

# Problem Solving—It Has to Begin with Noticing and Wondering

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**H**ave you ever heard mathematics teachers say that they never assign word problems? Some teachers tell me [Marie Hogan] that they never assign them because they are too hard, they take up too much time, and students simply cannot do them. How sad for those students since all students need opportunities to talk and write about their mathematical thinking. Communication, both in spoken and written language, is a skill that is needed for mathematics, for other subjects, and for life. Why would any teacher deprive his or her students of the opportunity to develop these important skills?

Problem solving is still the most challenging part of mathematics to teach to students. Yet, when students learn to solve problems, it empowers them in many ways. Students have more confidence in approaching problems, they ask questions that help further their understanding of more challenging problems, and they persevere in working towards solutions. Problem solving can be time-consuming for a teacher since it is necessary to find appropriate problems that give students the opportunity to discuss strategies and to develop connections to the skills that they are learning. Then it also takes time to provide feedback on each student's work. It helps to think of this problem-solving aspect of teaching as an adventure.

Last year, Suzanne Alejandre, Director of Professional Development at the Math Forum at Drexel University in Philadelphia, visited my classroom. Suzanne taught my classes for two consecutive Tuesdays and I was able to observe her modeling a full cycle of the Math Forum's Problems of the Week (PoWs). In particular, Suzanne modeled the "What do you notice/What do you wonder" activity that offers a great strategy, not only for the PoWs, but also for other mathematical activities.

## Noticing

"Noticing" allows all students to participate in problem solving. Whether I am using a PoW or simply working with a set of word problems from the textbook, I ask my students to answer the question, "What do you notice?" While noticing may seem like asking a student to list everything he/she needs to solve the problem, or to list items as "given," it becomes much more than that once students try it. Noticing can be done as a whole class or with partners since all students can notice what is being stated in the problem and write down or discuss what they notice. In my classroom, my quietest students are now raising their hands to tell me something they have noticed. My students use noticing so well now that I no longer have to wait for them to raise their hands to respond; I can simply call on any student.

## Wondering



"Wondering" is the planning part of problem solving in which we talk about the strategies students might use to try to solve a particular problem. Wondering can also be the restatement of the problem in a student's own words; it can be a little vague in the beginning. The first time I asked my class, "What do you wonder about this problem?" one of my students responded, "I wonder how to solve it." After talking with other students, they soon realized that wondering helps them pose questions about what they have noticed. Rather than racing to find a solution, wondering allows them to slow down, to think, and do mathematics. Wondering allows students to see patterns that they have seen before, and to compare problems and strategies that they have previously used. It creates great dialogue within the classroom and is where the students brainstorm, list, and discuss ideas. I have found that my students no longer say, "I don't get it," or "I don't know how to solve

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this problem.”

### The Oracle’s Oration Problem

Here is an example of how noticing and wondering might be used with a class. On the first Tuesday that Suzanne worked with my class, we had the students report to the computer lab. At the front of the lab, we projected the following scenario on the screen:

The Oracle’s Oration	
<p>Two travelers stop by the side of the road to eat. The first, Amani, has seven bread rolls, while the second, Biagio, has five. A third person, Caleb, stops and asks if Amani and Biagio would be willing to share their rolls. They are, and each of the three eats four rolls.</p> <p>Upon departing, Caleb leaves behind twelve silver pieces as payment for the rolls. Amani thinks that she should have seven of the pieces since she began with seven rolls, and that Biagio should get five, since he started with five rolls. Biagio, however, thinks that they should each get six pieces of silver, since they shared the rolls equally.</p> <p>Since they can’t agree, they go to the Oracle.</p>	 

After Suzanne read the story, she asked the students to list things that they had noticed. Some of their responses were:

- There are 12 bread rolls.
- Amani and Biagio have bread but Caleb doesn’t.
- Amani and Biagio share with Caleb.
- There are 3 people.
- Caleb gave 12 silver pieces.
- They can’t agree on who should get what.
- Biagio had 5 bread rolls.
- Amani had 7 bread rolls.
- Amani wants 7 silver pieces.
- Biagio thinks that they should each get 6 pieces of silver, because they shared the rolls equally.
- They each ate 4 rolls.
- They split the bread into fours and there were 3 people.

Next Suzanne asked them to tell her some things they were wondering about. These

were some of those thoughts:

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| <p>I wonder</p> <ul style="list-style-type: none"><li>• how many silver pieces they each get</li><li>• what an oracle is</li><li>• where they are going on their trip</li><li>• where the oracle is</li><li>• how far they are going because they have 12 breads</li><li>• how they will split the bread to be equal</li><li>• if they will each get the same amount of bread</li><li>• if the Oracle’s decision will be fair</li><li>• if they will share the 12 pieces of silver equally</li><li>• how much each person should get</li></ul> |
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After completing the activity orally, Suzanne asked them to write down everything they remembered from their conversation. For homework, she asked students to write up their solution and to attempt to answer this assignment:

**Extra:** Is there another decision the Oracle could have made about the payment? What might it have been and how would the Oracle have explained it?

### Using Noticing and Wondering

Among the key outcomes in using the noticing and wondering activity are the following.

- ✓ Student ownership and understanding of the question to be solved.
- ✓ Momentum toward a solution path stimulated by all of the mathematical quantities and relationships noticed.
- ✓ Slowing down the thinking process and surfacing all of the information and questions that are too easily passed over or dismissed.
- ✓ Articulation of specific sub-problems or questions students need to answer or learn more about in order to solve the problem.
- ✓ Identifying other questions and features of the problem that may be even more interesting and challenging for students.

- ✓ Reinforcing the expectation of problem solving as a process. The goal is not to be over and done but to think and express.
- ✓ Understanding what it means to revise an answer (remove noticing/wondering text and replace it with an answer in the form of a complete sentence and an explanation of the answer).

I use noticing and wondering whenever I begin a new PoW. The hardest part for me is to allow students to do the talking. Since good questioning techniques are very important, I try to pose questions that further class discussions. But it is difficult for me to move from just posing questions to encouraging students to work out problems without my assistance. It is much easier to give students the answer than to expect them to discover it for themselves.

With Suzanne working with my students, I was able to listen to what they were saying about their thinking. I could observe them when they interacted with one another and with Suzanne. As teachers, we are so used to seeking a correct answer that we forget that it is also important to see if a student understands the process.

Suzanne is a master at questioning and will wait longer than I ever thought possible. Wait time becomes very important if teachers are going to have students respond to questioning. Before this can happen, however, teachers must develop a sense of safety in their classrooms and students need to know that it is okay to make mistakes. Students

need to understand that they all have a vested interest in everyone being successful.

Suzanne and I worked in our school's new media center so that all of the students could do their work on computers and submit their responses by using the Math Forum's online PoW environment. After students submitted their answers to a given problem, we read and responded to their submissions.

I now respond to the solutions students submit by using noticing and wondering, as modeled by Suzanne. I use "I notice" to acknowledge and value something the student has written, and then I use "I wonder" to pose a question that may further the student's thinking or understanding of the problem. My students look forward to my responses. Students who submit before the due date now get impatient for my response. They keep asking me, "Did you read my submission yet?" Notice, they don't ask, "Did you correct my paper?"

Once the submission cycle is over, I share their work, especially work that I believe is exemplary, and we discuss the different strategies that students have used. I am beginning to see more students arriving at answers that are reasonable even if they are not correct.

Teaching problem solving continues to be a journey for me. My students still struggle with problem solving and they still want me to give them the answers. This problem-solving journey for both my students and me is far from over, but for now, please excuse me while I respond to another student's solution to the Problem of the Week. 