



Pre-Algebra PoW Packet

At Least a Mile

September 7, 2009 • <http://mathforum.org/pows/>

Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, a problem-specific scoring rubric, and some samples of the student work we received when we posted a very similar problem in September, 2000, named **Baseball Trivia** [Problem #1310].

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From your My PoW area use the link to “PoW Members” or use this URL to go to *prealgpow-teachers* directly: <http://mathforum.org/kb/forum.jspa?forumID=527> [Log in using your PoW username/password.]

The Problem

In **At Least a Mile**, students are asked to explain how many runs the Phillies have to score before the scoring base runners have covered at least a mile.

The text of the problem is included below. A print-friendly version is available from the “Print this Problem” link on the current PreAlgPoW problem page.

At Least a Mile

As you may know, the Math Forum office is located in Philadelphia, Pennsylvania and many of us enjoy attending the 2008 World Champion Phillies games. The game of baseball provides many opportunities to use our math skills. This week’s problem was presented as a trivia question at a recent game.

Question: How many runs do the Phillies have to score before the scoring base runners have covered at least a mile?



Extra: If the players all wore pedometers and measured their actual distance traveled, do you think that they could get to 1 mile with fewer players? Or might they need more players than you calculated? What factors might change their actual distance traveled?

Answer Check

The Phillies must score 15 runs.

If your answer **doesn’t** match ours,

- did you use the photo of the baseball diamond to think about how runs are scored?
- did you find out the distance between the bases on a professional baseball field?

- do you know how many feet are in a mile?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you try the Extra?
- have you clearly shown and explained the work you did?
- did you explain your work as well as you could?
- did you make any mistakes along the way? If so, how did you find and fix them?
- what hints would you give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

The key concepts are measurement, units, and operations with numbers.

Method 1: Calculations of distance and equivalent units

I need to find out

- the distance between each of the bases
- the number of feet in a mile

There are 90 feet between each base in baseball and starting at home, traveling to 1st, 2nd, 3rd and then back home again would be a distance of 90×4 or 360 feet.

I divided 360 into 5280 because there are 5280 feet in a mile. I got 14.6, but since you can't score a half a run in baseball, I rounded my answer up to 15.

Method 2: Make a Table

I know that there are 90 feet between each base. I know that there are 5,280 feet in a mile. I decided to make a table to think about how many runs it would take to have at least a mile of distance covered.

Number of runs	Number of bases	Number of feet
1	4	360
2	8	720
3	12	1080
4	16	1440
5	20	1800
6	24	2160
7	28	2520
8	32	2880
9	36	3240
10	40	3600
11	44	3960
12	48	4320
13	52	4680
14	56	5040
15	60	5400

The mile mark is between the 14th and 15th scored run and so "at least a mile" was covered after the fifteen runner scored.

Method 3: Guess and Check

A mile seems like it's pretty far and so I'm going to guess that 10 runners scored to cover that distance. I know that it's 90 feet between the bases and there are four segments to run from home plate and around the bases and then back home to score. Each runner has to run 360 feet each time they score a run and if there are 10 runs scored, that makes 3600 feet.

I wonder if 3600 feet is a mile? I looked up how many feet are in a mile and found out that is too low a guess! There are actually 5280 feet in a mile.

If I were to double my guess to 20, I would have to double the feet run and that would be 7200 feet. That's way over the needed 5280 feet. I'm going to try 15 runs next because 7200 is so much bigger than 5280. 360 times 15 is 5400 and that seems really close. I wonder what 14 is just right. When I multiply 360 times 14 I get 5040 and that's not enough. I've found the answer. It's 15 runs!

Teaching Suggestions

We chose this problem because it is nearing the end of baseball season and before we know if the Phillies might be defending their World Series Champion title from 2008 or, at least, we can optimistically hope that's the case. Some students may not be as familiar with the rules of baseball as others and it's also possible that students may not have the number of feet in a mile memorized. We're pretty sure, however, that once these two facts are discovered either by using text-based or computer-based references, the mathematics of the problem will be accessible.

One of our favorite classroom activities is to just "notice" and "wonder" about a problem with the final question removed. Using this activity with this particular problem may generate a variety of interesting thinking about how mathematics is present in baseball. Once the question is removed from this problem, the photo is pretty much all that is left. In case it's useful for classroom "noticing" we've made a page that includes more photos taken at a recent Philly game. You'll find a link to those photos from the "Other Links" section of the Online Resources Page.

It is unlikely that students will end up "wondering" the particular question we've posed in this problem but their conversations may lead to other interesting mathematics. In case you would like a copy of the problem to either project for the class or to print for individuals or groups, we'll be providing one for most of the problems this year, including this one. Just look for "Scenario Only" listed under the Teacher Support Materials for this problem.

The Problem Solving and Communication Activity Series document for this problem contains ideas and activities to help students experience generating their own understanding of the problem.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like one page to find all of the resources for the Current Problems as we add them throughout the 2009-2010 season, consider bookmarking this page:

<http://mathforum.org/pow/support/>

Sample Student Solutions

Focus on Interpretation

In the solutions below, we've focused on students' "interpretation" of the problem, meaning that they interpret the problem correctly and attempt to solve all of the parts. Since it is early in this year's Pre-Algebra PoW season, we've included more Novice and Apprentice solutions than we will later in the year. Our hope is that they help provide insight into conversations you might have with your students as they start their 2009-2010 year of problem solving and communication.

With our new PoW environment and our continued offering of both these Packets and the Activity Series documents, we invite you to consider registering to participate in one of our online professional development courses. View information about our courses here: <http://mathforum.org/pd/>

Also join us in conversations with the PoW community using our discussion: *prealgpow-teachers*: <http://mathforum.org/kb/forum.jspa?forumID=527> [Log in using your PoW username/password.]

Brian and Kevin age 13 Interpretation Novice

The answer is 14.

I used yahoo

Although it may seem that there isn't much here to go on, it is helpful that Brian and Kevin mentioned "yahoo" and I would use that as a starting point.

I might wonder aloud to them what they wanted to find out from Yahoo. What information were they looking for?

Sarah
age 13
Interpretation
Novice

8
i just guessed

When someone guesses you wonder if they have realized that guessing can actually be a great starting point.

I wonder if Sarah thought about how far the runners ran if they scored 8 runs. I wonder how to figure that out if that is my first guess?

Mike
age 13
Interpretation
Novice

I think it would be 22 runs.
I used an educated guess.

Mike has included "educated" as an adjective to his "guess" but similar to Sarah's guess, I wonder how far the runners ran. Is the guess too high or too low? What's next?

Guessing is a great strategy but once a guess has been made it helps to check it and, if necessary, revise.

Denman Team
average age 12

Interpretation
Apprentice

In order to run 1 mile you have to run 59 times around the bases. do 90 times 59 and you get 5,310 so you have to run at least 59 times around the bases in order to run one mile

I notice that the Denman Team multiplied by 90. I wonder what that number represents in their mind.

Interestingly 59 divided by 4 is 14.75 and so my guess is that they are more on the right track than I might think at first look.

Samantha
age 13

Interpretation
Apprentice

The Phillies would have to run 51 runs to make it a mile. First of all, I went to askjeeves.com. It asked me to type a question that I needed an answer to. I typed in what is the distance between the bases in a baseball diamond. It didn't really give me an answer. So I just fooled around and clicked on a bunch of links that I thought would have something to do with baseball. Finally I was about to sign off and try again tomorrow, so I clicked onto Answer Point thinking it would take me back to where I started. But it didn't. Instead it took me to a link with a bunch of questions that kids asked. I found a question someone typed in, it said, what is the distance in a baseball diamond. So I clicked on the answer. It said 90 feet are in between the bases. So then I took how many feet are in a mile which are 5,280. I divided that into 90 for 90 feet in between each base. That equaled to 58 60/90 and I just reduced that then since you can't make a fraction or a decimal in baseball, then I rounded it to 51. And it that's how I got my answer.

I notice that Samantha was resourceful in using the Internet to help her with the problem.

I wonder what her answer represents when she divides 5,280 by 90. I wonder if referring to the photograph to think about what happens as a runner scores a run, might help her think more about what to do next with the result of her calculation.

Christy
age 15
Interpretation
Apprentice

60 runners must score.

I multiplied 90 ft., the amount of feet between bases, by numbers until I got to 5,280 (amount of feet in a mile). I got 60 for my answer, so it is 60 runners.

Christy noticed the distance between bases and she's noted the number of feet in a mile. I wonder what she's thinking a baseball runner does to score a run.

Like Samantha it might help Christy to look at the photo of the baseball diamond to visualize what each runner does to score a run.

Carrie
age 15
Interpretation
Apprentice

14.6666667 runs must score.

I multiplied 90 x 4 and got 360 and I divided that into 5280 and got 14.6666667.

I notice that Carrie uses both the "90" and the "5280" indicating to me that she has a good start with her interpretation of the problem.

I wonder how a baseball runner scores a fraction of a run.

Jake
age 11
Interpretation
Practitioner

17 base runners need to score to have run a mile.

I found out that there is 90 feet between each base. If you score you have to run around all 4 bases and that is 320 feet. Also there is 5,280 feet in a mile. So I did 320 multiplied by 17 which equals 5,440. 16 runners to score is little. That equals 5,120.

Jake has interpreted the problem correctly but has one inaccurate multiplication fact.

I might ask him what numbers he used to determine that a runner would travel 320 feet.

Abby
age 11
Interpretation
Practitioner

The Phillies will have to score 17 runs for the scoring base runners to have covered at least a mile making scoring runs, assuming that no one overran a base or had to tag up.

According to the Major League Baseball Rules, the distance between each two bases in sequential order (i.e., first to second) must be 90 feet, so a runner who runs all four bases and does not overrun any bases or have to tag up will run 360 feet (four bases X 90 feet) to score a run. There are 5,820 feet in a mile. The Phillies will have to score 17 runs for the scoring base runners to have covered at least a mile (5,820 feet in a mile divided by 360 feet = 16.1666667, which has to be rounded up to the next whole number, 17, in order for the scoring runners to complete the whole mile).

Abby used the information needed to work the problem and she has explained her thinking very nicely.

I particularly like that she included why she "rounded up" which many students didn't stop to explain.

I wonder if she checked her division if she would get the same answer.

Jon
age 11

Interpretation
Practitioner

The Phillies need to make at least 15 runs so that the scoring base runners cover a mile.

The distance between any two bases is 90 feet.

A runner needs to pass between all 4 bases before scoring a run.

So that means in each run $90 \times 4 = 360$ feet is covered.

There are 5,280 feet in 1 mile.

Therefore, the number of runs needed for the scoring base runners to cover one mile = $5280/360$

$$= 528/36 = 14.667$$

The Phillies need to make at least 15 runs so that the scoring base runners cover a mile.

Jon has done a nice job explaining the numbers he used and how he calculated the number of runs needed to cover a mile.

NOTE: We've not included any Expert student solutions for this problem because the solutions are from **Baseball Trivia** and the Extra for that original problem is different from the Extra we've included in this newer, updated version **At Least a Mile**. We're looking forward to reading the solutions students write as they submit to our new Extra.

Scoring Rubric

On the last page is the **problem-specific rubric**, to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Pre-Algebra PoWs. Please let me know if you have ideas for making them more useful.

~ *Suzanne* <suzanne@mathforum.org>

Pre-Algebra Scoring Rubric for *At Least a Mile*

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	does none or one of the things listed under Practitioner	does two of the things listed under Practitioner	understands a runner runs from home plate to 1st base to 2nd base to 3rd base to home plate to score a run understands that the distance between each base is 90 feet knows that 5280 feet = 1 mile	is at least a Practitioner in Strategy and has successfully answered the Extra
Strategy	does not have any ideas about how to solve the problem	has some ideas about how to solve the problem, but isn't quite there	has a strategy that relies on skill, not luck might use calculations might use a table might use guess and check	uses two separate strategies
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	work is accurate and contains no arithmetic mistakes	[not normally available for this category]
Communication				
Completeness	has written nothing that tells you how they found their answer	shows work without an explanation or explains everything without showing the numbers doesn't include enough information for another student to follow	attempts to explain all of the steps taken to solve the problem, which might include <ul style="list-style-type: none"> • details of how they calculated and the measurement facts they used • a table showing the numbers they used • describing what they guess, how it checked and what they guessed next 	adds in useful extensions and further explanation of some ideas involved
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely long and written in one paragraph lots of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing
Reflection	<i>The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:</i> does nothing reflective	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer does one reflective thing	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used does two reflective things	comments on and explains the ease or difficulty of the problem revises their answer and improves anything does three or more reflective things or an great job with two

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