

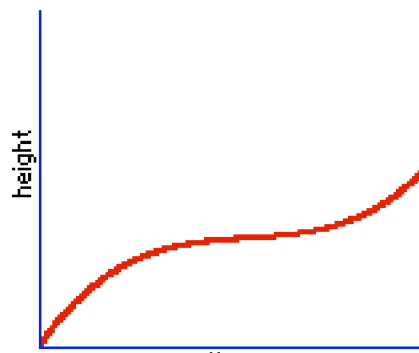


Using Constructed Response Problems to Enhance Assessment and Increase Students' Connections to Mathematics

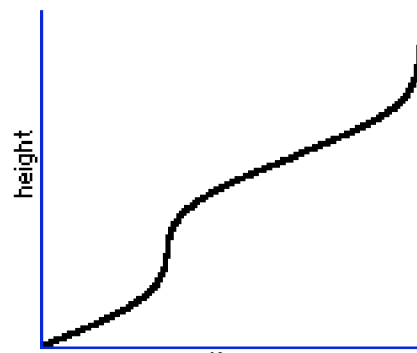
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Monday, April 7, 2008, 2:15 - 3:15
Grades 6 - 12

Learn more about all of the Math Forum's talks at NCSM and NCTM by visiting
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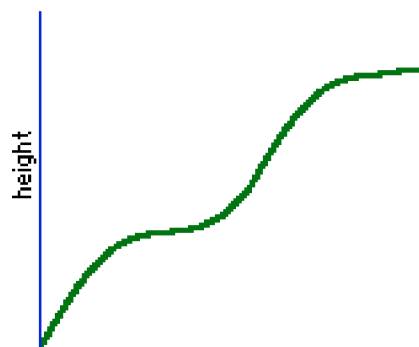
Filling Glasses



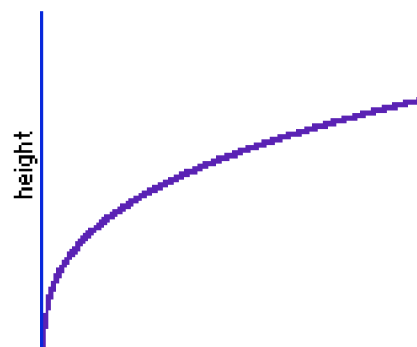
Graph 1



Graph 2



Graph 3



Graph 4

Filling Glasses for Student Solutions, part 1

Catecia, age 13

Answer:

glass c goes with graph 3

Explanation:

I think that glass c goes with graph 3 because the water in graph 3 has a lot of water, but not as much as graph 2. glass c is tall it is wavy like graph 3.

I think that glass b goes with graph 4 because glass b is skinny and tall but not as tall as glass c. graph 4 looks like it goes with glass b. I think that glass a goes with graph 1 because glass a is short and fat. graph 1 is short and it should go with glass a.

Kierra, age 14

Answer:

graph 1 goes with glass a/graph 3 goes with glass c/graph 4 goes with glass b

Explanation:

I think that glass b go with graph 4 because if you look at the glass, it's a little narrow at the top, and as you look at a glass b it got a little curve then it just go straight up. On graph 4 it shows that it is going up faster then what it had start's to go the same amount, but it's still increasing going to the right.

Glass a goes with graph 1, because if you look at the glass, you can see that it is skinny then it goes up wider and wider each time. The graph is showing the same exact thing. You can also see that it is going to the right and going up, then it's staying that same size until it starts to increase more.

I thought glass c go with graph 3, because glass c have a little skinny curve then it go up, going to the right. first it increase then it stop, then it start to increase some more then as it get to the top it start to stay the same amount. so really it go up, down, up, down each time.

Kirin, age 9

Answer:

My answer was that Glass A matches up with Graph 1, Glass B matches up with Graph 4, and Glass C matches up with Graph 2.

Explanation:

We know that if you are putting water into a glass at a constant rate the wider the glass the slower it will fill up. For Glass A it would get higher pretty slowly. Then we would reach a spot where the the glass has the same with for a little bit. Now the glass raises higher very slowly. this is shown on Graph 1.

For Glass B it keeps on getting higher. this is shown on Graph 4.

for Glass C it gets higher very slowly and then since the with is smaller it get higher faster. now it gets higher little by little. This is shown on Graph 2.

Filling Glasses for Student Solutions, part 2

Danielle, age 14

Answer:

graph 1 match up to glass a. graph 2 match up to glass c. and graph 3 match up to glass b.

Explanation:

I think that because the way they are shape and because of there size and the reason why i put glass b with graph 2 is because glass b is taller than a,c and a,c are the smaller glasses.

Ben, age 13

Answer:

the rate changes

Explanation:

the rate changes becuase each glass is different so they won't fill up the same time or start either becuase some of the glasses are lower than other

Gordon, age 14

Answer:

Glass A is described in Graph 1, Glass B is shown in graph 4, and Glass C is shown in graph 3.

Explanation:

The first step I took was to look at Glass A and its shape. I knew the glass had a shape that did not show much change, so I looked at a graph that would show water rising at a constant rate. Both graph 1 and 4 showed little change, so I looked at the top of Glass A, which had a slightly inward curve. This must be shown on the graph as an increase in height per unit of time. I looked at the two graphs that I had not eliminated and Graph 1 matched my description. Next, I looked at Glass B which clearly has a steady rise in water with no visible changes. I looked on the graphs, and graph 4 fit the description. Finally, I looked at Graph C, which had an outward curve, inward, then outward again. I looked on the graphs for a small, steady rise, then a higher rise in height, then smaller again. Graph 3 had that information.

Jeron, age 13

Answer:

graph 1 goes with glass a graph 3 goes with glass c graph 4 goes with glass b

Explanation:

Graph 1 goes with glass a because of the water rate starts fast then starts to get slower an slower from the glass getting wider an wider. Graph 4 goes with glass b because of the small stem it's small stem will mak the water rate go up faster an faster. Then it will get slower an slower from the glass getting wider an wider. Graph 3 goes with glass c because the stem it will start fast then slower then faster then slower from the glass curves

Samantha, age 14

Answer:

Glass A goes with graph one, glass B goes with graph four, and glass C goes with graph three.

Explanation:

To figure out these problems, I used height vs. time as the basis.

Glass A, Graph 1:

This glass would start to fill up fast for the first few seconds, but would slow down very soon after this because of the fact that the glass gets wider. It's small at the very bottom of the glass, but it gets very wide towards the middle. The glass starts to get a little bit smaller at the top, but not by much. I think graph one goes with glass A because at first, the water would start to fill up fast. The graph starts out with a steeper slope than the rest of the graph because of this reason. Towards the middle, the water would fill up slower because it's wider there and it would take more water to fill that part. Finally, towards the top of the glass, the water would fill up a little faster because the top is a tiny bit less wide than the middle. Since the water would fill up faster, the graph would start to develop more of a steeper slope.

Glass B, Graph 4:

This glass takes up little volume at the bottom, but gets only wider towards the top. At first, the water would start to fill up the glass fast because you wouldn't have to pour much water into the glass to occupy that space. Since the glass does not curve in at any point, it would make sense that the graph wouldn't either. As you fill up the glass with water, it would take longer because you would have to pour more water in to occupy the space. Since the water starts filling up fast in the beginning, the graph would start out with a steep slant. As the glass continues to fill, the graph would get less steep because the water wouldn't fill up as fast.

Glass C, Graph 3:

At first, this glass starts out filling fast because it's small at the bottom. After that, it starts to slow down a bit because the glass gets wider. The glass basically repeats this pattern again afterward.

I think graph 3 goes best with glass C because the graph repeats itself in the same way. At the smaller parts of the glass, the graph has a sharper rise. At the wider parts, the graph is not as steep as it is at the thinner parts.

How Submissions are "Scored"

We look for good problem solving and strong mathematical communication when reading submissions to our Problems of the Week. Solutions should include enough information to help another student understand the steps taken to solve the problem and the decisions made in the process.

Submissions are scored using the following categories:

Problem Solving

- Interpretation: Interpret the problem correctly and attempt to solve all of the parts.
- Strategy: Pick a good strategy and apply it well, achieve success through skill instead of luck.
- Accuracy: Get the calculations and details correct, including writing correct statements and equations.

Communication

- Completeness: Explain all the steps taken to solve the problem.
- Clarity: Explain the steps in such a way that a fellow student would understand, and make an effort to check formatting, vocabulary, and spelling.
- Reflection: Check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.

Submissions are scored using four levels of performance:

- Novice: Just starting out
- Apprentice: On the right track, but not quite there
- Practitioner: Got it
- Expert: Wow! Above expectations in some way

Scoring Grids

General versions of the grid are available for each of our services and can be accessed from the main page of each service. The Algebra rubric for this specific problem is included on the next page of this document. □

Math Forum mentors, volunteers, and Teacher Members who choose to mentor their own students' solutions use the problem-specific scoring guidelines to help them apply the rubric as consistently as possible.

A grid like the one shown on the right is used by mentors when viewing an individual submission. The "scores" are then automatically pasted into the reply to the student (and can easily be removed if desired) and are available to teachers when managing student work.

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation	🌟	🌟	🌟	🌟
strategy	🌟	🌟	🌟	🌟
accuracy	🌟	🌟	🌟	🌟
Communication				
completeness	🌟	🌟	🌟	🌟
clarity	🌟	🌟	🌟	🌟
reflection	🌟	🌟	🌟	🌟

Algebra Problem of the Week Scoring Rubric for *Filling Glasses*

For each category, choose the level that *best describes* the student's work

		Novice	Apprentice	Practitioner	Expert
Problem Solving					
Interpretation	doesn't seem to understand much of anything about the situation	compares the heights of the glasses with the heights of the graphs without considering volume compares the shapes of the graphs with the shapes of the glasses without considering the volume.	understands that each glass has a unique graph understands how to interpret the graphs in terms of height and time understands that the increase in the volume of the water is constant understands how the shape of a glass affects its volume understands how the speed of the rising of the water is related to the volume pays attention to the whole glass/graph, not just the beginning or the end	is at least a Practitioner in Strategy and has successfully answered the Extra, describing a glass that might generate the remaining graph	
Strategy	does not have any ideas about how to solve the problem	picks an invalid strategy, or relies on luck to get the right answer	might use some or all of the following sound mathematical ideas in trying to answer the question: <ul style="list-style-type: none"> correlates specific changes in the glasses to specific changes in the graphs connects slope to the speed of the height changing rules out candidates using comparison and logic divides the glasses and graphs into event segments 	does something that totally blows you away, such as talking about functional relations	
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	correctly describes what happens in the graph or the glass describes things with accurate and relevant math terms	[not normally available for this category]	
Communication					
Completeness	has written nothing that tells you how they found their answer	explains only one or two of their choices completely doesn't include enough information for another student to follow	explains each of the steps they took in sufficient detail that another student might be able to follow their work. In particular, they might mention: <ul style="list-style-type: none"> how to interpret the shape of the graph with regard to time and height why a narrower glass leads to a faster increase in water height sufficient details of the glasses and graphs why there can't be other matches 	adds in useful extensions and further explanation of some of the ideas involved	
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely lengthy warrants multiple paragraphs many of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing	
Reflection	<i>The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:</i> does nothing reflective	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer does one reflective thing	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used does two reflective things	comments on and explains the ease or difficulty of the problem revising their answer and improving anything does three or more reflective things or an great job with two	

Reflecting on Student Work

The Math Forum @ Drexel

1. Select the student work. Video clips of classroom work can be used for very young students. Usually plan on 1 piece (or set of pieces/drafts) for a 60-minute session. Pick a piece of work that represents a persistent or critical issue involving either a math concept or a student's approach to learning. Naturally the piece(s) selected should have enough material to support reflection on the student's thinking.
2. Do the problem/assignment that the students did. Solve the problem with as many different strategies as you can (e.g. estimation, with and without algebra, geometric reasoning, using data and probability tools, etc.).
3. Identify the main concepts and strategies involved, differentiating between those that have to be involved and those that are optional.
4. Use the following process to discuss what you see in the student work in order to understand what they did and were thinking.



















Work in groups of 3-5 people. Select a recorder to keep notes of what is said. Go around in a circle and each person shares one thing that they notice about the student work. Try to keep moving so that all ideas and voices are heard and keep discussion and judgment to a minimum for now. Keep going until there are no more noticings or time requires moving on. Example of types of noticing:

- The way the work is organized
- The understanding of the question or problem
- The techniques and problem-solving strategies used: from random guessing to development of mathematical models.
- Accuracy in calculations.
- Which parts of their solution process do they communicate/make visible and which not?
- What connections do they make to prior knowledge and experience?
-

Be careful not to impose your expectations on the students' work and not to judge it. Look closely at all aspects of a student's work and be prepared to have your view expanded or to be left with questions. If you find yourself making claims about what is going on in the student's head, turn those into questions or wonderings. Instead of "James doesn't know other factors", try "James did not write down any other factors. I wonder if James knows the other factors? I wonder if James thought about finding other factors."

5. Design a question or a task for the student that would help you understand more about what the student is doing and thinking. If the student appears to have made mistakes, do not try to teach the correct approach yet. Focusing on eliciting more information from the student. Use your wonderings from step 4 as good areas for investigation. Share your task with your colleagues in your group and have others describe how they think a student might approach such a task.

Example of the Teacher's View of One Student's Work

Puzzle	Submit Date	Submission	Score
Visiting Friends	Oct 7 2005 7:44PM	 answer  Karen	
Cutting Corners	Nov 7 2005 1:33AM	 answer  ans ck 1  revision  Karen	
Finding FAB *	Nov 14 2005 10:10PM	 answer  ans ck 1  revision  Karen	
Picking an Area Formula	Nov 28 2005 8:49PM	 answer  ans ck 1  Karen	
Mirror, Mirror	Dec 14 2005 12:11AM	 answer  ans ck 1  revision  Karen	
Building Bookshelves	Jan 3 2006 4:00PM	 answer  ans ck 1  revision  Karen	
A Minor Problem	Jan 18 2006 11:07PM	 answer  ans ck 1  Karen	
Filling the Hot Tub	Feb 16 2006 8:52PM	 answer  ans ck 1  revision  Karen	
Dendrochronology	Feb 27 2006 11:50PM	 answer  ans ck 1  Karen	
Exploding Shapes	Mar 21 2006 11:38PM	 answer	
Building a Regular Hexagon	Apr 6 2006 9:30PM	 answer  ans ck 1	
Broken Pottery	Apr 17 2006 12:34AM	 answer  ans ck 1	