1 *IMCePtion*

Hey, welcome to the class. We know you'll learn a lot of mathematics here—maybe some new tricks, maybe some new perspectives on things with which you're already familiar. A few things you should know about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, we haven't written the problem sets correctly.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How did others at your table think about this question?
- **Respect everyone's views.** Remember that you have something to learn from everyone else. Remember that everyone works at a different pace.
- **Learn from others.** Give everyone the chance to discover, and look to those around you for new perspectives. Resist the urge to tell others the answers if they aren't ready to hear them yet. If you think it's a good time to teach everyone about manifolds, think again: the problems should lead to the appropriate mathematics rather than requiring it. The same goes for technology: the problems should lead to appropriate uses of technology rather than requiring it. Try to avoid using technology to solve a problem "by itself". There is probably another, more interesting, way.
- **Each day has its Stuff.** There are problem categories: Important Stuff, Neat Stuff, Tough Stuff, and maybe more. Check out Important Stuff first. The mathematics that is central to the course can be found and developed in Important Stuff. After all, it's Important Stuff. Everything else is just neat or tough. If you didn't get through the Important Stuff, we noticed... and that question will be seen again soon. Each problem set is based on what happened before it, in problems or discussions.

At least one problem in this course is unsolvable. Can you find them all?

Every three days, go back and read these again.

PROBLEM

Get Sketchpad working on your computer. The actual problem in the box will be done later.

Point your web browser to `http://www.tinyurl.com/getgsp`.

The first day of class is a perfect time for a huge logistical undertaking! Woo.

Important Stuff.

1. Sameer tells you the perimeter and area of a rectangle. Is it possible to confidently determine the dimensions of the rectangle?

Or whatever his name is. The hat guy.

2. Here are some perimeters and areas. Find the dimensions of each rectangle.
(a) perimeter 24, area 36
(b) perimeter 24, area 35
(c) perimeter 24, area 32
(d) perimeter 24, area 27
(e) perimeter 24, area 34
(f) perimeter 24, area 37

3. Find all solutions to each equation.
(a) $x^2 - 12x + 36 = 0$
(b) $x^2 - 12x + 35 = 0$
(c) $x^2 - 12x + 32 = 0$
(d) $x^2 - 12x + 27 = 0$
(e) $x^2 - 12x + 34 = 0$
(f) $x^2 - 12x + 37 = 0$

I found 'em! They were right over there!

4. Get Sketchpad on your computer!

5. A rectangle has perimeter 36. What could its area be?

6. Chance tells you the surface area and volume of a rectangular box. Is it possible to confidently determine the dimensions of the box?

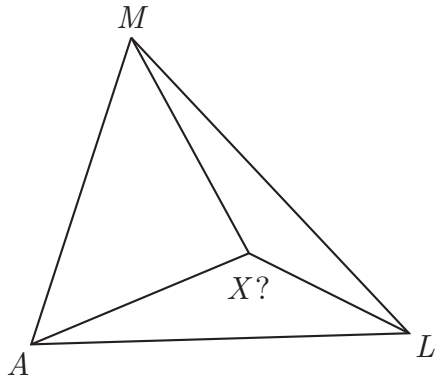
If you can confidently determine the dimensions, Chance will be sad: he'll have nothing left.

PROBLEM

Three cities located at points M, A, L get together to build an airport. Where should the airport be placed to minimize the lengths of the new roads that need to be built?

There's a problem in the box *inside* the problem in the box! MIND-blowing. Let us know if you need help using Sketchpad to measure anything.

Three computers are located at points M, A, L. Where should a router be placed so that the smallest amount of cable is needed to connect it to the computers?



Build this sketch, then use Sketchpad to figure out where point X should be placed.

Is there anything special about this point?

Neat Stuff.

7. For each point, decide if it is the same distance from $(7, 1)$ and $(-2, 9)$.

This problem has four parts, but it's *not* multiple choice! Mind-blowing.

(a) $(7, 10)$
(b) $(-1, 1)$
(c) $(-17, -17)$
(d) $(-2, 0)$

8. Find some rectangles whose perimeter and area have the same numeric value. More. MORE!

9. Find some rectangular boxes whose surface area and volume have the same numeric value. More?

10. Rina tells you the perimeter and area of a triangle. Is it possible to confidently determine the side lengths of the triangle?

11. Yesterday was 7/4/11, and $7 + 4 = 11$. Hooray for America. But...

(a) How many more times this century will there be a day like this? By *this* we mean the next one is August 3, 2011.

(b) How many times *next* century will there be a day like this?

(c) How can your second answer help you check the first?

At least one answer to this problem is totally awesome. Or found in the lyrics of a Talking Heads song. Or both.

Tough Stuff.

12. A triangle has perimeter 24. Find its maximum possible area, and explain how you know that this *must* be it.

13. Given positive integer n, the unit fraction $\frac{1}{n}$ can be written as the sum of two other unit fractions:

$$\frac{1}{n} = \frac{1}{a} + \frac{1}{b}$$

Find a rule for the number of ways to write $\frac{1}{n}$ as the sum of two unit fractions.

Like the blood type, a and b must be positive. Unlike the blood type, they must be integers.

14. Find a rule for the number of ways to write $\frac{1}{n}$ as the sum of *three* unit fractions.

15. Spin a top so that it does not stop spinning. Or *does* it?