



Math Fundamentals Problem Packet for Teachers: Wooden Legs

October 27, 2008

• <http://mathforum.org/funpow/>

Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, and a problem-specific scoring rubric. **Wooden Legs** has been chosen from the Problem Library (#3619), so I’m also including some sample student solutions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to “PoW Members” or use this URL to go to *funpow-teachers* directly:

<http://mathforum.org/kb/forum.jspa?forumID=526> [Log in using your PoW username/password.]

The Problem

Wooden Legs involves conceptual understanding of multiples. Solvers need to discover all the ways that Wendy can use exactly 31 legs to make 3-legged stools and 4-legged tables. A solver needs to demonstrate that she/he has found all possible solutions.

The Extra asks solvers to determine which of the three combinations would earn Wendy the most money if she sells stools for \$2 and tables for \$3.

A print-friendly version is available from the “Print this Problem” link on the problem page.

Wooden Legs

Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables.

She has a supply of 31 legs and wants to use them all to make stools and tables.

Find all the possible ways she can use all 31 legs.

Explain how you solved the problem and how you know you have found all solutions.

Extra: Wendy sells her furniture to the local toy store. She gets \$2 for each stool and \$3 for each table. Of all the ways you found, which would earn her the most money?

Be sure to explain how you know.



Answer Check

There are three ways Wendy can use up all 31 legs. One way uses just 1 table and some stools. Now can you find the rest?

If your answer doesn't agree with ours —

- did you try using toothpicks or triangles and squares to model the problem?
- did you make a list to help you keep track of the numbers you tested?
- did you try making a list of the multiples of 4? or of 3?

If you used guess-and-test, did you tell . . .

- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?

If your answer does agree with ours —

- have you clearly shown and explained the work you did, so that a fellow student would understand?
- did you try the Extra?
- did you show how you know you have found all the possible ways to use 31 legs?
- try to describe any special observations or patterns you noticed.

Our Solutions

Here are several examples of ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. In fact, we often receive solutions from students who have used approaches we've not anticipated. I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1 – Guess-and-check with toothpicks

I used 31 toothpicks to represent the wooden legs and tried to use all of them by putting them in groups of 3 and 4. First I made one group of 4 to represent a table, then found I could group all the rest, 27 toothpicks in groups of 3, so Wendy could have made 1 table and 9 stools.

$$4 + (9 * 3) = 4 + 27 = 31$$

Then I used 8 toothpicks to show 2 tables, but the 23 leftover toothpicks could not all be used to make groups of 3. I found that 3 tables didn't work either, because they used 12 legs, and the 19 leftover legs could not be grouped in 3s. I kept trying the multiples of 4 (12, 14, 20, 24, 28) and seeing if I could group the leftovers into 3s. Here are the ones that worked:

$$4 \text{ tables and 5 stools } (4 * 4) + (5 * 3) = 16 + 15 = 31$$

$$7 \text{ tables and 1 stool } (7 * 4) + 3 = 28 + 3 = 31$$

Strategy 2 – Triangles and Squares, with trading

I used Pattern Block triangles to represent stools and squares to represent tables. I tried to find the way that used the most tables first, so I counted by fours up to 28 (7 squares). I needed to use 3 more legs, so that was 1 triangle. I knew that 7 tables and 1 stool would use 31 legs.

Then I tried to see how to trade some tables for some stools that used the same number of legs. The first trade I found was 3 fewer tables for 4 more stools because that's 12 legs for each. That gave me a new combination: 4 tables and 5 stools.

Again I knew Wendy could build 3 fewer tables and 4 more stools, which gave me 1 table and 9 stools.

Strategy 3 - Multiples of 4, and make up the difference with 3s, in a table

I made a table listing the number of legs that would get used to make 1-7 tables (multiples of 4). Then I found how many legs would be left and found which ones could be grouped in 3s to make stools. There were 3 ways to do it: 1 table and 9 stools, 4 tables and 5 stools, 7 tables and 1 stool.

Tables	Legs	Leftover legs	Stools
1	4	27	9
2	8	23	
3	12	19	
4	16	15	5
5	20	11	
6	24	7	
7	28	3	1

I discovered that the number of stools has to be odd, because any number of tables will have an even number of legs. It takes an odd number to add to an even number to get an odd sum of 31.

Strategy 4 – Lists of multiples, with Extra

I listed all the multiples of 4 and 3 up to 31. I looked for pairs which added to 31. I realized that only odd multiples of 3 would work, so I crossed out the even multiples.

Multiples of 4	4	8	12	16	20	24	28			
Multiples of 3	3	6	9	12	15	18	21	24	27	30

The combinations that worked were

4 and 27, 1 table and 9 stools

16 and 15, 4 tables and 5 stools

28 and 3, 7 tables and 1 stool

Extra: For each combination above I multiplied the number of tables by \$3 and the number of stools by \$2. Wendy would get the most money from 7 tables and 1 stool.

Tables	Table \$	Stools	Stool \$	Total
1	\$3	9	\$18	\$21
4	\$12	5	\$10	\$22
7	\$21	1	\$2	\$23

Teaching Suggestions

Solving this problem involves two kinds of understanding:

1. The constraints of the problem – including the fact that Wendy wants to use all 31 of the legs, and that she'll construct 4-legged tables and 3-legged stools with them.
2. What constitutes a solution – in this case finding all three solutions and demonstrating why there are no more, whether directly stated or implied by the solution strategy.

Some students will use a guess-and-check strategy. Younger students will likely use a manipulative to test numbers. Using 31 items, e.g., toothpicks or cubes, to model the legs is a useful way to solve the problem. Triangles and squares, as in Pattern Blocks, could also be used. Some children will draw pictures or diagrams to model the story. Making a table to keep track of test results is an excellent strategy, since it leads the solver to see patterns and discover insights, especially if the student tests numbers in a systematic way.

Older students may take advantage of their knowledge of multiples to arrive at a solution more directly. Students who have learned how to write expressions with variables might bring that skill to the problem.

Wooden Legs offers an opportunity to make students more aware of the role of parity (odd vs even) in problem solving, as illustrated by strategies 3 and 4 in the Expected Solutions above.

In any case, encourage students to find a representation that makes sense to them. Sharing and discussing strategies after solving the problem may help children who are ready to move forward in their thinking to have an “Aha!” moment, which they can apply to future problems.

The Teacher Support Page for this problem contains links to related problems in the Problem Library and to other web-based resources: <http://mathforum.org/funpow/puzzles/supportpage.ehtml?puzzle=405>.

Scoring Rubric

On the last page is the **problem-specific rubric**, to help those who are assessing student solutions. It specifies what we expect from students in three areas of problem solving and three areas of communication. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

Sample Student Solutions

Focus on Strategy

I've chosen the samples below to illustrate a range of strategies demonstrated by submitters. This category of our rubric addresses whether the student has approached the problem with a sound method and in a systematic way, relying on skill and not luck. A guess-and-test strategy must involve good reasoning and informed guessing. Those who solve the problem more directly need to use valid processes based on well-grounded concepts. In this problem the chosen strategy must allow the child to be confident that she/he has found all possible solutions.

It is not my purpose here to give a definitive judgment, but rather to highlight the range and variety of work done by students and suggest ways I might encourage them to take next steps. The best way to elicit more writing from students is to ask them specific questions about the missing details.

You can view commentary and solutions that were highlighted in 2006 at this Library page: http://mathforum.org/librarypow/office/teachers/index.ehtml?puzzle=3619&page=lib_solution

Kira Age 10

Strategy Novice

All the ways that you could make are 1) Four tables and zero stools. 2) Seven tables and zero stools. 3) Zero tables and 12 stools. The answer for extra credit is #1.

I solved the problem by grouping. I first wrote down 31 lines. Then I grouped them together and added them up to the number of tables and stools. I then had to check to make sure that they were right. Like the first one I would have to do four times nine and get 36 now that's over so you have to change your answer to four. If you do the second one then it would be 3 times 9 and get twenty seven. There for you have to change your answer to 7.

Drawing 31 legs is a good first step. It's difficult to imagine how Kira grouped them, or why she checked them by multiplying by 9. This strategy clearly "didn't work." None of Kira's three solutions use 31 legs. I'd ask Kira to paraphrase the problem to check for understanding, and then to show drawings of her solutions to see if she can find her own errors.

Kenneth
age 9

Strategy
Apprentice

1: The ways are 7 four legged tables and 1 three legged stool, 9 three legged stools and 1 four legged tables, and 4 four legged tables and 5 three legged stools.

Extra: The most money she could make is with the 7 tables and 1 stool.

For 1 I played with the number 4 and the number 3 until I got my answers. For the extra I counted the different kinds of tables and stools she could have had and added up those amounts.

Kenneth's correct answer indicates that his strategy was sound, but he doesn't give evidence of what he tried and how he knows he's found them all. Guess-and-Test approaches must involve good reasoning and informed guessing. I'd use the prompts in the answer check to move him forward.

Ryan
age 11

Interpretation
Apprentice

There are three possible answers. 4 tables, 5 stools. 1 table, 9 stools. 7 tables, 1 stool.

I charted out the possible ways to make the chairs and tables. Then I tried making the right amount. I randomly put down four tables (16 pegs) then I did $31 - 16 = 15$ and that is divisible by three!

So five stools and four tables is one possible way. Then I saw another one! Nine stools and one table. I put it on the chart.

Then I found one stool and seven tables. I couldn't find anymore possibilities.

For the extra I found that the most money she could make was by selling 1 stool and seven tables for a total of \$23.

Ryan's solution is also correct. Looking for multiples of three is a sound approach, but he appears to depend too much on luck. By approaching the problem "randomly" and just trying to "see" combinations that work, he can't be sure he has found them all. I'd encourage him to make a table to organize what he's found so far, and use that to explore whether there can be other solutions.

Taylor
Age 9

Strategy
Apprentice

She can make 5 stools and 4 tables.

I first made 31 legs. Then I circled 3 than 4 than 3 than 4 ... Then I got 5 stools and 4 legs. Then I added $4 + 5 = 9$ So she can make 9 items.

Extra: $\$2.00 \times 5 = \10.00
 $\$3.00 \times 4 = \12.00
 $\$12.00 + \$10.00 = \$22.00$

but if she wants more money she would do

$\$3.00 \times 5 = \15.00
 $\$2.00 \times 4 = \8.00
 $\$15.00 + \$8.00 = \$23.00$

So if you multiply the more stools to the highest price you will get more money than doing the tables times the highest amount of money. She will also get more money if she buys more legs and sells twice the much and get twice as much money.

Taylor either missed, or didn't understand, that he was to "Find all the possible ways she can use all 31 legs." His Extra is more evidence that he didn't understand that. Taylor's strategy of alternating was an effective way to find one solution, but it "isn't enough to solve the whole problem." Since he seems to understand the key math ideas of the problem, asking him to read the problem aloud could allow him to realize the extra requirement.

Kate
age 9

Strategy
Practitioner

The different possibilities I found were 1.7 tables and 1 stool, 2.1 table and 9 stools, and 3.4 tables and 5 stools.

I went through all 3 and 4 times tables that were under 31 and took the number 4 or 3 was multiplied by and said that's how many tables I made or that's how many stools I made then took the answer to that problem and subtracted it from 31. If the answer I got was divisible by 3 or 4 I knew it could be a complete table or stool using all the legs without leftovers.

"all the 3 and 4 times tables" indicates a systematic search that led Kate to find all solutions. Her description of using subtraction and then testing for divisibility of the remainder is further evidence of her systematic mindset. Her explanation would be more complete if she showed some of the math she described. She might need help in creating a number model.

Emma
age 14

Strategy
Practitioner

The different ways that Wendy can build dollhouse furniture are:

$4+4+4+4+4+4+4+3=31$ * 7 tables, 1 stool
 $4+4+4+4+3+3+3+3=31$ * 4 tables, 5 stools
 $3+3+3+3+3+3+3+4=31$ * 1 tables, 8 stools

I got these answers by doing this;

I started by adding only fours. I saw that the closest I could get to 31 was 28 without going over. I figured, hey, lets take away a four, and put in a 3. From that I got 31. I

continued this process by doing this:

$4+4+4+4+4+4+3=31$, check
 $4+4+4+4+4+3+3=30$, nope
 $4+4+4+4+3+3+3=32$, nope
 $4+4+4+3+3+3+3=31$, check
 $4+4+3+3+3+3+3=30$, nope
 $4+3+3+3+3+3+3=29$,nope
 $4+3+3+3+3+3+3=31$, check
 $3+3+3+3+3+3+3+3=30$, nope.

These are all of the possible answers.

Extra

The scenario containing the most tables is the one with 7 chairs and one stool, making \$31!!

*Emma gradual method, starting with the maximum number of tables and decreasing one at a time is very systematic. I would ask her to check her 1-table solution for accuracy. That error could be avoided by an alternate equation, e.g., $(1 * 4) + (9 * 3) = 27$. She could improve her Strategy to Expert by explaining her Extra method and showing the math.*

Archit
age 11

Strategy
Expert

Answer: The possibilities are:

TABLES|STOOLS

1 | 9

4 | 5

7 | 1

To solve this, I used the following equation:

$$(31 - 4 * \text{amount of tables})/3 = \text{amount of chairs.}$$

If you fill in the value for amount of tables, a chart will look like this:

Tables|Stools

1|9

2|7 2/3

3|6 1/3

4|5

5|3 2/3

6|2 1/3

7|1

8|-1/3

As you cannot have a negative amount of stools, and you are trying to make only whole stools, the only possibilities are 1 table and 9 stools; 4 tables and 5 stools; and 7 tables and 1 stools.

Extra: If you try the values:

With seven tables and one stool she makes \$23.

With four tables and five stools she makes \$22.

With one table and nine stools she makes \$21.

So the highest money will be made by selling seven tables and one stool.

Archit's approach is very sophisticated for his age, as well as very algebraic. He generalized a formula, and then evaluated it for each number of tables, producing a corresponding number of chairs. He is ready to learn about spreadsheets! He exhibits good number sense to determine which of the combinations is a true solution. I'd ask to see the math he used to solve the Extra.

We hope these packets are useful in helping you make the most of FunPoWs. Please let me know if you have ideas for making them more useful.

~ Claire

claire@mathforum.org

The Math Fundamentals Problem of the Week Scoring Rubric — Wooden Legs (posted 27 October 2008)

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that each solution must use all 31 legs. Attempts to find all possible ways to use them. Each stool must have 3 legs; each table must have 4. Answers the main problem.	Solves the main problem correctly. Understands and solves the Extra. Achieves at least Practitioner in Strategy.
Strategy (NB: based on their interpretation of the problem)	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Strategy is not evident. Might verify a correct answer, but fail to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Guess-and-Check approach must involve good reasoning and informed guessing. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does one or more of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology or algebra.
Accuracy (NB: based on chosen strategy)	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness (NB: an incorrect solution can be complete)	Writes very little to explain how the answer was achieved.	Provides explanation but does not include calculations; OR Shows calculations without explanation about why they were done.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes key calculations with rationale. Shows how she/he knows all solutions have been found. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does one or more of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem. Includes a table of data.
Clarity (NB: incomplete and incorrect solutions can be explained clearly)	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Show critical calculations. Uses level-appropriate math language, including correct units: legs, tables, stools. Shows effort to use good formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful diagram or image. (A diagram or image alone doesn't qualify for Expert status.)
Reflection (See list below.)	Does nothing reflective.	Includes one reflective thing. • Revises and improves the submission. • Checks the answer using a different method. • Explains a hint she/he would give someone.	Includes two reflective things. • Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them.	Includes 3 or more reflective things or does an exceptional job with 2 of them. • Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used.
	<p>The items to the right are considered reflective, and could be in the solution OR in the comment they leave after viewing our answer:</p>			