Math Fundamentals PoW Packet

Dad's Cookies

February 28, 2011 • http://mathforum.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in September 2003. This is Library Problem #2959. The text of the problem is included below. A print-friendly version is available using the "Print" link from the blue-shaded box on the problem page.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. To access the discussions [log in using your PoW username/password], choose one of these methods:

- use the link to "PoW Member Discussions" from your **My PoW Work as a Teacher** area
- go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526
- from the blue-shaded box, use the **Tips/Ideas from Teachers** link.

Are you making the most of your PoW Membership? If you have an Individual Teacher Membership, consider registering for one of our (free) Orientation Sessions to learn more about the features of your membership. Teachers with Class, School, or District Memberships are welcome to take the free Orientation Session but also are encouraged to register for one of our online courses. View information, dates, and links to register at http://mathforum.org/pd/.

Standards

In ***Dad's Cookies***, students are asked to figure out how many cookies Dad baked for the family before they ate them all. The **key concepts** are fractions and addition.

If your state has adopted the Common Core State Standards, this alignment might be helpful:

Grade 3: Number & Operations–Fractions

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.

Grade 4: Number & Operations–Fractions

Extend understanding of fraction equivalence and ordering.

Grade 5: Number & Operations–Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Additional alignment information can be found through the Write Math with the Math Forum service, where teachers can browse by NCTM and individual state standards, as well as popular textbook chapters, to find related problems.

The Problem

Dad's Cookies

Dad bakes some cookies. He eats one hot out of the oven and leaves the rest on the counter to cool. He goes outside to read.

Dave comes into the kitchen and finds the cookies. Since he is hungry, he eats half a dozen of them.

Then Kate wanders by, feeling rather hungry as well. She eats half as many as Dave did.

Jim and Eileen walk through next, and each of them eats one third of the remaining cookies.

Hollis comes into the kitchen and eats half of the cookies that are left on the counter.

Last of all, Mom eats just one cookie.

Dad comes back inside, ready to pig out. "Hey!" he exclaims. "There is only one cookie left!"

How many cookies did Dad bake in all?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Dad made 22 cookies.

If your answer **doesn't** match ours,

- did you try working backwards?
- did you test your answer in the problem to see if it works?
- did you check your arithmetic?

If you used guess and check, did you tell . . .

- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- are you confident that you could solve another problem like this successfully?
- did you include each step you took to solve the problem?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- are there any hints that you would give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Work Backwards

Since I know how many cookies there were at the end and I know how many people took along the way, I'm going to work backwards.

At the end of the story, Dad found 1 cookie left on the counter. Mom ate one, so before she came along there were (1 + 1) or 2 cookies. This means there were 2 cookies left after Hollis took half.

If Hollis took half, half was also left – so there were twice as many before Hollis came along. That's (2 • 2) or 4 cookies. This means there were 4 cookies after Jim and Eileen took theirs.

They each took 1/3, so there was also 1/3 remaining (1/3 + 1/3 + 1/3 = 1). If 4 is 1/3, there were (3 • 4) or 12 cookies before Jim and Eileen came along. This means there were 12 cookies after Kate left.

Since she ate 3 (half of half a dozen = 1/2 • 1/2 • 12 = 1/2 • 6 = 3), there were (12 + 3) or 15 cookies before she came along. This means there were 15 cookies after Dave left.

He ate 6 (half a dozen), so there were (15 + 6) or 21 cookies before Dave came along.

That brings us back to Dad, who ate one hot out of the oven. Add this one to the rest, and Dad started with (21 + 1) or 22 cookies.

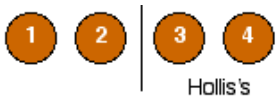
Method 2: Make a Picture while Working Backwards

I read the problem once through and then as I thought about what I noticed I drew a picture to help me think more about it:

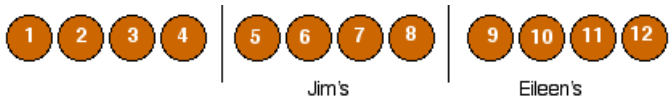
At the end of the story there's one cookie left for Dad:

Mom ate one cookie and so I can add that as the #2 cookie in my picture:

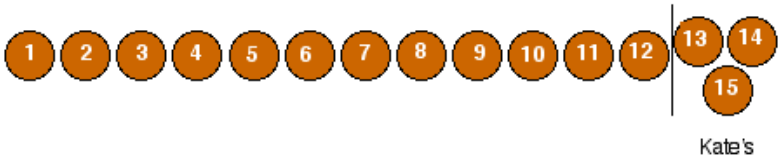
I think of Hollis's half like this:



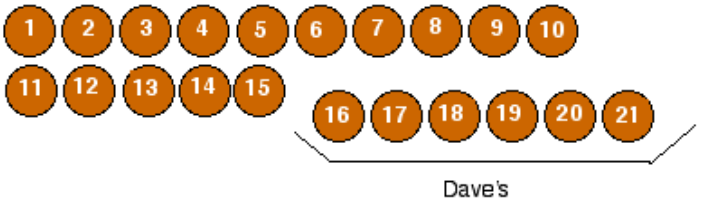
I think of the thirds like this:



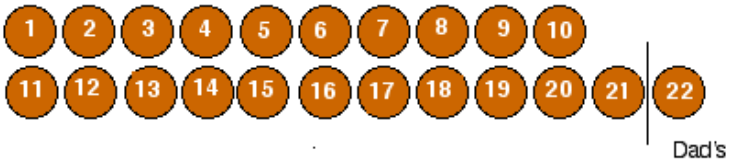
Kate ate half of what Dave did. He ate half a dozen, and since a dozen is 12, that means he ate 6. So Kate must have eaten 3.



Then I add in Dave's half a dozen (or six) cookies:



And then there's the first cookie that Dad ate when he took them out of the oven:



Dad baked 22 cookies!

Method 3: Guess and Check

When we make cookies I think the recipe is in dozens, so I decided to try starting with 12 cookies.

Before I started, I made a note of what happened:

Dad - ate one cookie
Dave - ate 6 cookies (half a dozen)
Kate - ate 3 cookies (half of Dave's amount)
Jim & Eileen - 1/3, each, of what's left - 2/3
Hollis - 1/2 of what's left (after J & E)
Mom - ate one cookie
Dad - one left

Just looking at this list, I realized that 12 wouldn't be enough because, leaving out the fractions we don't yet know, we have:

Dad + Dave + Kate + Mom + Dad
1 + 6 + 3 + 1 + 1(left) = 12 cookies

So I decided to start with 2 dozen cookies instead. I used the same list as above, but I added information about what was left after each person took their cookies:

24 cookies baked
Dad - ate one cookie: 23 left
Dave - ate 6 cookies (half a dozen): 17 left
Kate - ate 3 cookies (half of Dave's): 14 left
Jim & Eileen - 1/3, each, of what's left - 2/3: 4 2/3 left
Hollis - 1/2 of what's left (after J & E): 2 1/3 left
Mom - ate one cookie: 1 1/3 left Dad - one left!: 1/3 remaining

I suppose it's possible that Jim & Eileen ate thirds of cookies, but it doesn't seem likely. I suppose it's also possible that Dad said there was one left but only mentioned the whole cookie... so there might have been 1/3 cookie there as well. However, I think they all probably took whole cookies, so I'm going to try another guess.

I ran into trouble with the 14 cookies which Jim & Eileen split into thirds (1/3 for each of them plus 1/3 remaining). 14 wasn't divisible by 3 and I had a part of a cookie left at the end, so I'm going to pick a smaller number here that is divisible by 3 – that would be 12. This means I'd start with 22 cookies.

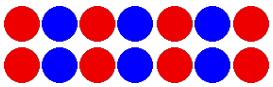
Let's see if that works:

22 cookies baked Dad - ate one cookie: 21 left
Dave - ate 6 cookies (half a dozen): 15 left
Kate - ate 3 cookies (half of Dave's): 12 left
Jim & Eileen - 1/3, each, of what's left - 2/3: 4 left
Hollis - 1/2 of what's left (after J & E): 2 left
Mom - ate one cookie: 1 left
Dad - one left!

So Dad started with 22 cookies.

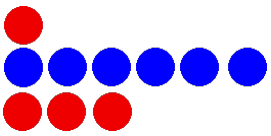
Method 4: Using Manipulatives to Guess and Check

We have red and blue counters that we could use to think about the cookies. We talked about how many counters we would try first and so we thought about how many people were in the story. There are 7 people. We thought that maybe on average each person might get 2 cookies. So we put 14 counters out on our table:



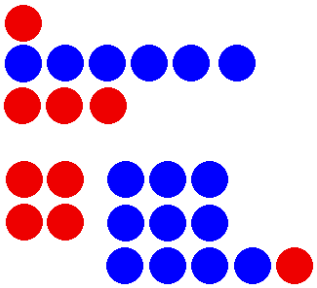
As we read each clue, we moved the counters to see if the numbers worked. The first part wasn't too hard because we knew:

Dad - 1 cookie
Dave - half a dozen or 6 cookies
Kate - half of what Dave ate - 3 cookies



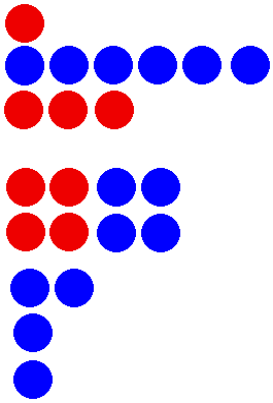
We've already used up all but 4 of our counters and we still have to have cookies for Jim, Eileen, Hollis and Mom. We need more counters! We stop to think if we can use what we know about how many cookies each of those people eat. We know Mom eats 1 cookie and we know that there was 1 left at the end. It's the third of what's left idea and then the half of what's left idea that seems tricky. We decide to not worry about it and just put in 10 more counters to see what happens.

We continue where we left off but now we have 24 counters. So, once Kate ate her cookies we have 24 - 10 cookies left which is 14. Uh, oh. that number doesn't divide into thirds. If we had 1 more it divides evenly because that would be 15 left. We add 1 more counter so now we have 25 cookies at the beginning and we keep thinking. Now our counters are arranged like this:



If Jim eats 5 cookies (1/3 of 15 that are left) and Eileen eats 5 cookies, then there are 5 left and we have a problem again because Hollis ate half of that. We can't just add a cookie to make it even because that messes up the thirds idea.

We decide to think again about a number close to 14 that can be divided evenly into 3s. We decide to use 12. So, now we have 22 counters and we're back to the step of thinking about what's left after Kate eats her cookies. 22 - 10 is 12. A third of 12 is 4 and so Eileen ate 4 cookies and Jim ate 4 cookies. The 1/3 that's left is 4 cookies. If Hollis ate half of that then that leaves 2 cookies – 1 for Mom to eat and one for Dad to find on the counter. We did it! Dad made 22 cookies and here's how our counters looked once we figured it out:



Method 5: Make a Table

After reading the problem, I made a list of what happened:

Dad - ate one cookie
Dave - ate 6 cookies (half a dozen)
Kate - ate 3 cookies (half of Dave's)
Jim and Eileen - 1/3, each, of what's left - 2/3
Hollis - 1/2 of what's left (after Jim and Eileen)
Mom - ate one cookie
Dad - one left

I wondered if I could arrange this information in a table.

Person	Cookies eaten	How many are there now?
Dad	1	1
Dave	6	7
Kate	3	10
Jim	1/3 of what's left	10 + Jim's
Eileen	1/3 of what's left	10 + Jim's + Eileen's
Hollis	1/2 of what's left	10 + Jim's + Eileen's + Hollis's
Mom	1	11 + Jim's + Eileen's + Hollis's
left on counter	1	12 + Jim's + Eileen's + Hollis's

I can see from my table that there were 2 cookies left when Hollis ate his and so he must have eaten 2 cookies. I fill that in my table. Once I have that number I can see that once Jim and Eileen took their cookies there were 4 left (2 for Hollis, 1 for Mom and 1 on the counter) and so if 4 is 1/3 of what's remaining then Jim ate 4 cookies and Eileen ate 4 cookies. Now my table looks like this:

Person	Cookies eaten	How many are there now?
Dad	1	1
Dave	6	7
Kate	3	10
Jim	4	14
Eileen	4	18
Hollis	2	20
Mom	1	21
left on counter	1	22

Dad baked 22 cookies.

Method 6: Algebraic Reasoning

After reading the problem I knew that 1 cookie was eaten at the beginning by Dad.

I added the 6 (1/2 dozen) that Dave ate to the 1 that Dad ate and got 7.

Next I added 3 more that Kate ate because the story said that Kate ate half of what Dave had eaten.

At this point there had been 10 cookies eaten so far.

If Jim and Eileen each had 1/3 of the remaining cookies, I know that there are "1/3 of the remaining cookies" left.

Hollis ate 1/2 of the "1/3 of the remaining cookies". I know that 1/2 of 1/3 is 1/6.

That means that 1/6 of the cookies are remaining after Hollis eats his cookies.

If Mom ate one cookie and Dad said that there was only one cookie left, that means that the number of cookies remaining after Hollis eats his cookies is 2 and it also means that 1/6 of the remaining coolies is 2 cookies. Since 2 is 1/6 of 12, the remaining number of cookies when it's Hollis turn to take cookies is 12.

When I add 10 (the number of cookies eaten so far before I started thinking of the fractional parts of the "remaining cookies") to 12, I get 22.

Dad baked 22 cookies.

Now, I'll check my thinking. If Dad baked 22 cookies and he ate 1, that's 22 - 1 = 21. Dave ate 6 and 21 - 6 = 15. Kate ate 3 and 15 - 3 = 12. Jim and Eileen each ate 1/3 of 12 (or 4 cookies) and 12 - 8 = 4. Hollis ate 1/2 of 4 (or 2 cookies) and 4 - 2 = 2. That leaves 1 for Mom and 1 on the table. Everything checks!

Teaching Suggestions

When we first offered this problem we found that submitters had a good overall understanding of what was going on in the problem. Some submitters used guess and check to work through the puzzle, trying different numbers of cookies Dad might have baked. Manipulatives or drawings can help students think through their guesses and checks as we've modeled above.

Other submitters realized they could work backwards to find the number of cookies Dad baked. While most submitters did find the correct number of cookies baked, they usually didn't show enough work. Often we would get a list of how many cookies each person ate including the correct final number, but no indication of how those numbers were found.

The part of the problem that caused the most confusion for submitters was calculating the number of cookies that Jim and Eileen ate. This was because

- this part of the problem involved think about fractions,
- some submitters were confused as to whether Jim and Eileen ate their cookies at the same time or one after the other,

- it was sometimes confusing which amount of cookies was supposed to multiplied by 1/3 or 2/3 (sometimes the submitter would add up the numbers of cookies given before that clue and then multiply that sum by 1/3), and
- this step was in the middle of the problem and so sometimes the submitter would lose track of which cookies had already been added together.

In the same respect, submitters sometimes had problems with the number of cookies eaten by Hollis. Again, the fractions seemed to be the stumbling block here.

One other difficulty we saw only a few times was that some submitters found their answers by adding up fractions of cookies. It's true that in real life people eat parts of cookies sometimes, especially if they are sharing. However, that's not usually the case (would you really walk by and take two and two-thirds cookies?). In this problem, the cookies were to be kept whole. This seemed obvious to us when writing the problem, but obvious to one person may not be obvious to another.

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Using the Noticing/Wondering activity might help students notice what is happening in this problem. Encourage them to actually make a list of those noticings. Writing them down is an important part of the process. Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are several ways to approach this problem. And keep in mind that we may not have thought of them all!

I also encourage you to explore activities in the Understand the Problem and the Work Backwards strategies in our Activity Series. Something there may provide the impetus that your students need.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the Current Problems as we add them throughout the 2010-11 season, including a calendar, consider bookmarking this page (a link to the page is always available in the left menu when you're logged in):

http://mathforum.org/pow/support/

Sample Student Solutions

focus on **Interpretation**

In the solutions below, I've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands that • the problem asks to find the total number of cookies there were before anyone ate any. • a dozen equals 12. • Dad ate 1, Dave ate 1/2 dozen, Kate ate 1/4 dozen, Mom ate 1, and there was 1 left. • Jim and Eileen each ate a 1/3 of what was left after Kate ate her cookies. • Hollis ate 1/2 of what was left after Jim and Eileen ate their cookies.	There is no Extra, and no way to be an Expert in this category for this problem

Brett
age 10

Interpretation
Novice

My final answer is 23 cookies.

I added up the cookie that dad ate=1

I notice that Brett says that Dad ate one cookie. That gives us a starting point.

I would point out to Brett that there are other people in the story who ate cookies. I'd ask him to list the names. Also, I'd ask him if he knows how many cookies they each ate. If he does, he could include that number as he did with Dad and if he isn't sure, he could put a question mark next to the name.

Becky
age 12

Interpretation
Novice

Dad baked 15 cookies.

I wrote by each step how much each person ate. Then I added them all together.

I notice that Becky refers to making a list and how many cookies each person ate. Without seeing her list it's difficult to know whether she realized what was happening in the problem.

I would ask her to add that step-by-step list in her explanation so that we can look at it and think about it together.

Miranda
age 11

Interpretation
Apprentice

26 cookies

first i added dads cookie to the six dave ate. 7

then i added six that kate ate. 13

then jim and eileens third . 4

and mom and dad togther. 2

I notice that Miranda correctly understands that Dad ate 1 cookie and Dave ate 6. When she lists that Kate ate 6 I'm wondering if she skipped over the "half" idea.

I also notice that she ignored Hollis! He doesn't get listed.

I would ask her how she decided Kate ate 6 cookies and I would also ask her what happened to Hollis.

Letizia
age 12

Interpretation
Apprentice

I didnt get the entire problem but I got up to step 4. My answer until there was 10 cookies.

First I read that the dad ate 1 of his cookies. I put 1 on a paper. Then Dave ate half a dozen cookies. A dozen is 12 and half a dozen is 6. So I added 1 and 6 and got 7. Then when Kate comes she eats half the amount of cookies Dave ate and half of 6 is 3. Then I added the 7 cookies from before plus these 3 and got my 10.

I notice that Letizia realizes that she only understands up to the part when Jim and Eileen are mentioned.

It's great that she submitted what she had to that point and also let us know that was her stopping point. Having students use the idea of a "draft" is great!

I might suggest to Letizia that she start from the end of the problem and see if she can work her way up to where she left off.

She might decide to use a full Work Backwards strategy or she might decide to use it for just the second half and then combine her two numbers. I would encourage her to submit her second draft so that we could work from there.

Steven
age 13

Interpretation
Apprentice

The number of cookies dad baked is 30 cookies.

First, i took the one cookie that dad ate and added it to Dave's share which was six which now makes 7. It says Kate's share was half of Dave's which is 3 and 3 added to 7 is 10. Mom eats one cookie and when dad comes in there is 1 cookie so 2 plus 10 is 12. Now Jim and Eileen ate 1/3 of the cookies left which is 4, and 4+4 is 8, and 8 added to 12 equals 20. It also says that Hollis eats half the cookies left on the counter and half of 20 is 10. 10+20=30 and there is ur solution

I notice Steven gets things a little out of order when he goes from the number of cookies Kate ate to the number that Mom ate and that Dad found. I would point out to Steven that order matters in this problem.

I wonder if using manipulatives or drawings or even a table might help Steven think about how the order matters in this problem.

Patrick
age 12

Interpretation
Practitioner

Dad baked 22 cookies.

I started out with a random number of 22.
Dad eats one. 1
Dave eats half a dozen. 6
Kate eats half as many as Dave. 3
Jim and Eileen had 1/3 of the cookies. jim=4 Eileen=4
There are 4 cookies.
Hollis eats half of the cookies remaining. 2
Mom eats one cookie. 1
Dad eats 1.
22-1=21 -6=15 -3=12 -4=8 -4=4 -2=2 -1=1 -1=0
1+6+3+4+4+2+1+1= 22

I notice that Patrick's strategy of picking a "random number" that just happens to be 22 is suspect. I also notice that he hasn't completely explained how he found the numbers of cookies each person ate. And, last but not least, his use of equations in his next to last string of numbers is something to improve.

That all being said, however, Patrick has shown that he interpreted what is going on in the problem correctly.

Suz
age 11

Interpretation
Practitioner

I thimk that the answer is 22.

I started by going backward and the last thing it says was that dad came in and there was only one cookie so I wrote down one. Then it says that Mom ate one cookie so I wrote down that so far there was 2 cookies. Then it says that Hollis came in and ate half the cookie on the counter. If you add 2+2 than that equals 4 so so far there are 4 cookies.Next Jim and Eileen come in and eat one-third of the cookies each. I did 4 times 3 which equals 12 and so now there are 12 cookies.It says that Kate walks in and eats half as many Dave did so before you figure out how much Kate ate you have to now how much Dave ate. It says that Dave ate half a dozen which is 6. Half of 6 is 3 so Kate ate 3 cookies and Dave ate 6. It also says that Dad ate one hot out of the oven.Now add all those numbers together and you get 22.

Suz picked a better strategy than Patrick. However, she also could improve her Clarity score. In her case I might just mention the idea of including some paragraph breaks.

Having a few breaks in the text would provide the reader with more opportunity to think about each part before going on to the next.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Suzanne <suzanne@mathforum.org>