The Math Forum: Problems of the Week

Problem Solving and Communication Activity Series

Guess and Check

Guess and Check is a popular and important problem-solving strategy, though it often gets a bad rap and may not be developed into the strong and powerful resource it could be for students. The *Guess and Check* strategy has at least three purposes: (1) to understand a problem thoroughly, (2) to home in on a goal, and (3) to discover efficient ways to jump to a solution by noticing patterns and developing related algebraic representations.

In this round of activities, we focus on two layers of guessing and checking. The first is making sure students can generate a *guess*, identify any necessary *calculations*, and decide how to *check* whether or not their guess fits the constraints of the problem. The second layer is helping students notice how different ways of recording their work help them notice patterns that allow them to improve their guesses. Throughout the *Guess and Check* activities, we emphasize the three processes of guessing, calculating, and checking, and encourage students to focus their thinking and recording in these three areas.

In subsequent *Guess and Check* lessons, we will focus on strategies for recording guesses, calculations, and checks in ways that make patterns in the calculations clear, with the goal of helping students represent their work in an efficient, algebraic manner.

Problem-Solving Goals

Guess and Check strategies can help problem-solvers:

- More fully understand the problem.
- Find patterns.
- Discover and generalize important relationships.
- Solve complex problems.

Communication Goals

Guess and Check strategies are enhanced when students focus on writing to learn. Good problem-solvers record their works in ways that:

- Make clear what they guessed, what calculations they did, how they checked their guesses, and what did or did not work.
- Make it easy to notice patterns and repetition.
- Help them analyze their attempts so that they can make their next guess even better.
- Allow them to share their guesses, their calculations, and their checks with others.

Activities

I. Guess, Calculate, Check

Format: students working in small groups (3 students to a group).

We encourage all students to begin problem solving by using some form of the *Understanding the Problem* strategy covered in the first two rounds of the Activity Series, but *Guess and Check* can be another good method for familiarizing oneself with a problem.

One of the first requirements of the *Guess and Check* strategy is to determine the quantities or aspects of the problem they can "play with." This might be equivalent to "Doing it Wrong" or noticing quantities and relationships between quantities in *Understanding the Problem.* Once they have chosen an aspect to guess about, students need to see the effect of the guess on as many other quantities and relationships as they can. On the way, they will also think about how to check their guesses.

Sample Activity

Each group will generate 3 guesses. For each guess, students will identify and perform the necessary calculations to check their guess. Each student will have a chance to be the group's scribe for one guess. The scribe should make sure to include as much as they can of:

- what was guessed,
- the calculations that were completed based on that guess (it should be clear what the results of each calculation represent), and
- how the guess was checked.

One reasonable outcome from this effort may be for the students to get clear about what they may not understand or need to first figure out before they can effectively apply *Guess and Check*. Some students may have trouble doing calculations, thinking of ways to check guesses, or even understanding what to guess. In that case, they should spend the time listing what they do know and what they need to figure out. They might return to one of the *Understanding the Problem* strategies, or find other resources in order to figure out these sub-problems.

Key Outcomes

- Focus conversations and writing on three distinct aspects of the solution path (guesses, calculations, checks).
- Develop awareness of where difficulties arise as students use *Guess and Check*; is it deciding what to guess, figuring out the calculations, or checking guesses?
- Generate record keeping methods for guess and check that can evolve into organized and systematic approaches.

II. Class Discussion on Guess, Calculate, and Check

Format: small groups present their guesses; the audience has the task of asking certain kinds of questions and/or giving certain kinds of feedback (Herrenkohl & Guerra, 1998).

Having the opportunity to present their own work, see others' work, ask questions of others, and discuss similarities and differences will help students become aware of their own guess and check process. With each presentation they should note:

- Clarity of organization and labeling.
- Completeness of the work.
- Similarities and differences among groups.

NOTE: In order to facilitate discussion and keep all students engaged, in the sample activity, we have provided roles for audience members to take as they listen to groups.

Sample Activity

Students were asked to focus on generating guesses, performing and recording necessary calculations, and checking their guesses against the constraints in the problem. As students present their work, audience members can be assigned the following roles:

- Noticing guesses: Was it clear what guesses were made and how they relate to the desired answer? Can you tell how the guesses were generated? Once you've seen multiple groups' presentations, do you notice similarities or differences among the guessing strategies?
- Noticing calculations: Are the calculations organized or labeled so you can tell what is being calculated and how? Are all the necessary calculations shown? Are there calculations that are not necessary? Do you notice patterns among the calculations? Once you've seen multiple groups' presentations, do you notice similarities or differences in the calculations that are being done?
- Noticing checks: Can you tell what constraints in the problem were used to check guesses? Are all the necessary constraints used? Do unnecessary or added constraints exist? Did you notice similarities and differences among the constraints that different groups used?

Audience members can brainstorm (grouped by role or as a whole class) what questions they might ask the presenters for clarification, completeness, or to highlight similarities and differences in their assigned category. After all the presentations, students in each role should be encouraged to share further noticings and wonderings that will help everyone reflect on and improve their guesses, calculations, and checks.

After groups have made their presentation and listened to feedback, and the class has discussed what they have learned about improving their *Guess, Calculate, and Check* notes, each student should note what specific

changes they would like to make to improve their guesses and their recording of guesses, calculations and checks. Students should think about why they believe those changes will be helpful, so they can advocate for them in their group.

Key Outcomes

- Develop a better understanding of the problem, the mathematical relationships that must be recorded and carried out, and what counts as an answer.
- Gain perspective on the guess and check process; what parts were difficult for you? Were your calculations and checks complete?
- Improve your recording of your thinking; notice different ways of recording guesses, calculations, and checks.
- Identify multiple ways to guess, and how they effect calculations and results.

III. Improve

Format: students form new groups of three, working with members from other groups. Each group will report back to the larger group.

Students have two goals in this activity. The first is to try to make better guesses by observing the results of previous guesses, which might include looking for ways to jump straight to a solution, using patterns or algebraic representations. The second is to be aware of how different ways of keeping track of guesses, calculations, and checks make it easier to notice and analyze patterns that lead to improved guesses.

Sample Activity

In your small group, select one of the recording methods that were presented in the previous activity or create a modified version that improves on it. Brainstorm ways of improving the guesses, and then try them out.

- Are you getting closer?
- What did you notice about each guess that helped you decide how to improve it? (i.e., Did you notice if you were too high or too low? Did you notice how far off you were?)
- Did you notice patterns in what changed as a result of each guess? Did your new guess get you get closer or further by the same amount as the last guess?
- Are there ways to reorganize your work to see patterns more clearly?
- Did you notice doing certain calculations repeating?

Prepare to report back to the class on how the group's method of recording helped you figure out how to improve the guesses, calculations, and checks, even enabling you to find a calculation or equation that you can use to solve the problem directly; was there something you would improve about how you recorded your calculations for next time?

Key Outcomes

- Recognize how to change guesses in the right direction; in some problems, this can lead to a more efficient solution.
- Gain a sense of control over the guesses, getting closer and closer to a solution that is more strategic.
- Develop effective methods for recording guesses, calculations, and checks that comes from students' own noticings of what is useful.

References

Herrenkohl, L. R., & Guerra, M. R. (1998). Participant structures, scientific discourse, and student engagement in fourth grade. *Cognition and Instruction, 16*(4), 431-473.