another student?
* did you have any "Aha!" moments? Describe them.

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did - you might answer one or more of the questions above.

Our Solutions

Here are a few ways that we envision students might solve the problem. There is always the possibility they will come up with another method, and if they do please email me with the thread number and I'll add it to this document. Note that the accompanying Problem Solving and Activity Series document for this problem talks about the strategy of 'Tables and Patterns'. We'll model one such solution here, but that document contains a lot more about what that strategy might look like in the context of this problem.

Strategy 1: Single variable approach

I saw that all the apple amounts are expressed in terms of each other, so I decided to start with one of them being x and seeing if I could use the relationships to define all of them using x. It seemed like the fact that there were 50% more Fuji than Rome was a good place to start, since once I had Fuji and Rome defined, the other two apples were just twice those two. To it easier to do 50% more, I chose to let the Rome be 2x instead of just x. Then I was able to write my expressions:

```
Let 2x = the number of Rome apples picked
    3x = the number of Fuji apples (1.5 times Rome, so 1.5 * 2x = 3x)
    4x = the number of Cortland (twice Rome, so 2 * 2x = 4x)
    6x = the number of Braeburn (twice Fuji, so 2 * 3x = 6x)
```

I know the total number of apples picked is 360, so I wrote an equation adding up my expressions for each kind and having it equal 360. I then solved it for x:

```
2x + 3x + 4x + 6x = 360
              15x = 360
                x = 24
```

Then I took x = 24 and substituted it into each of my variable expressions to find how many of each apple had been picked:

```
Rome = 2x = 2(24) = 48
Fuji = 3x = 3(24) = 72
Cortland = 4x = 4(24) = 96
Braeburn = 6x = 6(24) = 144
```

The Nomials picked 48 Rome, 72 Fuji, 96 Cortland, and 144 Braeburn apples.

Strategy 2: Multiple variable approach

There were four different kinds of apples, so I chose four variables to represent how many of each were picked. I used the first letter of the apple to make it easy to keep track:

```
Let b = the number of Braeburn apples picked
    c = the number of Cortland apples picked
    f = the number of Fuji apples picked
    r = the number of Rome apples picked
```

There were various relationships between apples given in the problem, so I used each item in the list to make an equation:

```
b + c + f + r = 360  (the total number of apples was 360)
b = 2f               (there were twice as many Braeburn as Fuji)
c = 2r               (there were twice as many Cortland as Rome)
f = 1.5r             (the number of Fuji was 50% more than the number of Rome)
```

Now I had four equations and four variables, so I started to make some substitutions to try to get down to a single variable in an equation. I saw that I already knew both c and f in terms of r, so I thought that if I could get b in terms of r I'd be all set. Since f = 1.5r, I substituted 1.5r for f in the equation b = 2f:

```
f = 1.5r and b = 2f so b = 2(1.5r) or 3r
```

Now I had everything in terms of r, so I went back to my total equation and substituted:

```
  b  +   c  +    f   + r = 360
(3r) + (2r) + (1.5r) + r = 360
                    7.5r = 360
                       r = 48
```

Then I took r = 48 and substituted it into each of my expressions to find how many of each kind of apple had been picked:

```
Rome = r = 48
Fuji = 1.5r = 1.5(48) = 72
Braeburn = 3r = 3(48) = 144
Cortland = 2r = 2(48) = 96
```

The Nomials picked 48 Rome, 72 Fuji, 96 Cortland, and 144 Braeburn apples.

Note that there are other substitution paths that would work as well. We've just modeled one.

Strategy 3: Making a table to look for patterns and get a sense of the problem

I wasn't sure how to start the problem, so I made a table and made some guesses to start looking for patterns. My table had rows for each kind of apple and the total, and I started by guessing that they had picked 20 Rome apples:

Guess	1	2	3	4
Rome	20			
Fuji	30			
Cortland	40			
Braeburn	60			
Total	150			

I filled in the rest of that column by knowing that there were 50% more Fuji than Rome, so 1.5(20) = 30. There were also twice as many Cortland as Rome and twice as many Braeburn as Fuji. My total was too low, so I moved up to 40 Rome for my second guess:

Guess	1	2	3	4
Rome	20	40		
Fuji	30	60		
Cortland	40	80		
Braeburn	60	120		
Total	150	300		

I noticed that when I doubled the Rome the total doubled, too. That made me think there is a linear relationship between the change in Rome and the change in total. Increasing the Rome by 20 increased the total by 150. I still need to increase the total by 60 more, so I wrote a proportion using that ratio of change to see how much change in Rome I'd need to get a change of 60 in the total:

```
Let x = the number of Rome to change by

Change in Rome     20    x
---------------   --- = ---
Change in total   150    60
```

I solved the proportion:

```
(20)(60) = (150)(x)
   1200  = 150x
      8  = x
```

Now I knew that I needed to increase the Rome number by 8 to have the total increase by 60 and get to 360. I went back to my table and filled it in for Rome being 40 + 8 or 48:

Guess	1	2	3	4
Rome	20	40	48	
Fuji	30	60	72	
Cortland	40	80	96	
Braeburn	60	120	144	
Total	150	300	360	

It checked with the total being 360, so I know I'm right. *The Nomials picked 48 Rome, 72 Fuji, 96 Cortland, and 144 Braeburn apples.*

Extra

As I did in the main problem, I decided to see if I could define each of the apple amounts in terms of the same variable. Since Fuji was defined in terms of Rome, Braeburn in terms of Fuji, and Cortland in terms of Fuji and Rome, the best choice to start with is Rome. I used r for that one and then wrote expressions for each of the others based on the clues:

```
Let r = the number of Rome apples picked
   4r = the number of Fuji  (four times as many as Rome)
   5r = the number of Braeburn  (25% more than the number of Fuji)
   3r = the number of Cortland  (the difference between Fuji and Rome)
```

To find 25% more, I thought of 25% as 1/4. Then 1/4 of 4r is r, so there are r more Braeburn than Fuji. That makes Braeburn be 4r + r or 5r. To find the difference between the numbers of Fuji and Rome for the Cortland, I just subtracted Fuji and Rome, and 4r - r = 3r.

Now that I know how many of each kind there were, I can answer the question. I added them up to find the total number of apples picked:

```
r + 4r + 5r + 3r = 13r
```

If there are 13r apples and 4r of them are Fuji, the fraction of the total that are Fuji would be 4r/13r. The r factors cancel out and leave 4/13 as the fraction. No matter what r is, the ratio of Fuji to total will always be 4/13.

4/13 of their total apples are Fuji.

Teaching Suggestions

One of the things that I find most interesting about this sort of problem is the large number of kids who immediately choose to use four variables, one for each kind of apple. They do this even if they've not had experience with multi-variable equations or solving systems of equations. Possibly they are so conditioned to define variables when they start a problem that they see that as simply a discrete step where you label all the unknowns with a variable rather than as an important solving step where you can make good decisions about how you define your variables which may well lead to less work in the actual solution.

This sort of thing happens frequently. How many of your students, told that two numbers sum to ten, would label the numbers x and y and write x + y = 10 rather than simply labeling the two numbers x and 10 - x and not needing an equation at all at that point? Once you start to think of using one variable, many classic 'two-variable, two-equation' system problems can easily be solved with a single variable.

While the substitution involved in solving this particular problem with multiple variables is certainly not complicated, it's still a good bit more work than the single variable solution shown above. The problem presents an opportunity for kids to think about solving it both ways and then discussing which method seems easier and why. You might encourage students who solve it one way to then try and solve it the other way so they can make that comparison.

The Extra question provides a context where choosing to use a single variable is considerably easier than using multiple variables. It's our hope that as kids look at both approaches in the main problem they'll choose to approach the Extra in the most efficient way.

One way to get students thinking about the 'scenario' of the problem instead of jumping right to finding 'the answer' is to remove the question when you present the problem. This year we are making a version of the problem without the question available for most of our Current Problems. Just look for the Scenario Only on the Problem page when you're logged in as a Teacher. In general, using the Scenario provides a context for students to engage in 'Noticing and Wondering', an activity presented in 'Understanding the Problem' in the first two rounds of the Problem Solving and Activity Series.

For this problem, the scenario page presents the relationships between the types of apples but does not ask the question or include the fact that the total number of apples is 360. This will give kids a chance to focus on expressing those relationships, using multiple variables or a single variable and hopefully trying both.

While understanding the problem is a good first step in any solving process, the Activity Series document accompanying this problem focuses on the strategy of 'Tables and Patterns' and includes examples of how that strategy would apply to this problem.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the resources for the Current Problems as we add them throughout the 2009-2010 season, consider bookmarking: http://mathforum.org/pow/support/

Scoring Rubric

A **problem-specific rubric,** to help in assessing student solutions, is available in the Teacher Support Materials on the Problem page when you are logged in as a teacher. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of the Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Riz