Assessing Problem Solving and Writing With Constructed Response Problems

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Reflecting on Online Student Work

1. Select the student work. Usually plan on 1 piece (or set of pieces/drafts) for a 60-minute session. Pick a piece of work that represents a persistent or critical issue involving either a math concept or a student's approach to learning. Naturally the piece(s) selected should have enough writing to support reflection on the student's thinking.

2. Do the problem/assignment that the students did. Solve the problem with as many different strategies as you can (e.g. with and without algebra, geometric reasoning, using data and probability tools, etc.).

3. Identify the main concepts and strategies involved, differentiating between those that have to be involved and those that are optional.

4. Review student work and try to understand what they did and were thinking. Work in groups of 3-5 people. Select a recorder to keep notes of what is said. Go around in a circle and each person shares one thing that they notice about the student work. Try to keep moving so that all ideas and voices are heard and keep discussion and judgment to a minimum for now. Keep going until there are no more noticings or time requires moving on. Example of types of noticing:

    - The way the work is organized
    - The understanding of the question or problem
    - The techniques and problem-solving strategies used: from random guessing to development of mathematical models.
    - Accuracy in calculations.
    - Which parts of their solution process do they communicate/make visible and which not?
    - What connections do they make to prior knowledge and experience?
    - ….

    Be careful not to impose your expectations on the students' work and not to judge it. Look closely at all aspects of a student's work and be prepared to have your view expanded or to be left with questions. If you find yourself making claims about what is going on in the student's head, turn those into questions or wonderings. Instead of "James doesn't know

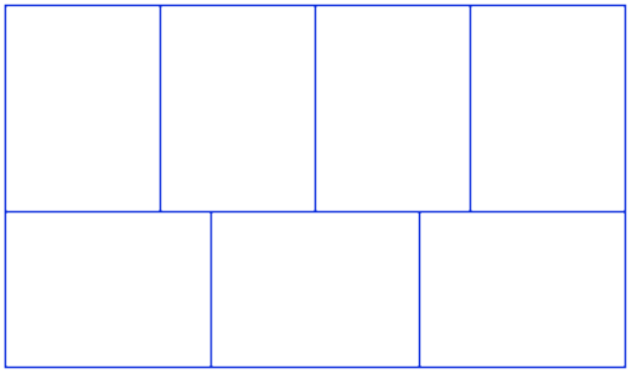
other factors", try "James did not write down any other factors. I wonder if James knows the other factors? I wonder if James thought about finding other factors."

5. Design a question or a task that would help you understand more about what the student is doing and thinking. If the student appears to have made mistakes, do not try to teach the correct approach yet. Focusing on eliciting more information from the student. Use your wonderings from step 4 as good areas for investigation. Share your task and have others describe how they think a student might approach such a task.

6. Work with a partner to identify one or two key aspects of the student work to respond to through email with the main goal of getting the student to come back and write some more—ideally revising some of their solution. Focus on connecting to the student's thinking, concretely honoring it, and ask a question or pose a consideration or suggest a way to organize their process that helps them think about that key aspect. Then work individually to draft an email response.

Congruent Rectangles

These seven congruent rectangles form a larger rectangle.



Question: If the area of the larger rectangle is 756 units2, what is its perimeter?

James

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The perimeter is 166 units.

I knew that area was length times width. So I knew that 27 times 28
gave me 756. So i added 28 plus 28 plus 27 plus 27 to get my answer
of 166.
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Erika

The perimeter would be 114 units.

I am trying to find the solution to the perimeter of the figure shown. Since I knew the rectangle was divided into 7 congruent rectangles, I took the total of the area, 756 un^2, and divided it by 7 and got that each smaller rectangle had an area of 108 un^2. After I did that, I saw that the 4 smaller sides had to equal the 3 bigger sides becasue they were opposite sides and they had to be equal becasue it was a rectangle. I also saw that the smaller number and the larger number multiplied together had to equal 108 un^2(which would be the area). Then I made the equation 4S=3B and that S*B=108 (S being the smaller side and B being the bigger side). I then used guess and check and plugged in various numbers. I started by plugging in 3 for S and 6 in for B and got 12=18, which was wrong. Then 6 in for S and 9 in for B and got 24=27, which was also worng. Then I plugged in 9 for S and 12 for B and gor 36=36, which was correct. So I found that 9 was equal to S and 12 was equal to B. Then, looking back at the rectangle, there are 5 larger sides(B) on the perimeter and 6 smaller sides(S) on the perimeter. So (5*12)+(6*9) would equal the perimeter, which is 114 units.

Dre

The perimeter of the rectangle is 111.2 units^2.

First I tried to solve it algebraicly. I used xy=756 but there were too many answers so I tried finding the area of one of the smaller squares. I did this by dividing the area of the big square by seven because there are seven little triangles in the big one. 756/7 = 108 but that got me no where.

Then I saw the answer. It takes 3 of the rectangles layed across the long way to go from one side to the other the long way but it takes 4 rectangles stacked up the tall way to get from one side to the other the long way.

Hmmm I thought I now have 2 equations and 2 unknowns. They are 3/4x = y and xy = 756.

All I need to do now is solve. I plugged in 3/4x into y in the seccond equation. I now have x(3/4x) = 756. I then multiplied x*3/4x and got 3/4x^2 = 756. Then I divided 756 by 3/4 and got x^2 = 1008. Next I took the square root of 1008 and got x = aprrox. 31.8. To solve for y you must multiply by 3/4 because 3/4x = y. Therefore y = approx 23.8.

There are 2 x's and 2 y's for both sides of the rectangle. When you add those up you get 31.8*2+23.8*2 = 111.2. The perimeter of the rectangle is 111.2 units^2.

The Pre-Algebra Problem of the Week Scoring Rubric for Students

For each category, choose the level that *best describes* your work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	I do not understand much of the problem.	I understand some of the math in the problem. I attempted part of the problem.	I understand all of the math in the problem. I attempted all parts of the main problem.	I understand the **Extra** question and solved it correctly. I am at least a Practitioner in Strategy.
Strategy	I didn't know how to set up the problem. My strategy didn't work.	I tried a strategy that makes sense, but it isn't enough to solve the whole problem. My strategy relied on luck.	I picked a sound strategy. I solved the problem through skill, not luck.	I used two separate strategies *or* I used an unusual or sophisticated strategy.
Accuracy	My work contains many errors.	Most of my work is accurate. I may have a couple of errors. I didn't use correct units.	My work is accurate and contains no arithmetic mistakes. I used appropriate units.	[not possible for most problems]
Communication				
Completeness	I wrote very little to explain how I solved the problem.	I included an explanation but none of my calculations. *or* I included calculations without any explanation. I didn't explain why I did several steps.	I explained almost all of the steps taken to solve taken to solve the problem. I explained why I used my equations, expressions, and calculations.	I included some extra ideas or explanation about some of the concepts in the problem.
Clarity	My explanation is very difficult to read and follow.	My explanation isn't entirely unclear, but another student wouldn't be able to follow it easily. My explanation is long and is written in a one paragraph. My spelling and typing errors make my explanation hard to understand.	I explained all of the steps in such a way that another student could understand. I made an effort to check my grammar, formatting, spelling, and typing.	My answer is very readable and it looks good! My organization makes my ideas especially clear. Optional: I included a diagram to help a reader understand.
Reflection	*These items are reflective. I can put them in my solution or in the comment I leave after viewing the answer provided by the Math Forum.*	I showed how I checked my own answer. I explained why I think my answer is reasonable. I summarized the process I used.	I connected the problem to another problem or experience. I explained where I'm stuck. I suggested a hint that I would give to another student.	I explained why I think the problem is easy or difficult. **I *revised* and improved my work.**
	I did nothing reflective.	I did one reflective thing.	I did two reflective things.	I did three or more reflective things or I did an exceptional job with two of them.