Math Fundamentals Problem Packet for Teachers:

Growing Worms

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Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, teaching suggestions, and a problem-specific scoring rubric. **Growing Worms** has been chosen from the Problem Library (#3283), so I'm also including some sample student solutions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username/password.]

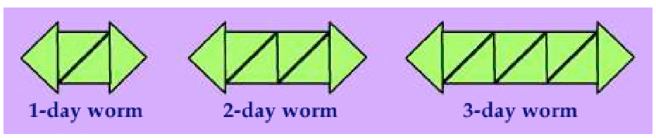
The Problem

In **Growing Worms** students discover a pattern in a growth rate that can be applied to answer questions about future stages of the worm. This type of thinking helps lay the foundation for understanding linear functions and graphing, which students will encounter in algebra.

The Extras ask them to apply the rule in reverse and to describe a general rule or formula that can be used to determine the makeup of a worm of any age.

A print-friendly version is available from the "Print this Problem" link on the problem page.

Growing Worms



In the land of Trianglia the worms are made of isosceles right triangles — and they grow fast! As you can see above, a worm that is 1 day old is made of 4 of these triangles. You can also see worms that are 2 days old and 3 days old. If that growth rate remains constant, how many triangles will be needed for a 4-day-old worm? a 10-day-old worm? a 63-day-old worm?

Be sure to explain your strategy.

Extra: I found a worm that was made of 60 triangles. How old was it? Explain how you know.

Super Extra: Can you make a rule that uses a worm's age (in number of days) to find out how many triangles it is made of? You may express your rule with words or with numbers and symbols.

[Inspired by *Navigating through Algebra in Grades 3-5*, National Council of Teachers of Mathematics]

Answer Check

A 4-day-old worm is made of 10 triangles. A 10-day-old worm is made of 22 triangles. Can you use what you learned to find the number of triangles in a 63-day-old worm?

If your answer **doesn't** match ours,

- do you realize that all the triangles count, including head and tail?
- did you try drawing a 4-day-old worm? a 5-day-old worm?
- did you make a table listing the age and number of triangles?
- did you check your arithmetic??

If your answer **does** match ours,

- have you clearly shown and explained the work you did?
- did you try the Extra?
- did you explain any patterns or insights you discovered while solving the problem?
- did you make any mistakes along the way? If so, how did you find and fix them?

Our Solutions

Here are several examples of ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. In fact, we often receive solutions from students who have used approaches we've not anticipated. I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1 – Drawing Worms and Counting by 2s:

I noticed that the worm grows one more square in the middle every day. Each square consists of two triangles. The head and tail stay the same. I drew a 4-day-old worm. It had 4 squares in the middle and was made of 10 triangles. I drew a 10-day-old worm. It had 6 more squares in the middle. I started at 10 and counted on six more 2s and got to 22 triangles.

I did not want to draw a 63-day-old worm, so I skip counted on a number line, using the numbers to represent the number of days. I pointed at the 10 for the 10- day-old worm. I began with 22 (triangles) on the 10th day and counted by 2s on each number on the line until I got to number 63. My skip count was at 128, so a 63-day-old worm is made of 128 triangles.

Strategy 2 – Using a table and adding 2s:

I made a table listing the age of the worm in days and the number of triangles it was made of. Then I drew a 4-day-old worm and added it to the table.

Each day the worm grew by one square, or two triangles. In 6 more days the worm would grow 6 more squares, or 12 more triangles. I added that to 10 triangles (at 4 days). 12 + 10 = 22 triangles.

Age	**Triangles**
1	4
2	6
3	8
4	10
.	.
.	.
.	.
10	22

After 53 more days the worm would be 63 days old. 53 days * 2 triangles per day = 106 more triangles
22 triangles (after 10 days) + 106 more triangles = 128 triangles for a 63-day-old worm.

Extra: I knew that a 4-day worm needed 10 triangles. 60 triangles is 50 more triangles than that. The worm grows by one square, or two triangles, per day. 50 triangles represent 25 days of growth because
50/2 = 25.
4 days + 25 days = 29 days, so the worm would be 29 days old.

Super Extra: I looked at the numbers in my table and saw that you can find the number of triangles by doubling the age and adding 2 more for the head and tail. For example, to find the number of triangles for a 4-day-old worm:

(2 * 4) + 2 = 10 triangles

Strategy 3 - Direct observation of the pattern:

From the diagram I could see that each worm contains a square of 2 triangles for each day of its age, and two extra triangles (one head and one tail). From that information I could tell that a 4-day-old worm would be made of 10 triangles:

(2 * 4) + 2 = 10 triangles

A 10-day-old worm would be made of 22 triangles:

(2 * 10) + 2 = 22 triangles

A 63-day-old worm would be made of 128 triangles:

(2 * 63) + 2 = 128 triangles

Extra: I worked backwards. First I subtracted 2 triangles for the head and tail. That left 58. I divided 58 in half, since the triangles are paired to make squares, and one pair of triangles represents one day of age.

58/2 = 29 days old

Super Extra: If you use n to stand for the age of the worm in days, you can find the number of triangles (t) by using the formula $t = 2 * n + 2$. I discovered this by looking at how I solved the main part of the problem.

An alternate and equivalent way to express the rule would be: Find the number of triangles, add one to the age in days and then double that sum. This works because the head and tail are equivalent to one square, or one more day's growth.

OR

Using n as the age in days, the formula for triangles is $2(n + 1)$.

Teaching Suggestions

Some students may simply notice that the number of triangles increases by two each day and use a counting strategy. We call this the *recursive* form, as each result is based on the previous result. It is the pattern most easily discovered by students. While it is a legitimate strategy, it is limited in that, in order to find the number of triangles for any given age, we need to find that number for all the ages prior to it. Finding the number of triangles for a 63-day worm with this method becomes quite tedious.

A more sophisticated solver takes advantage of a rule based on the age of the worm, called the *closed* form. The advantage of this method is that the number of triangles can be directly calculated for any given age. Students who demonstrate understanding of the problem as a recursive process might be encouraged to take advantage of what they have learned in order to move toward discovery of the closed form. Consider asking questions such as: What parts of the worm change as the worm grows older? What parts remain the same no matter the age?

Students may use different methods of representing the problem and organizing their thinking, such as drawing pictures, making a table or graph, or using a spreadsheet. As they generalize any type of pattern or rule from the data in a table, they should support the continuation of that pattern by relating it to the physical model. Encourage students, for example, to explain why the total number of triangles increases by two each day based on the diagram, not simply on data in a table.

The Teacher Support Page for this problem contains links to related problems in the Problem Library and to other web-based resources: http://mathforum.org/funpow/puzzles/supportpage.ehtml?puzzle=404.

Scoring Rubric

On the last page is the **problem-specific rubric,** to help those who are assessing student solutions. It specifies what we expect from students in three areas of problem solving and three areas of communication. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

Sample Student Solutions

Focus on Completeness

I've chosen the samples below to illustrate the range of Completeness demonstrated by submitters. This category of our rubric addresses whether the student has explained most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. The explanation should include key calculations. See the problem-specific rubric for more information on scoring.

It is not my purpose to give a definitive judgment, but rather to highlight the range and variety of work done by students and suggest ways I might encourage them to take next steps. The best way to elicit more writing from students is to ask them specific questions about the missing details.

You can view commentary and solutions that were highlighted in 2004 at this Library page:
http://mathforum.org/librarypow/office/teachers/index.ehtml?puzzle=3283&page=lib_solution

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William
Age 13

Completeness
Novice

4th day will be 10 triangles and The 10thday will be 22 triangles, and the 63 day will be 128

The channleage was esay. You have to find a patern and see how much the worm grew

William's short answers are correct, but he wrote nothing to explain how he found them. It's possible that he understood the pattern. He needs to describe the pattern and how he discovered it, how it relates to the diagram, and include any calculations he used.

Christina
age 11

Completeness
Apprentice

There are 10 isosceles right triangles in a four day old worm.

For every additional day, the worm increases by 2 isosceles right triangles. If a 2 day old worm is 6 triangles than a 4 day old worm would be 10 triangles.

A 10 day old worm would be 22 triangles.

A 63 day old worm would be 128 triangles.

Christina understood how the worm grows. She explained how she found the 4-day worm, but I'd like to know how she found the others. Did she draw the worms? Make a table? Use a rule? I'd ask to see any calculations.

Austin
Age 10

Completeness
Apprentice

1. 4 days=10 isosceles triangles
2. 10 days=22 isosceles triangles
3. 63 days=128isosceles triangles

Extra: The worm was 29 years old

Super Extra: 2n+2=number of triangles n=nuber of days

1. I did 2x4+2 and got 10 as my answer.
2. I did 10x2+2 and got 22 as my answer.
3. I did 63x2+2 and got128 as my answer.

Extra: I subtracted 2 from 60 and got 58 and divided 58 by 2 and got 29 as my answer.

Super Extra: I knew I had to use a variable for the days, multiply the days by 2, then add 2.

Austin clearly has a good mastery over the math. He generalized a good formula and used his rule inversely to solve the Extra. I'd ask for his reasons for multiplying the days by 2, and then adding 2. What do the two 2s in his formula represent in terms of the diagram and the growth of the worm? I'd remind him about correct use of the equals symbol, a Clarity issue.

Robin
Age 10

Completeness
Apprentice

For a four day worm you will need ten triangles, for a ten day old worm you wiil need 22 triangles and for 63 day old worm you will need 128 triangles.

Days Triangles
1 > < 4
2 > < 6
3 > < 8
4 > < 10
5 > < 12
6 > < 14
7 > < 16
8 > < 18
9 > < 20
10> < 22

I made a chart up to ten and then I noticed that if you add the number of days to itself plus two you get the answer. So to figure out 63 days I did 63 x 2 + 2 = 128.

Extra- I took 60 and subtracted two so it was 58. After that I divided 58 by two and that made 29.

Robin did a good job of including a table and finding a formula. She applied the formula accurately to the 63-day worm. I'd like to know how she carried out the table beyond the 3-day worm in the diagram. If from the +2 pattern in the table, how does the diagram support that? She carried out her inverse steps correctly in the Extra. I'd ask why she did those steps and why in that order.

Masha
age 10

Completeness
Practitioner

A 10 day old worm is 22 triangles. A 63 day worm is 128 triangles. A 4 day old worm is 10 triangles. The rule I have is: multiply the day # by 2, then add 2. The 60 triangle worm was 28 days old.

I figured out that there were the # of squares in the worm as the # of days, and that each square has 2 triangles. Using that, I multiplied each of the day old numbers and then added two triangles, then, because there were two at the ends of each one. Knowing that I did that, I knew what stradegy I used. To figure out the 60 triangle one, I just did the stradegy backwards: divide by 2, subtract 2.

Masha makes a strong connection between her rule and the physical model. I'd ask her to clean up some Clarity issues, e.g., # for the word "number". I'd ask her to check the 28-day worm with her rule. This will help her understand that, when applying inverse operations (great strategy!) one needs to reverse the order as well (subtract first and then divide by 2.

Daniel
age 12

Completeness
Practitioner

In four days the worm will have 10 triangles. In 10 days it will have 22 triangles. And in 63 days the worm will have 128 triangles.

The first thing I did was look at the worms and counted how many triangle they had. Then I tried to figure out a pattern. I noticed that as the days went up by one, each worm got 2 more triangles. On the third day the worm had 8 triangles, so I added two and got 10 triangles for the 4th day.
Then I had to figure out how many triangles there would be in 10 days. So I counted up by two each time. Day five had 12 triangles, 6 had 14, 7 had 16, 8 had 18, 9 had 20, so 10 would have 22.
Now I had to figure out how many triangle there would be in 63 days. I wasn't going to count like I did for 10 all the way up to 63, so I had to figure out a different way. I was going to go up by tens. 10 had 22 so 20 would have 42. So if I went by that by the time I got to 60 it would have 122 triangles. I figured this out because 10 had 22 and that was 2 more than 10 *2. So 20*2=40 then I added 2 to 40 and got 42. So now that I new that 60 had 122 triangles, I could count up like I did with 10. So 61 has 124, 62 has 126, so 63 would have 128 triangles.
I am sure I am correct because, the answer is 2 more than double the age.

Daniel described in detail his method of gradually working his way up by twos, always relating his numbers to the worms. His process of constructing the 63-day worm demonstrated good number sense. His final statement generalizing a rule would count as a reflection, as he uses a different method to check his calculations. Daniel is ready to tackle the Extra!

Emily
age 10

Completeness
Expert

The answer is a 4 day old worm has 10 triangles, a 10 day old worm has 22 triangles, and a 63 day old worm has 128 triangles. The answer to the extra is a worm that has 60 triangles is 29 days old.

I know that the pattern adds two triangles each day. So I will multiply the day by two, then add two because of the triangles on the outside of the worm. Also, all of my answers will be even because the first worm started out even and adds even numbers each day.

1) 4x2=8+2=10 2) 10x2=20+2=22 3) 63x2=126+2=128

So the answer is the 4 day old worm is 10 triangles, the 10 day old worm is 22 triangles and the 63 day old worm is 128 triangles.

I checked my work by subtracting 2 then dividing it in 2.

1) 10-2=8 8/2 =4 2) 22-2=20 20/2=10 3) 128-2=126 126/2=63

EXTRA: Using the first method in reverse, I did 60-2=58. Then I did 58/2. So my answer is the worm is 29 days old. I checked my work by doing 29x2=58+2=60.

*Emily explained her rule and how it related to the diagram. She included her calculations for the main problem and the Extra. She provided exceptional insight into the problem by explaining why the numbers of triangles need to be even. Her EXTRA explanation could be improved by connecting it to the diagram. I would model for her a more accurate way to notate her calculations, e.g., 4 * 2 + 2 = 10.*

Daniel
age 7

Completeness
Expert

A 4 day old worm will need 10 triangles
a 10 day old worm will need 22 triangles
a 63 day old worm will need 128 triangles

Extra: A worm made of 60 triangles will be 29 days old.

SuperExtra: If the number of days old = x, then the number of triangles needed will be 2x+2.

I noticed that the number of triangles needed for each worm followed an arithmetic sequence with a difference of +2.

I also noticed that if you removed the 2 end triangles, the number of triangles needed was twice the number of days which gave me the formula 2x+2 when x was the number of days old.

Therefore a 4 day old worm was 2(4)+2 = 10
a 10day old worm was 2(10)+2 = 22
a 63 day old worm was 2(63)+2 = 128

Extra: To find how old a worm was that was made of 60 triangles, I started with the formula 2x+2=60. I need to get x by itself so I used inverse operations. First I subtracted 2 from both sides so I had 2x=58. Then I divided both sides by 2 which gave me 29=x.

Daniel's thinking and his communication skills are excellent. He justifies his formula with details from the diagram. He clearly understands inverse operations. I would ask him to explain how his inverse procedure relates to the physical model. Why does it make sense in terms of the diagram to subtract 2 before dividing?

We hope these packets are useful in helping you make the most of FunPoWs. Please let me know if you have ideas for making them more useful.

~ Claire

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The Math Fundamentals Problem of the Week Scoring Rubric — **Growing Worms** (posted 13 October 2008)

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that the total number of triangles increases by 2 each day. Understands that the 2 head and tail triangles remain constant. Uses correct order of operations. Answers all parts of the main question.	In addition to solving the main problem correctly: Understands and solves both of the Extras correctly Provides a correct rule for the growth pattern as a function of days, using either words or an algebraic expression. Achieves at least Practitioner in Strategy.
Strategy *(NB: based on their interpretation of the problem)*	Does not know how to approach the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Verifies a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill and understanding, not luck. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., algebra or an effective and appropriate use of technology.
Accuracy *(NB: based on chosen strategy)*	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness *(NB: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Describes the steps but does not include calculations or numbers. OR Shows calculations without rationale or explanation.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes key calculations with rationale. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem.
Clarity *(NB: incomplete and incorrect solutions can be explained clearly*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language and notation, including units: days, triangles. Shows effort to use good organization, formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful table. (A table alone doesn't qualify for Expert status.)
Reflection (See list below.**)**	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment they leave after viewing our answer:	• **Revises and improves the submission**. • Checks the answer using a different method. • Explains a hint she/he would give someone.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them.	• Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used.