The Impact of The Math Forum's *Problem(s) of the Week* on Students' Mathematical Thinking

K. Ann Renninger, Laura Farra, and Claire Feldman-Riordan
Swarthmore College, Swarthmore, PA 19081-1397

Introduction

The present study was undertaken to evaluate the impact of The Math Forum's *Problem(s) of the* Week (PoW; http://forum.swarthmore.edu/pow/) on students' mathematical thinking, where mathematical thinking refers to habits of thinking common to mathematicians. Among these habits are the use of strategies and reflection in the process of working to understand numerical relationships and symbolic forms. In its developed form it refers to, and includes, the use of strategies and reflection as part of the process of problem-solving (Schoenfeld, 1992). Development of students' abilities to think mathematically has been found to require both ongoing work with a variety of problem types, and instruction that is systematically organized to facilitate strategy use and reflection (Lester, Garofalo, & Kroll, 1989).

The PoWs are designed to provide users with individualized mentoring on non-routine word problems that will enhance their mathematical thinking. An extension of students' mathematics instruction, the PoWs are typically built into the existing curriculum, assigned as extra credit, or undertaken as independent mathematical enrichment. Each week, a new PoW is posted in each of the following six categories: an elementary, a middle school, a geometry, an algebra, a discrete math, and a calculus/trigonometry PoW. To receive credit for submitting to a PoW, students must submit not only answers but explanations of their solution strategies within the week that the problem is posted. Mentor teachers respond to each submission by acknowledging students' work, engaging the students in discussing their answers, and encouraging students to revise and resubmit an answer when work is incomplete or inaccurate. Following the posting of the problem and the period during which students submit and resubmit answers, a solution page that summarizes a variety of accurate solutions to the problem is posted. These problem and solution pages are then archived and made available to everyone.

Method

Subjects

A random sample of student submissions was drawn from the Math Forum archives of the Elementary PoW (ElemPoW), the Middle School PoW (MidPoW), and the Geometry PoW (GeoPoW) during the 1998-1999 school year. It consisted of 40 students (20 boys, 20 girls) for each service studied. Data from a control group of students are now being collected.

Procedure

Data collection. Students' work with PoWs was studied over a 10-month period that began with their initial work on the PoW. Because completion of the PoW is voluntary and/or because teachers sometimes assign it one week but not the next, the number of problems completed varied. Also, students did not necessarily work on the same problems. The students' first 3 problems were identified as their Time 1 problems and the last 3 completed were identified as their Time 2 problems.

Data reduction. In order to control for differences in the level of difficulty of the problems on which students worked, all problems were rated for problem difficulty. These ratings were used to weight students' scores on mathematical thinking (connectedness, strategy use, and autonomy). Following this, students' patterns of submission and resubmission were studied in relation to mentor feedback.

Problem difficulty was identified using a 5-point scale that focused on the mathematical challenges represented by the problem, the difficulty of the mathematical concept, and the difficulty of mathematical calculations for students at this level. Inter-rater reliability for identification of problem difficulty was 98%.

Mathematical thinking was coded using a two-step procedure. First, prior to coding a given student's problem-solving, all of that student's submissions and the responses of the mentor were reviewed. Thus coding was undertaken with full knowledge of the range of the individual student's work. Second, each submission/response was again individually reviewed and rated using a one-through-five scale for connectedness, followed by coding for strategy use and autonomy. Inter-rater reliability for coding mathematical thinking ranged from 88-90%. Coding of *connectedness* assessed students' abilities to (a) make real world connections, including links between math topics; (b) identify what the problem involved and demonstrate an understanding of the problem as posed, and (c) recognize alternative mathematical solution strategies. Coding of *strategy use* assessed students' abilities to (a) explain mathematical concepts, (b) work with mathematical terms, and (c) explain decision-making processes in working on a problem. Finally, coding of *autonomy* assessed the extent to which the mentor: (a) supported the development of student responses in future submissions; (b) focused mathematical thinking through the use of tools such as models, examples, and scaffolding; and (c) emphasized problem-solving.

Narrative description of students' mathematical thinking involved review and narrative description of each student's patterns of submission and resubmission, in relation to a classification of mentor feedback.

Results

All reported results are significant at the .05 level, or better. MANOVAs were employed to evaluate differences in student performance from Time 1 to Time 2 for each of the three PoW student groups with respect to their connections to, strategy use, and autonomy on the problems. There were two independent variables: sex and student identification as a person who revised and resubmitted (or as a person who did not). T-tests were used to evaluate differences between groups.

Overall findings for change in levels of connectedness, strategy use, and autonomy indicate that for each PoW student group: (a) students' connectedness (ability to make real-world connections, identify what a problem involved, and choose between strategies; (b) students' strategy use (ability to explain mathematical concepts, work with mathematical terms, and explain decision-making); and (c) students' autonomy (independence in explaining their mathematical thinking; need for models, example, or scaffolds; and ability to focus on problem-solving) all increased from Time 1 to Time 2.

MANOVAs were also employed to examine change, where the independent variables were student preparation and identification as a person who revised and resubmitted (or did not). Student preparation for making connections, using strategies, and autonomy were each evaluated as strong or weak, using students' ratings at Time 1 as the indicator. These analyses produced three primary findings. First, on both the ElemPoW and the GeoPoW, weaker students made even more gains than did their stronger counterparts on connectedness, strategy use, and autonomy. Similar results are suggested by main effects for time and for student preparation in the MidPoW data. Second, a two-way interaction indicates that weak students who revised and resubmitted answers to the GeoPoW over the 10-month period made a similar number of connections at Time 2, as did strong students at Time 1, whereas weak students who did not revise and resubmit answers did not. In addition, narrative descriptions suggest that students were more likely to revise and resubmit when the specific conceptual, explanatory, or computational issues on which they needed to focus were addressed.

Discussion

This study was designed to investigate the impact of the PoW(s) on students' mathematical thinking. Although data from the study of a control group are currently being compiled, it appears that the mathematical thinking of students in elementary, middle school, and high school is enhanced through work with the PoWs. Furthermore, students who are weak with respect to their ability to connect to and use strategies to solve problems, and/or need structured facilitation (lack autonomy) at the outset of their work with the PoW, make even more dramatic gains than do their counterparts who have stronger preparation. At Time 2, weaker GeoPow students who revised and resubmitted solutions made connections to mathematics as effectively as students who had stronger backgrounds at Time 1. Due to random sampling, there were too few weak students in the ElemPoW and MidPoW groups to permit further analysis.

Narrative data suggesting that students are most likely to use feedback that specifically indicates what they need to do when they revise and resubmit their work, also suggests that students may need to understand more about what they have to gain from developing their abilities to think mathematically in order to make effective use of the feedback they receive.

References

Lester, F., Garofalo, J., & Kroll, D. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven classes.* Final report to the National Science Foundation of NSF Project, MDR 85-50346.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 334-370). New York, NY: Macmillan Publishing Co.

Acknowledgements

Support for this study was provided by National Science Foundation Grant # 9618223.