



Assessment that Guides Instruction Using Writing with Problem Solving

see more at <http://mathforum.org/workshops/nctm2007/>

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Room 302, Georgia World Congress Center

Level: 6 - 12

Session type: Extended

- Introduction
- Do Math: Hagrid's Height
- Identify Strategies
- Do Careful Review of Student Work
- Plan Intervention and Feedback
- Review a Group of Student Work Against Reading, Problem Solving, and Writing Strategies and Criteria
- Design a Classroom-Level Program/Response

Niydeira, age 14

Answer:

Hagrid is 2.75m and Baby Norbert is 1.80m because $2.75 + 1.80 = 4.55$. I got Baby Norbert's height by subtracting 2.75 from 4.55.

Explanation:

You need to know that Hagrid is the tallest. You also need to know that when subtract their height you will get 2.75 and when you add their height you will get 4.55. TO solve this problem I sbtracted 2.75 from 4.55.

Linsey, age 13

Answer:

Norbert's height is 1m and Hagrid's height is 2.55m.

Explanation:

first I subtracted the difference in height from the total height.

$$4.55\text{m} - 2.75\text{m} = 1.8\text{m}$$

then I needed to make it so their heights have 2.75m difference. I did this by trial and error. first i subtracted 1.00 from the answer 1.8m and added it to 2.75m $1\text{m} + 2.75\text{m} = 3.75\text{m}$

then i subtracted 0.8m(Norbert's height) from 3.75m to see if the difference is 2.75m

$$3.75\text{m} - 0.8\text{m} = 2.95\text{m}$$

It was wrong so i subtracted 0.8m from 1.8m and then added 0.8m to 2.75m and got 3.55m. then i subtracted 1m from the total height and got 1m.

Inez, age 13

Answer: prob

I think hagrid height is 2.75 and the dragon height is 1.80

Explanation:

The reason why i think is that is because i tried 2.32 and add 2.23 and it gave me 4.55 and took away 2.23 and it gave me 2.32 then i tried another solution but that didn't work so i sat there and think and think then i finally came up with a solution and i took 2.75 and add 1.80 and it gave 4.55 and took away 1.80 and it gave me 2.75 that's how i got my answer

Fellicia, age 14

Answer: reading

Hagrid is 2.35m. And Norbert is 1.85m

Explanation:

THE process of this problem is Hagrid is half of 455. And half of 455 is 227.5. And Norbert is 1.85.

Christian, age 13

Answer: writing

hagrid is 3.65 and norbert is .9

Explanation:

First step I thought that it was one fourth of norbert and three fourth of hagrid so I subtract $4.55 - 2.75$ and it equal 1.80 so I thought that if there's one fourth then it have to be 4 numbers so I divided 1.80 by 4 and you have .45. So I made a equation $2n + 2.75 = 4.55$ so now you have to figure what's 2n but since you already know that all of it put together equal 4.55 you subtract now to get 2n. So it equal 1.80 after this you divide this by two because there's 2n so it is .9. But now you add .9 to hagrid cause hagrid is three fourth so now you add and get 3.65 now .9 is left and it have to be norbert height and he is one fourth too.

Wilson, age 14

Answer: reading

Hagrid heights is 2.75m and the baby norbert is 1.80m.

Explanation:

I know that the height of hagrid and the baby together is 4.75. I also knew that if you subtract the baby norbert heights you get 2.75 which is hagrid height.

Isaiah, age 13

Answer: writing

the height of h is 3.65, and the height of n is 0.9

Explanation:

First I added 4.55 and 2.75, which gave me 7.30. Then I divided it by 2, giving me 3.65.

After that I subtracted that by 2.75 and gave me 0.9. Thus, I can conclude that $H=3.65$ and $N=0.9$

Patricia, age 14

Answer: reading/prob

My answer is 180 and 0.95 meters

Explanation:

First I did 455 take away 275 and I got 180 then I did 275 take away 180 so hagrid is 180 meters and baby norbert is 0.95 meters

Colin, age 13

Answer: prob

Hagrid is 3.75 m tall and baby norbert is 0.8 m tall .

Explanation:

I solved this question by adding many numbers together knowing that one number is 2.75m taller than the other.

Developing Students' PoW Participation

Reading Comprehension

1. **PoW I.Q.** – Make a list and/or drawing of the key **Information** you will use from the problem and the **Question** you have to answer. Compare and discuss which information is essential and which is not.
2. **Wonder-Like** - A student reads the problem out loud and then go around the group sharing one thing you like/understand and one thing you wonder about. Record the “wonder abouts” and after you have finished going around, see which “wonder abouts” are still active. Brainstorm strategies for figuring out answers to those. Each student goes away with responsibility for figuring out the answer to [2?] key “wonder abouts” for him or her. (At some point brainstorming strategies will no longer be necessary except for the occasional new situation.)
3. **MASV** (pronounced “massive”. Make a simpler version.) – Write a problem like this one that is easier. Swap problems with a buddy. Compare and discuss if your problems are the same, just with different numbers, or do they reflect different understandings of the original problem. Resolve differences.











Problem-solving

1. **Do It Wrong** – Give an example of the situation using numbers. Don't worry about whether the numbers work out right completely; just point out what should happen. (e.g. Hagrid is 3 ft. tall, Norbert is 1.55 ft tall. $3+1.55=4.55$, $3-1.55=1.45$ but should equal 2.75.).
2. **Strategy RoundUp**: As a group, brainstorm possible strategies that could be used with this problem. Over the course of the year, shift this to having students identify strategies on their own and then have them list these and try to identify strategies that could have been used. Discuss the merits of different strategies, why some are getting used a lot, etc.
3. **Working Backwards**: Have students guess or give the numerical answer (i.e. There were 14 bananas.) and ask them to figure out how it works.

Writing

1. **First draft**:
 - a. **Summarize** what is going on in the situation and conclusions you can easily draw from the information given.
 - b. **What and Why?** *Show* what you have done so far and *explain* why you did that, why your steps make sense for this situation.
 - c. **Note** where you have questions or still need to figure something out.
2. **Feedback review**: Have a partner read the mentor feedback sentence by sentence. After each one, identify what is being affirmed, corrected, or requested. Make a note of the corrections and tasks for the revision.
3. **Over My Shoulder**: Type in your draft. Then explain what you wrote to a partner who is looking over your shoulder at what you wrote while you explain. The partner should point out information that you are adding while you talk, and you should type in these additions as you go.

Example of the Teacher's View of One Student's Work

Puzzle	Submit Date	Submission	Score
Visiting Friends	Oct 7 2005 7:44PM	 answer  Karen	
Cutting Corners	Nov 7 2005 1:33AM	 answer  ans ck 1  revision  Karen	
Finding FAB *	Nov 14 2005 10:10PM	 answer  ans ck 1  revision  Karen	
Picking an Area Formula	Nov 28 2005 8:49PM	 answer  ans ck 1  Karen	
Mirror, Mirror	Dec 14 2005 12:11AM	 answer  ans ck 1  revision  Karen	
Building Bookshelves	Jan 3 2006 4:00PM	 answer  ans ck 1  revision  Karen	
A Minor Problem	Jan 18 2006 11:07PM	 answer  ans ck 1  Karen	
Filling the Hot Tub	Feb 16 2006 8:52PM	 answer  ans ck 1  revision  Karen	
Dendrochronology	Feb 27 2006 11:50PM	 answer  ans ck 1  Karen	
Exploding Shapes	Mar 21 2006 11:38PM	 answer	
Building a Regular Hexagon	Apr 6 2006 9:30PM	 answer  ans ck 1	
Broken Pottery	Apr 17 2006 12:34AM	 answer  ans ck 1	

How Submissions are "Scored"

We look for good problem solving and strong mathematical communication when reading submissions to our Problems of the Week. Solutions should include enough information to help another student understand the steps taken to solve the problem and the decisions made in the process.

Submissions are scored using the following categories:

Problem Solving

- Interpretation: Interpret the problem correctly and attempt to solve all of the parts.
- Strategy: Pick a good strategy and apply it well, achieve success through skill instead of luck.
- Accuracy: Get the calculations and details correct, including writing correct statements and equations.

Communication

- Completeness: Explain all the steps taken to solve the problem.
- Clarity: Explain the steps in such a way that a fellow student would understand, and make an effort to check formatting, vocabulary, and spelling.
- Reflection: Check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.

Submissions are scored using four levels of performance:

- Novice: Just starting out
- Apprentice: On the right track, but not quite there
- Practitioner: Got it
- Expert: Wow! Above expectations in some way

Scoring Grids

General versions of the grid are available for each of our services, which can be accessed from the main page of each service. The Pre-Algebra rubric is included on the next page of this document. □

Math Forum mentors, volunteers, and Teacher Members who choose to mentor their own students' solutions use problem-specific scoring guidelines to help them apply the rubric as consistently as possible.

A grid like the one shown on the right is used by mentors when viewing an individual submission. The "scores" are then automatically pasted into the reply to the student (and can easily be removed if desired) and are available to teachers when managing student work.

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation	☺	☺	☺	☺
strategy	☺	☺	☺	☺
accuracy	☺	☺	☺	☺
Communication				
completeness	☺	☺	☺	☺
clarity	☺	☺	☺	☺
reflection	☺	☺	☺	☺

The Pre-Algebra Problem of the Week Scoring Rubric for Students

For each category, choose the level that *best describes* your work

		Apprentice		Practitioner		Expert	
Problem Solving							
Interpretation		I do not understand much of the problem.	I understand some of the math in the problem. I attempted part of the problem.	I understand all of the math in the problem. I attempted all parts of the main problem.	I understand the Extra question and solved it correctly. I am at least a Practitioner in Strategy.		
Strategy		I didn't know how to set up the problem. My strategy didn't work.	I tried a strategy that makes sense, but it isn't enough to solve the whole problem. My strategy relied on luck.	I picked a sound strategy. I solved the problem through skill, not luck.	I used two separate strategies or I used an unusual or sophisticated strategy.		
Accuracy		My work contains many errors.	Most of my work is accurate. I may have a couple of errors. I didn't use correct units.	My work is accurate and contains no arithmetic mistakes. I used appropriate units.	[not possible for most problems]		
Communication							
Completeness		I wrote very little to explain how I solved the problem.	I included an explanation but none of my calculations. or I included calculations without any explanation. I didn't explain why I did several steps.	I explained almost all of the steps taken to solve the problem. I explained why I used my equations, expressions, and calculations.	I included some extra ideas or explanation about some of the concepts in the problem.		
Clarity		My explanation is very difficult to read and follow.	My explanation isn't entirely unclear, but another student wouldn't be able to follow it easily. My explanation is long and is written in a one paragraph. My spelling and typing errors make my explanation hard to understand.	I explained all of the steps in such a way that another student could understand. I made an effort to check my grammar, formatting, spelling, and typing.	My answer is very readable and it looks good! My organization makes my ideas especially clear. Optional: I included a diagram to help a reader understand.		
Reflection		<i>These items are reflective. I can put them in my solution or in the comment I leave after viewing the answer provided by the Math Forum.</i>	I showed how I checked my own answer. I explained why I think my answer is reasonable. I summarized the process I used.	I connected the problem to another problem or experience. I explained where I'm stuck. I suggested a hint that I would give to another student.	I explained why I think the problem is easy or difficult. I revised and improved my work.		
		I did nothing reflective.	I did one reflective thing.	I did two reflective things.	I did three or more reflective things or I did an exceptional job with two of them.		

Reflecting on Student Work *The Math Forum @ Drexel*

1. Select the student work. Video clips of classroom work can be used for very young students. Usually plan on 1 piece (or set of pieces/drafts) for a 60-minute session. Pick a piece of work that represents a persistent or critical issue involving either a math concept or a student's approach to learning. Naturally the piece(s) selected should have enough material to support reflection on the student's thinking.
2. Do the problem/assignment that the students did. Solve the problem with as many different strategies as you can (e.g. estimation, with and without algebra, geometric reasoning, using data and probability tools, etc.).
3. Identify the main concepts and strategies involved, differentiating between those that have to be involved and those that are optional.
4. Use the following process to discuss what you see in the student work in order to understand what they did and were thinking.

Work in groups of 3-5 people. Select a recorder to keep notes of what is said. Go around in a circle and each person shares one thing that they notice about the student work. Try to keep moving so that all ideas and voices are heard and keep discussion and judgment to a minimum for now. Keep going until there are no more noticings or time requires moving on. Example of types of noticing:

- The way the work is organized
- The understanding of the question or problem
- The techniques and problem-solving strategies used: from random guessing to development of mathematical models.
- Accuracy in calculations.
- Which parts of their solution process do they communicate/make visible and which not?
- What connections do they make to prior knowledge and experience?
-

Be careful not to impose your expectations on the students' work and not to judge it. Look closely at all aspects of a student's work and be prepared to have your view expanded or to be left with questions. If you find yourself making claims about what is going on in the student's head, turn those into questions or wonderings. Instead of "James doesn't know other factors", try "James did not write down any other factors. I wonder if James knows the other factors? I wonder if James thought about finding other factors."

5. Design a question or a task for the student that would help you understand more about what the student is doing and thinking. If the student appears to have made mistakes, do not try to teach the correct approach yet. Focusing on eliciting more information from the student. Use your wonderings from step 4 as good areas for investigation. Share your task with your colleagues in your group and have others describe how they think a student might approach such a task.

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Problem Solving Strategies

How do you solve problems? Lots of different ways! This list of strategies provides some ideas for different methods that can be used, presented in no particular order.

- Check your understanding/Look up new vocabulary
- Calculate as you go; don't always worry about how to solve the whole problem
- Estimate
- Act it out
- Systematic guessing and checking
- Solve a simpler problem
- Divide into cases (solve it for specific situations)
- Consider extreme cases
- Draw a picture/diagram
- Make a list/table/chart
- Label, assign variables, formulate expressions and equations
- Look for a pattern
- Work backwards
- Use logical reasoning (e.g. show the opposite can't be true)
- Make a model
- Take advantage of symmetry
- Set up an equation
- Use odd/even to analyze the situation (parity)

General approaches:

- Share your work with others; compare approaches
- Ask questions
- Write out what you know and what you don't know
- Identify your assumptions
- Take a break and work on something else. Come back to it later

<http://mathforum.org/pow/support/strategies.html>

Scaffolding for the Math Writing (and talking) Process

The Math Forum @ Drexel

Writing to figure out a solution path:

What is this problem about? What does the information in the problem mean?
What do you notice? What questions do you have?

Writing to explain a solution:

What did you do? Why did you do it?

What did you figure out?

Why does it work/not work?

What does your result mean?

Learning to ask good questions when stuck: Here are some models to follow throughout the talking/writing process:

Seeking specific information

e.g. "What is the meaning of _____?"

A problem-solving strategy to try

e.g. "I've tried making a diagram and estimating, but I think I need another approach. Do you have a suggestion?"

Review of a particular section of your work

e.g. "I think I might be making a mistake here because I am confident of everything before that. What can I do that would show me where I am going wrong?"

Is this approach ok?

e.g. "I did it differently but got the same answer, is that ok?"

Why does this work?

e.g. "I know how to do this but I can't explain why it works. How do I figure that out?"

Reflection that improves and extends solutions

- How did you check your answer? Did you use a different approach or line of reasoning from your main solution path?
- Does the solution make sense?
- Are there new problems that this one makes you think of?
- What other types of problems or situations does this remind you of?
- What parts of the problem solving process were the most satisfying or interesting? Least satisfying?

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- What did you learn about problem-solving or about the math that you would do differently the next time?

Typical learning and writing stages, somewhat developmentally organized:

- Completeness: both in terms of doing all tasks in the problem and in terms of reporting all the relevant work.
- Explaining the thinking behind the results and calculations.
- Giving good mathematical reasons.
- Convincing explanation for knowing why a solution or an approach is correct (proof).
- Imagining the implications, connections, and questions prompted by a given problem and its solutions.

Responding to reluctance to work on writing out solutions: “It’s exhausting”

- **Unique and critical benefits:** provides perspective and connects students to their own thinking, leads to self-correction and the development of ideas, makes it possible for others to engage and appreciate, etc.
- **Math is a language:** Symbols and diagrams can communicate a lot. Don’t focus only on words.
- This is meant to be only **one element of math instruction**, not a recommendation to focus on this exclusively.
- There are times when it makes sense to **differentiate instruction**. For some students, the writing task is too big and you can focus on one aspect, such as organizing the presentation of their work. For other students, the task is mastered more easily and detailed explanation becomes annoying. You can back off of the level of completeness required but make it clear that there are times when it’s helpful and expect them to begin to recognize that (when stuck, or when the audience will need it, or when looking for new insights and clarity).
- It can be **more tiring when the work is done on paper and has to be transferred**, rather than done electronically in the first place, although even this has its benefits, particularly when trying to craft something with care.