



Math Fundamentals Packet 2: *Eating Grapes*

September 10, 2007

• <http://mathforum.org/funpow/>

Welcome!

We have been thinking about ways to share implementation ideas for the Problems of the Week among our fellow educators, and to help students and professionals alike get the most from the PoW experience. We offer this packet as a step in that direction.

It contains a problem from our Library, the "answer check", our solution and scoring rubric, teaching suggestions, and a note about common mistakes we have seen. Perhaps best of all, using a past Problem of the Week allows us to include a range of student solutions from our archives. Each of the elements in this packet is described below in more detail.

We'll still be writing new problems this year, just not so many, so that we can focus on providing more support for your work in problem solving and writing. These packets will accompany all problems that we use this year, although there will be no student work for new problems when they are first published.

The Problem

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Eating Grapes is Problem 3247 from the Library. It first ran live in October 2004. I've chosen it because it lends itself to a discussion about interpretation. Any correct solution requires understanding a growth pattern: Angela eats 6 more grapes each day than she ate the previous day. See our solution and the scoring rubric for more detail.

The text of the problem is included here. A print-friendly version is available from the "Print this Problem" link on the problem page on the website.

Answer Check

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After students submit their solution, they can choose to "check" their answer by looking at the answer that we provide. Along with the answer itself (which doesn't explain *how* to find the answer), we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Our Solution

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These are examples of several ways I could imagine children solving the problem. They are not meant to be prescriptive or comprehensive. In fact, we often receive solutions from students who have used approaches we've not anticipated. Those are cause for celebration! My purpose in illustrating a variety of approaches is to demonstrate that the problem is accessible to a wide range of developmental stages. Children with no experience in algebra will apply their number sense and reasoning in ways that are meaningful to them. This kind of thinking prepares them for more formal algebraic thinking in the future.

Scoring Rubric

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We write a **problem-specific rubric** for every problem to help those who are assessing student solutions. It spells out what we expect from students in three areas of problem solving and three of communication. The goal is to look at each category separately when evaluating the students' work. This way the assessment process can provide us with more focused information regarding the areas of strength and weakness in the student work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand the criteria we think make for good problem solving and communication.

Sample Solutions
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The sample student work included in this packet represents a broad range of writing and problem solving skills. We believe strongly in the power of written explanation not only to communicate one's thinking to others, but also to develop the student's own mathematical thinking. Sharing our selected solutions with your students after they have written their own affords an opportunity to discuss what they value in someone else's explanation, leading them to add more detail in their revision. Students often omit information because they don't know what's expected, or they are unable to put themselves in the reader's shoes and imagine what information the reader needs. Seeing a variety of work by other students gives them ideas for what they might include in their own writing.

I've included the Interpretation rubric score and my thoughts about each of the sample solutions.

Common Errors

Most of the **Eating Grapes** solutions that did not reach Practitioner status in Interpretation resulted from faulty understanding of the scenario. Many students interpreted "six more grapes than the day before" as merely "six more grapes" and ignored "than the day before" or thought that Angela ate 100 grapes on Friday alone. These errors might stem from difficulty in reading comprehension or from jumping to conclusions and failing to reflect on whether the final answer works in the original problem.

Before having solvers work independently on a problem, it is a good idea to check their understanding. You might ask children to paraphrase the problem. You could also pick a hypothetical number for the number of grapes Angela ate on Monday and ask students to tell you how many she would have eaten on Tuesday and how they know. "What if Angela ate 20 grapes on Monday? How many would she have eaten on Tuesday?" Refrain from imposing your interpretation, but rather, ask questions that help them clarify their own thinking.

Good luck!

We're excited about providing these new resources to you. We hope you'll share your feedback and ideas in the funpow-teachers discussion group.

~ Claire

Problem

Eating Grapes

On Monday Angela ate some grapes. On Tuesday she was hungrier and ate six more grapes than she ate on Monday. Each day that week she ate six more grapes than the day before. After she had eaten her grapes on Friday she had eaten 100 grapes in all.

How many grapes did she eat on Monday?

Extra: If she continues this pattern, on each day eating six more grapes than the day before, on which day would she eat her 300th grape?

Answer Check

Angela ate 8 grapes on Monday.

If your answer doesn't match ours,

- did you remember that the number of grapes eaten each day after Monday is six more than was eaten the day before?
- did you understand that the total of the grapes eaten over five days is 100?
- did you check your arithmetic?

If your answer does match ours,

- have you clearly shown and explained the work you did?
- did you make any mistakes along the way? If so, how did you find and fix them?
- have you tried a different method to verify your answer?
- did you try the Extra?

Our Solution

Strategy 1 – Using Manipulatives:

I used a red block to represent the number of grapes Angela ate on Monday. For Tuesday I used a red block and six green blocks to show she ate six more grapes than on Monday. For Wednesday I used one red and 12 greens. For Thursday, a red and 18 greens, and for Friday, a red and 24 greens. At the end of the week the total was 5 red blocks and 60 green blocks. Since Angela ate a total of 100 grapes, the 5 red blocks had to stand for 40 grapes. 40 divided by 5 is eight, so each red block represented 8 grapes.

I checked my answer by figuring out how many grapes Angela ate on each day and adding them to make sure they totaled 100.

	Grapes Eaten
Monday	8
Tuesday	14
Wednesday	20
Thursday	26
Friday	32
Total	100

Strategy 2 – Systematic Guess-and-Test:

I took a guess that Angela ate 10 grapes on Monday. Then I figured out how many she ate on the other four days. 16 on Tuesday, 22 on Wednesday, 28 on Thursday, 34 on Friday.
 $10 + 16 + 22 + 28 + 34 = 110$

That told me 10 was too many for Monday. Next I tried 9 grapes on Monday and added six more grapes each day:

$$9 + 15 + 21 + 27 + 33 = 105$$

All these numbers are odd as is the total 105. I noticed that changing Monday from 8 to 9 made a difference of 5 in the total. I needed to find a way to get a total that was 5 less and an even number, so I decreased Monday's grapes by one and tried 8:

$$8 + 14 + 20 + 26 + 32 = 100 \text{ grapes}$$

Strategy 3 - Direct Approach:

I counted up all the 6s that would be added. One 6 on Tuesday, 2 more 6s on Wednesday, 3 more on Thursday, 4 more on Friday. That totals 10 sixes, or 60. 100 total grapes - 60 = 40 grapes, which Angela ate in equal portions over five days. $40 / 5 = 8$. Angela must have eaten 8 grapes the first day.

Strategy 4 - Algebraic:

I let x represent the number of grapes Angela ate on Monday. On Tuesday she ate $x + 6$, on Wednesday $x + 2(6)$, Thursday $x + 3(6)$, Friday $x + 4(6)$.

$$5x + 10(6) = 100$$

$$5x + 60 = 100$$

$$5x = 40$$

$$x = 8$$

Angela ate 8 grapes on Monday.

Extra strategy:

Starting on Saturday, I kept adding 6 new grapes to the number of grapes Angela ate the day before. I kept track of the totals until the total became higher than 300. It happened on Wednesday, the tenth day, when the total reached 350.

	Grapes Eaten	Total
Saturday	38	138
Sunday	44	182
Monday	50	232
Tuesday	56	288
Wednesday	62	350

The Math Fundamentals Problem of the Week Scoring Rubric — Eating Grapes (posted 10 September 2007)

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that the number of grapes eaten each day increases by 6 per day. Understands that the total number of grapes eaten over five days is 100. Completes the main problem.	Understands the Extra; the pattern of 6 more grapes per day continues; the 300 th grape is eaten during the 10th day. Achieves at least Practitioner in Strategy.
Strategy <i>(NB: based on solver's interpretation of the problem)</i>	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that seems to make sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically, OR doesn't make it apparent.	Picks a sound strategy; approaches the problem systematically and logically, achieving success through skill, not luck. (See Expected Solutions for guess-and-test standards.) Chosen strategy accounts for any answer(s) that changed in revision after checking our answers.	Does one or more of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology.
Accuracy <i>(NB: based on chosen strategy)</i>	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work is on main problem is accurate and contains no arithmetic mistakes.	Not available for this problem.
Communication				
Completeness <i>(NB: an incorrect solution can be complete)</i>	Writes very little to explain how the answer was achieved.	Provides explanation but does not include calculations; OR Shows calculations without rationale or explanation.	Explains most of the steps taken to solve the problems, and the rationale for them, with enough detail for another student to understand. Includes key calculations with rationale. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem.
Clarity <i>(NB: incomplete and incorrect solutions can be explained clearly)</i>	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language and notation; uses units appropriately: grapes, days. Shows effort to use good formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful diagram. (A diagram alone doesn't qualify for Expert status.)
Reflection (See list below.)	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment they leave after viewing our answer:	<ul style="list-style-type: none"> • Revises and improves the submission. • Checks the answer using a different method. • Explains a hint she/he would give someone. 	<ul style="list-style-type: none"> • Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them. 	<ul style="list-style-type: none"> • Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used.

Student Solutions

Regardless of strategy, scoring at Practitioner level in Interpretation requires understanding the key math ideas of the main problem: that the number of grapes eaten increases by 6 each day, and that after Friday, Angela has eaten a total of 100 grapes. Solvers who show understanding of the key ideas of the Extra usually earn Expert status.

I've included the Interpretation score for each sample below. It is not my purpose to give a definitive judgment on scoring, but rather to highlight the range of thinking done by these students and suggest ways I might encourage them to take next steps. Assigning an Interpretation score is often complicated by the degree of detail that the student provides, which we call Completeness. The challenge is to look for evidence of their thinking without reading too much between the lines. See the problem-specific rubric for more details.

Eric age 12 Interpretation Novice	Angela ate 4 grapes on Monday. All I did was 6 divided by 100 hundred and I found the remainder was the number of grapes that angela ate on monday.	<i>Eric lacks understanding of the key concepts. I'd ask him to paraphrase the problem for me. Then, "If Angela ate 4 grapes on Monday, how many did she eat on Tuesday? How do you know? Can you make a list of how many she eats each day?"</i>
Grace age 11 Interpretation Novice	Angela ate 70 grapes on monday. The stradigy I used was multiplying and subtracting. First, I multipliyed 6 grapes by the 5 days from Monday to Friday to get the number of grapes that she didn't eat on Monday. I got 30. Then I subtracted 30 (the number of grapes she didn't eat on Monday) from 100 to get 70, my final answer.	<i>Grace writes a fairly detailed explanation of what she did and why. I'd ask, "Did you remember that Angela eats six more grapes each day than she ate the day before?"</i>
Rodney age 13 Interpretation Apprentice	On Monday Angela ate 76 grapes. Since we know that on Friday she ate a total of 100 grapes, I just work backwards from there. Since she ate 100 on Friday, that means she ate 94 on Thursday. Since she ate 94 on Thursday, she ate 88 on Wednesday. 88 on Wednesday, 82 on Tuesday. 82 on Tuesday, 76 on Monday.	<i>Rodney understands that Angela eats six more grapes each day than the previous day. He thinks she ate 100 grapes on Friday alone. I'd ask him to read the last sentence of the problem and paraphrase it. "Did you understand that 100 is the total of all the grapes she ate over five days?"</i>
Alyssa age 9 Interpretation Apprentice	She ate 76 grapes on monday. First I mulltipliyed 6 times 4. to find out the toatal of grapes she ate tuesday through friday. Then I subtracted the 24 I got for the answer from onehundred [How many she ate in the five days.] I got 76 grapes total. Thats how I figured out she ate 76 grapes.	<i>Alyssa arrives at the same answer as Rodney. She understands that 100 is the total for the week, but thinks that Angela eats only 6 grapes on each of the 4 days after Monday. I'd ask her what she thinks the second sentence of the problem means.</i>

Student Solutions

<p>Tim age 11 Interpretation Practitioner</p>	<p>She ate eight grapes, on Monday.</p> <p>First I knew that if I did six grapes on Monday, that I would get six *15. Which equals 90. I knew that I would need ten more. So I tried ten. Then I got 110. Then I knew I needed ten less. So I tried nine. Then I got 105. So I thought that every time I go down by one my total goes down by five. So since I needed 100 I went down by one more to 8. I began with 8 and added 6 to it to get 14 and continued to add 6 three more times until I got to my answer of 100.</p>	<p><i>Tim uses good number sense in testing 6 by clustering the week's worth, 15 groups of 6 or 6 * (1+2+3+4+5), although he needs to explain why. He adjusts his trials thoughtfully. I'd like him to show his other test calculations.</i></p>
<p>Jordan age 9 Interpretation Practitioner</p>	<p>Angela ate 8 grapes on Monday.</p> <p>First, I made a chart and put down 6 grapes on Monday, 12 on Tues., 18 on Wed., 24 on Thurs., and 30 on Fri. Then I added it all up. The sum was 90. After that, I realized that if I added another grape for Monday and did a pattern adding 6 every day there would be 5 more grapes in the sum.</p> <p>Technically speaking, it's a matter of multiplication: 1 grape X 5 days. So that means, if I added 2 grapes to Monday and did the add 6 pattern I'd get the answer. (100 grapes on Friday) This is what the correct list looked like: Mon. 8 grapes, Tues. 14 grapes, Wed. 20 grapes, Thurs. 26 grapes, Fri. 32 Grapes.</p>	<p><i>Jordan gets great mileage from his first guess. He uses what he learns to find the answer directly. I'm glad he shows his first and his final lists of numbers. I wonder what prompted him to start with 6.</i></p>
<p>Michael age 12 Interpretation Practitioner</p>	<p>On Monday, Angela ate 8 grapes.</p> <p>I drew out a timeline, monday-friday, picked a logical number for Friday and subtracted 6 from that number for each day and added them together. I repeated the process until the total came to 100.</p>	<p><i>Michael's clever guess-and-test begins with Friday and works backward. To make his solution complete, he needs to show some of his tests and tell how he adjusted his guesses.</i></p>
<p>Megan age 11 Interpretation Practitioner</p>	<p>The total number of grapes Angela ate on Monday was 8 grapes.</p> <p>First I substituted the number of grapes angela ate on monday with x. Then for tuesday i put x+6,wednesday x+12 because you are adding 6 tuesday then 12 wednesday then thursday you have to add x+18 because since the number x is representing isn't changing yet because we don't know what x represents you just add six to the number you added before. Froday you would add x and 24. Then you have to add the numbers up. 6+12+18+24=60. To find out how many grapes angela ate monday you have to do 60+(5*_)=100. To find what blank equals which is the number of grapes angela ate on monday you have to divide 40 by 5 because if you subtract 60 from 100 you get 40. The answer to the division problem is 8, the number of grapes angela ate on monday.</p>	<p><i>I doubt that Megan has been introduced to formal algebra, but she exhibits good algebraic thinking. It's always a good idea to verify algebraic solutions by testing the numbers in the original problem. A list of the grapes eaten each day would do that.</i></p>
<p>Paul age 11 Interpretation Expert</p>	<p>Angela ate 8 grapes on Monday.</p> <p>She ate 8 grapes on Monday. She ate 8 because each day after Monday she ate 6 more grapes than the day before. So on Tuesday she ate 14 grapes (8+6=14). On Wednesday she ate 6 more grapes than she did on Tuesday so she ate 20 grapes because 14+6=20. So far she's eaten 42 grapes. On Thursday she ate 6 more grapes than she ate on Wednesday so she ate 26 grapes because 20+6=26. So far she's eaten 68 grapes. On Friday she ate 6 more grapes than Thursday so she ate 32 grapes because on Thursday she ate 26 grapes and 26+6=32. Since Friday was the last day she has eaten 100 grapes because if you add up all the numbers (8,14,20,26,32) you get 100.</p> <p>She would eat her 300th grape on the 10th day because in 5 more days she'll of eaten 350 grapes which is more than 300. She would eat her 300th grape on the 10th day because if she keeps eating 6 more grapes each day on the 6th day she'll eat 38 grapes, the next day 44 grapes, the day after that 50 grapes, then 56 grapes, and finally on the 10th day she'll eat 62 grapes. If you add it all up you'll get 350. It has to be 350 and not 300 because if you subtract one day, the last day which is 62 grapes, she'll only of eaten 288 grapes which is less than 300.</p>	<p><i>Paul does a fine job of verifying his answer and convincing us that he is correct. Since he fails to explain how he found his answer, his solution is not complete, but he does show good understanding of the key math ideas of the problem. That, along with his demonstrated grasp of the Extra earns him Expert in Interpretation. I'd ask how he arrived at 8 in the first place.</i></p>