Geometry PoW Packet

Boxing Up Harry's Broom

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Welcome!

We hope you've been keeping up with the packets we've been writing and that they've been useful to you. We'd love it if you'd post to the geopow-teachers discussion board and share how you've been using the packets. What's the most useful part to you?

You can access the discussion group via the link to "PoW Members" in your Teacher Office, or use this URL to go to geopow-teachers directly: http://mathforum.org/kb/forum.jspa?forumID=529 [Login to the discussions using your PoW username/password.]

This packet contains a new problem, the "answer check," our solution and scoring rubric, a note about possible common mistakes we may see, and ideas for implementing the problem in the classroom.

The Problem
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Boxing Up Harry's Broom is a new problem this year. While it is in some ways a fairly straightforward application of the Pythagorean theorem, understanding what the question is asking will be the first hurdle.

If students aren't sure where to start, suggest that they draw a picture. It's possible that you haven't covered the Pythagorean theorem yet in Geometry, though students saw it in middle school. If this is the case, you might start by giving them the problem without the question and ask them to tell you some things about problem. Getting them to draw and talk about the fact that the box has to be wide enough and that the broom should be the diagonal is a great starting point.

For each problem, we will pick one category from the scoring rubric (see below) on which we'll focus. For *Boxing Up Harry's Broom*, we're choosing "Completeness", which generally means whether or not the student has explained each of the steps they've done and why they did it. In this case, it would mean explaining how they know there is a right triangle, why they used the measurements they did for which sides of the right triangle, and why they can use the Pythagorean theorem.

Answer Check
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After students submit their solution, they can choose to "check" their answer by looking at the answer that we provide. Along with the answer itself (which never explains *how* to actually get the answer, and in this case only gives one of two possible answers), we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Our Solution
page 2

We've not made "our solution" available to anyone but mentors in the past, feeling that the student solutions are much better examples of how kids actually solve the problem! But sometimes having the solution ahead of time can be helpful, and we often include tips for mentors about how to support students in different areas of their work or in thinking about different parts of the problem. When appropriate, we also include multiple ways to solve the problem (which isn't the case with this problem).

Scoring Rubric
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The problem-specific rubric is something we write for every problem for use by those who are assessing student work. It spells out what we expect from students in three areas of problem solving and three of communication. The goal is to look at each category separately when evaluating the student work. This way the assessment process provides more focused information regarding the areas of strength and weakness in the student work. Completeness, especially, can affect other scores. For example, incomplete or unclear communication can lead to lower scores in strategy because it's harder to understand what the strategy is if it's not explained well.

Keep in mind also that a "Novice" score is not indicative of no work or a zero. It simply indicates that the student is at a beginning level in that category.

Common Mistakes right here!

Most students will find the right triangle. Some, however, will have trouble figuring out the situation from reading the problem. Encourage them to draw a picture and talk it over with their partner. Once they've figured it out in general, recommend that they draw the triangle by itself, without anything that resembles "the box". Others will see the triangle but then put the known lengths on the legs—they're used to knowing the legs and finding the hypotenuse.

Good luck!

We're excited about providing these new resources to you. We hope to get feedback and ideas from you on the geopow-teachers discussion group starting October 15, 2007.

– Annie

Problem

Boxing Up Harry's Broom

Harry needs to put his 4' long broom in a box. The first box he found was only 36 inches long, but he was able to fit his broom flat on the bottom of it. What's the smallest the width of the box could be?

Extra: Harry found another box that was 2' long by 3' wide, and while his broom didn't fit on the bottom, it did fit in the box. What's the shortest the height of the box could be?

Answer Check

The box has to be at least 2.65 feet wide.

If your answer **doesn't** match our answer,

- did you notice that the broom is the hypotenuse of a right triangle?
- don't worry about the height of the box until the Extra question.
- remember the Pythagorean theorem!

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- did you make and correct any mistakes along the way? If so, how did you find them?
- did you try the Extra question?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solution

The key concept for this problem is the Pythagorean theorem.

In the main question, Harry puts his broom in a box that is 36 inches long. The broom fits flat on the bottom of the box, and we have to figure out how wide the box needs to be so that the broom just barely fits.

The broom forms a right triangle with the length and width of the box. We know the hypotenuse as one leg. We can use the Pythagorean theorem to find the other leg.

$$4^2 = 3^2 + w^2$$
$$16 = 9 + w^2$$
$$7 = w^2$$
$$\sqrt{7} = w$$

The width is about 2.65 feet, or about 31.7 (or 32) inches.

Extra: This box has a length of 3' and a width of 2'. The broom won't fit in the bottom, so we have to find the shortest possible height of the box so that the broom fits.

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The broom will be on the diagonal of the box, and forms a right triangle with the height of the box and the diagonal of the bottom of the box. So the first thing I have to find is the diagonal of the bottom of the box.

The diagonal is the hypotenuse of a right triangle with legs of length 2' and 3'. We use the Pythagorean theorem again.

$$c^2 = 2^2 + 3^2$$
$$c^2 = 4 + 9$$
$$c^2 = 13$$
$$c = \sqrt{13}$$

I'll leave it in exact form for now. Now we can tackle the other right triangle. The hypotenuse of the triangle is the broom, which is 4' long. The legs are the height, h, and the bottom of the box, which is $\sqrt{13}$ feet.

$$4^2 = h^2 + (\sqrt{13})^2$$
$$16 = h^2 + 13$$
$$3 = h^2$$
$$\sqrt{3} = h$$

The height of the box is about 1.74 feet (rounded up, since the broom has to fit), or about 20.8 inches.

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Geometry Problem of the Week Scoring Rubric for *Boxing Up Harry's Broom*

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	does one or none of the things listed under Practitioner	does two or three of the things listed under Practitioner	understands that the broom fits in the bottom of the box, so the question is two-dimensional understands that the broom is the hypotenuse of a right triangle notices that there are two different units used attempts to find the smallest possible width of the box	is at least a Practitioner in Strategy and has answered the Extra question correctly
Strategy	has no ideas that will lead them toward a successful solution	has a strategy that somehow relies on luck	has a strategy that relies on sound reasoning, not luck	it's not clear what this would look like for this problem, as there aren't a lot of different ways to tackle the problem
Accuracy	has made many errors	has made several mistakes or misstatements, or has used vocabularly incorrectly uses incorrect units	makes no mistakes of consequence and uses largely correct vocabulary and notation uses correct units whenever they use units	[generally not possible]
Communication				
Completeness	has written almost nothing that tells you how they found their answer	shows work with no explanation, or gives an explanation and shows no work	shows and explains the steps that they take and why they are reasonable steps explains that they are using the Pythagorean theorem and why they know there is a right triangle explains which numbers they are putting where on the triangle and why	includes additional helpful information, doesn't just add more for the sake of adding more
Clarity	explanation lacks clarity and organization	explanation is difficult to follow length warrants separation into more paragraphs lots of spelling errors/typos	explains the steps that they *do* explain in such a way that another student would understand (needn't be complete to be clear) makes an effort to check their formatting, spelling, and typing (a few errors are okay)	answer is clearly written and well-organized formats things exceptionally clearly
Reflection	*The items in the columns to the right are considered reflective, and could be in the solution or their comment:*	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer comments that broom isn't really two-dimensional, or that we're not allowing for the bushy part at the bottom	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used	comments on and explains the ease or difficulty of the problem **revising their answer and improving anything counts as reflection**
	does nothing reflective	does one reflective thing	does two reflective things	does three or more reflective things or great job with two

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