

Birthday Trip – Expected Solutions and Scoring Guidelines

Method 1 - Working Backwards:

Jude's family travelled 145 miles on the third and last day of the trip. Since they travelled $\frac{2}{3}$ of what they had left on the day before, this is $\frac{1}{3}$ of what they had left on day two. I can multiply by 3 to find out how far they had left to travel on day two:

$$\begin{aligned}\frac{1}{3} * 3 &= \frac{3}{3} = 1 \\ 145 \text{ miles} &= \frac{1}{3} \text{ of distance left day two} \\ 145 \text{ miles} * 3 &= 435 \text{ miles}\end{aligned}$$

Now I know that they had 435 miles left to cover at the beginning of day two. Since they covered half the distance on the first day, the remaining distance is also half the distance. I can multiply by two to find the total distance of the trip:

$$\begin{aligned}\frac{1}{2} + \frac{1}{2} &= 2 * \frac{1}{2} = \frac{2}{2} = 1 \\ 2 * 435 \text{ miles} &= 870 \text{ miles}\end{aligned}$$

This is the entire trip, so it's **870 miles** from Jude's house to Aunt Mazie's house.

Method 2 - Guess and Check (Hypothesize, Test, and Revise):

Three days is a long time for driving, so I'm going to test and see if 1000 miles works:

$$\begin{aligned}\text{first day, half the distance} &= \frac{1}{2} * 1000 \text{ miles} = 500 \text{ miles} \\ \text{second day, } \frac{2}{3} \text{ of remaining } 500 \text{ miles} &= \frac{2}{3} * 500 = 333 \frac{1}{3} \text{ miles} \\ \text{third day, what's left? } 500 - 333 \frac{1}{3} &= 166 \frac{2}{3} \text{ miles}\end{aligned}$$

This is more than the 145 miles that the puzzle says are left on the third day, so my starting mileage was too high. I'll try 750 miles next:

$$\begin{aligned}\text{first day, half the distance} &= \frac{1}{2} * 750 \text{ miles} = 375 \text{ miles} \\ \text{second day, } \frac{2}{3} \text{ of remaining } 375 \text{ miles} &= \frac{2}{3} * 375 = 250 \text{ miles} \\ \text{third day, what's left? } 375 - 250 &= 125 \text{ miles}\end{aligned}$$

This is less than 145 miles, so my starting mileage was not enough this time. I'm going to show my tests in a table:

total (guess)	1st day $\frac{1}{2}$	2nd day $\frac{2}{3}$ of remaining	3rd day	correct?
1000	500	333 $\frac{1}{3}$	166 $\frac{2}{3}$	too high
750	375	250	125	too low
875	437 $\frac{1}{2}$	291 $\frac{2}{3}$	145 $\frac{5}{6}$	close!
870	435	290	145	yes!!!

The trip to Aunt Mazie's is **870 miles**.

Method 3 - Logical Reasoning (combo of working backwards with algebraic thinking):

On the last day, Jude's family travelled 145 miles. This is $\frac{1}{3}$ of the previous day's distance since they covered $\frac{2}{3}$ of what they had left.

However, the previous day's distance was half the total trip, so 145 miles is really $\frac{1}{3}$ of $\frac{1}{2}$ the distance or $\frac{1}{6}$ the total distance. I can just multiply by 6 to find the total:

$$6 * 145 \text{ miles} = 870 \text{ miles}$$

Jude and her family travelled **870 miles** to Aunt Mazie's house.

Method 4 - Algebraic:

We have to find the total distance between Jude's house and Aunt Mazie's house. I'm going to use d to represent the total distance.

The family travelled half the distance the first day, $\frac{2}{3}$ the remaining distance the second day, and 145 miles the third day. If I add all these up, I'll get the total distance. I can represent this algebraically with the following equation:

$$\frac{1}{2}d + \frac{2}{3}\left(\frac{1}{2}d\right) + 145 = d$$

$\frac{2}{3}$ the remaining distance, the remaining distance is $\frac{1}{2}$ the total

Now I just need to do the algebra to solve for d (simplify):

$$\begin{aligned} \frac{1}{2}d + \frac{1}{3}d + 145 &= d && \text{(multiply)} \\ \frac{3}{6}d + \frac{2}{6}d + 145 &= \frac{6}{6}d && \text{(common denominator)} \\ \frac{5}{6}d + 145 &= \frac{6}{6}d && \text{(add)} \\ 145 &= \frac{1}{6}d && \text{(subtract } \frac{5}{6}d \text{ from each side)} \\ 870 &= d && \text{(multiply both sides by 6)} \end{aligned}$$

The total distance is **870 miles**.

Extra:

The family averages 50 miles per hour while driving on the return trip. To find out how many hours this takes, divide the total miles by the rate:

$$870 \text{ miles} / 50 \text{ mph} = 17.4 \text{ hours or } 17 \text{ hours } 24 \text{ minutes}$$

It will take the family **17.4 hours** or **17 hours 24 minutes** of driving time to make the trip.

Note: submitters at this age might also make a list/table of how many miles are covered hour by hour, giving an approximate number of hours (or switching to smaller units to find minutes). This *is* an acceptable strategy for the Extra. Either way, they must still say something about the trip being made in one day or more.

Reasoning: I don't think they will make the trip in one day. While the time needed is less than 24 hours (the number of hours in one complete day), 17+ hours is a long time for someone to drive. It would seem more reasonable if there were several drivers taking turns (Mom, Dad, a child of driving age), but this would still be a long time to be buckled up in the car!

Submitters may say they think they will make the trip in a day (and receive credit), but be sure to ask them to think about how reasonable this would be.

To be considered a **practitioner** in **problem solving interpretation**, the student must

- interpret the problem correctly (this includes interpreting the math concepts correctly)
- attempt to solve all of the parts

Specific to this problem, the submitter must:

- realize that Aunt Mazie's age is not an important fact in this puzzle
- understand that the trip was made over 3 days
- understand that 145 miles is the distance travelled on day three
- understand the basic fractions that are included in describing the distances travelled on the first and second days

If they have a sense of the trip being made in 3 days but don't seem to understand the fractions or misunderstand what portion of the trip 145 miles represents, they are an **apprentice**. If they don't seem to understand the puzzle, they're a **novice** (and may have just copied the answer check).

Expert: would understand what the Extra question asks.

To be considered a **practitioner in problem solving strategy**, the student must

- pick a good strategy **based on their interpretation of the problem** and apply it well - success should be achieved through skill instead of luck

Specific to this problem, the submitter might:

- use guess and check (hypothesize, test, and revise)
- use knowledge of fractions to logically reason through the puzzle
- use knowledge of fractions to work backwards through the puzzle
- apply knowledge of algebra and use variables to work the puzzle
- use some other appropriate problem-solving method

An **apprentice** might have started with guess and check but be unsure how to narrow the selection (random as opposed to organized progression of hypotheses) -OR- might know how to start with one of the above mentioned approaches but not be sure how to carry the method to completion (note that this is different from an incorrect answer due to a calculation error). A **novice** will have no reference to how they found their answers.

Expert: would have an appropriate strategy for working through the Extra (divide, make a table), would show advanced skill with an algebraic approach (because this is the FunPoW level - algebra is not expected here), or include more than one way to correctly solve the puzzle.

To be considered a **practitioner in problem solving accuracy**, the student must

- have the "correct" answer **based on their interpretation of the problem and the strategy they've applied**
- get the calculations and details correct of what they included
- please be sure you base this on the work the submitter has included, **not** the work you expect to see

Specific to this problem, the submitter must:

- work accurately with fractions, when used
- have correct units *if / when* included
- if attempted, have Extra calculations correct (and error in the calculations in the Extra would move Accuracy to Apprentice).

An **apprentice** might have some calculation mistakes along the way, a **novice** would have many calculation mistakes.

Expert: none for this problem.

To be considered a **practitioner in communication completeness**, the student must

- explain all of the steps taken to solve the problem based on their interpretation and strategy (in other words, based on the work they've done - so some of the following might not apply if the submitter misinterpreted the problem or picked a poor strategy for solving it)
- if the submitter has understood the problem and picked an appropriate strategy we'd expect to see the following:
- if guess and check, some incorrect guesses that show movement closer to the correct distance (i.e., it's not enough to say, "I used guess and check," but only include the final guess)
- if algebraic, what the variable represents and some of the steps taken to solve (including how the pieces are put together - what they represent)
- please remember, if they have misunderstood the problem or applied an inappropriate strategy, you need to evaluate how they have communicated what they **did** try to do

An **apprentice** might have an explanation but not include any of the calculations -or- might have mostly calculations without explanation of what the calculations represent. A **novice** will have little or no explanation and/or calculations.

Expert: might state every single step in this problem and include every single calculation.

To be considered a **practitioner** in **communication clarity** , the student must

- explain all of the steps in such a way that a fellow student would understand (again, based on the work they've done - this is NOT dependent on correct interpretation, strategy, or accuracy)
- make an effort to check formatting, vocabulary, and spelling

Clarity - **Apprentice** would be used when a submitter has a few spelling and/or grammar errors that make the solution difficult to follow... or they've got formatting (or lack thereof) that doesn't work for tables, charts, or equations.

Clarity - **Novice** would be when the spelling and/or grammar is SO bad, it really gets in the way of understanding much of anything the submitter did.

Note: A solution does not have to be complete or correct in order to be clear.

Note that an incorrect solution can be clear. An **expert** might format things almost flawlessly and present a very readable and well- presented explanation. While an expert in communication-complete would have all the steps, an expert in communication-clarity would have presented them in a way that is easy to read and follow.

To be considered a **practitioner** in **problem solving reflection**, the student needs to do at least **two things** that are "reflective"

- check their solution/answer
- reflect on its reasonableness
- summarize the process they used
- discuss any errors they made along the way and how they found and fixed them
- connect the problem to prior knowledge or experience - an older relative, special birthdays, long trips they've taken etc.
- write a comment after checking the answer check
- explain where they're stuck

The comment that the student leaves after checking their answer might count as reflective.

If they only do one reflective thing, they're an **apprentice**. If they do none (and their comment might say "my answer is right" or something even less thoughtful), they're a **novice**.

Expert: more than two of the above -or- developed versions of more than one of the above.